Linear algebra ja matrices I, fall 2009 Exercise 2 (21.-25.9.2009)
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1. Determine the solution sets of the following equations:

$$\text{(a) } 2x_1 + 3x_2 = 5, \qquad \text{(b) } 4x_1 + 3x_2 + 2x_3 = 1.$$

2. Let $k \neq 0$. Consider the following linear system of equations:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2. \end{aligned}$$

   (a) Show that the row operation kR_1 leads to an equivalent system.

   (b) Show that the row operation $R_2 + kR_1$ leads to an equivalent system.

   (Two systems are equivalent if they have the same solution set.)

3. Which of the following matrices are in reduced row echelon form? Why?

$$\begin{bmatrix} 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Transfomr the following matrix to reduced row echelon form using elementary row operations:

$$\begin{bmatrix} -2 & -4 & 7 \\ -3 & -6 & 10 \\ 1 & 2 & -3 \end{bmatrix}$$

5. Solve the following system of equations using Gauss-Jordan elimination:

$$\begin{array}{rcrcrcrcl} 2w & + & 3x & - & y & + & 4z & = & 0 \\ 3w & - & x & & & + & z & = & 1 \\ 3w & - & 4x & + & y & - & z & = & 2 \end{array}$$

6. Show that matrices A ja B are row equivalent:

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 1 & -1 \\ 3 & 5 & 1 \\ 2 & 2 & 0 \end{bmatrix}.$$

   Present the elementary row operations needed in the proof.