1. Use the Gauss-Jordan method to find the inverses of the following matrices. Check your result by multiplication.

$$\text{(a)}\quad \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}, \qquad \text{(b)}\quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

2. Let B be a matrix of size 3×3. Denote by B' the result of applying the elementary row operation $R_3 + 2R_1$ to B. Construct a matrix C satisfying $B' = CB$. Is C invertible? Why?

3. Check the validity of the following identities using block matrix multiplication:

$$\text{(a)}\quad \begin{bmatrix} A & B \\ 0 & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} & -A^{-1}BD^{-1} \\ 0 & D^{-1} \end{bmatrix},$$

$$\text{(b)}\quad \begin{bmatrix} I & B \\ C & I \end{bmatrix}^{-1} = \begin{bmatrix} (I-BC)^{-1} & -(I-BC)^{-1}B \\ -C(I-BC)^{-1} & I + C(I-BC)^{-1}B \end{bmatrix}.$$

(You can assume that all needed inverses exist.)

4. Use the Gauss-Jordan method to find the inverses of the following matrices.

$$\text{(a)}\quad \begin{bmatrix} 2 & 3 & 0 \\ 1 & -2 & -1 \\ 2 & 0 & -1 \end{bmatrix}, \qquad \text{(b)}\quad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

5. (a) Prove: if A is invertible and $AB = 0$, then $B = 0$.
   (b) Construct 3×3 matrices $A \neq 0$ and $B \neq 0$ satisfying $AB = 0$.

6. Square matrix A is *idempotent* if $A^2 = A$.
   (a) Find three idempotent 2×2-matrices.
   (b) Prove that the only invertible idempotent $n \times n$ matrix is the identity matrix.