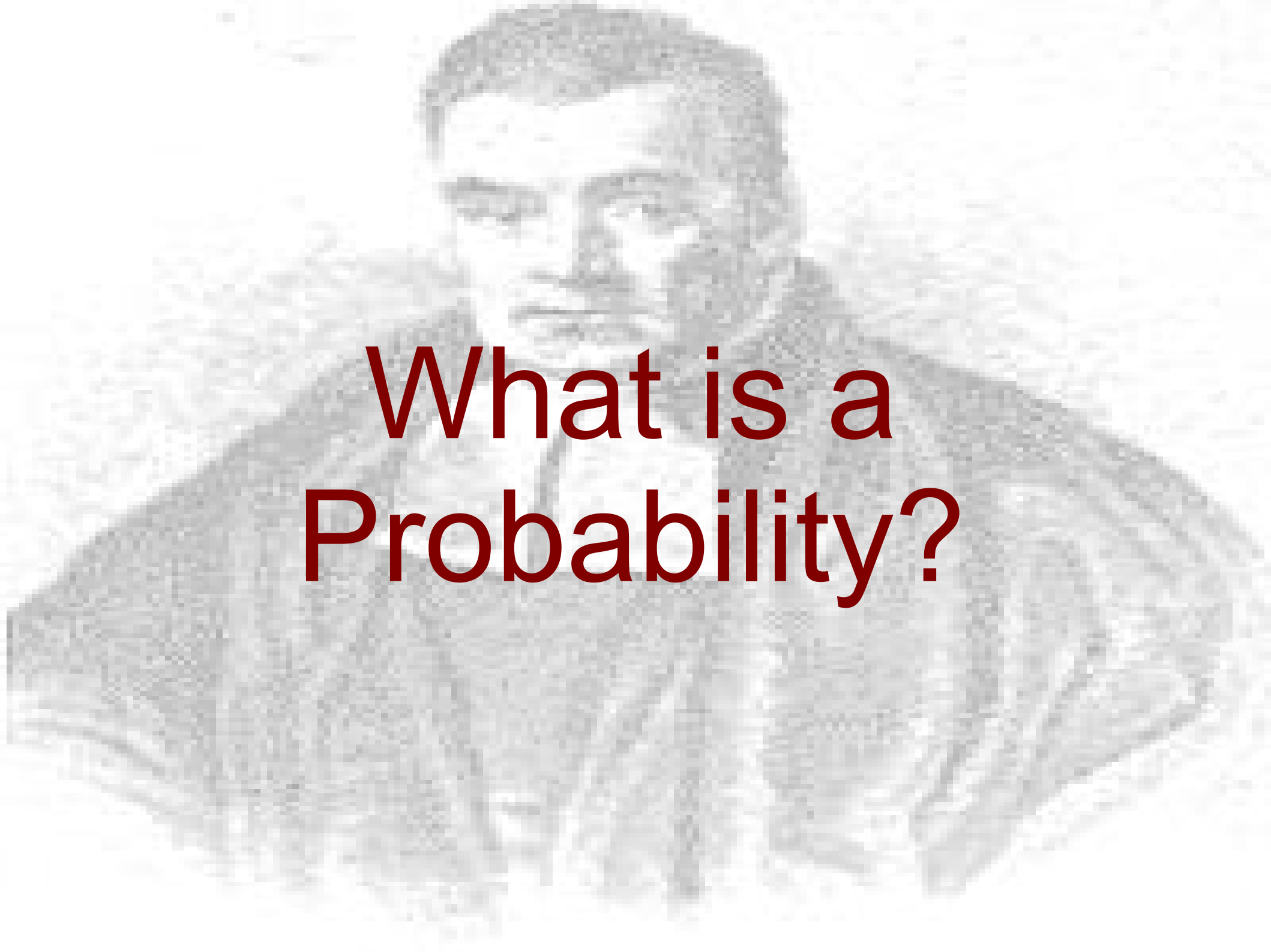

What is a Probability?

We are Uncertain

- Probability is to do with uncertainty
- If I throw a die, I am not sure what number will come up.
- I can talk about the chances of the different outcomes

The Traditional Answer

- If I role a die an infinite number of times, then 1 in every 6 rolls will be a 3
- More generally, the proportion of 3s will get close to 1 in 6.
  - frequency interpretation of probability
  - used in “traditional statistics”
- i.e. maximum likelihood
- Strictly, it does not tell us about individual events

Singular Events

- Probabilities are not only used in discussing dice
- e.g. we can talk about the probability of there being a typo on the next slide
- This does not have an easy frequency interpretation
  - there is only one slide coming up next

Gambling

- An alternative interpretation can come from gambling
- Imagine a betting game between A and B
  - toss a fair coin
    - if it comes up heads, A pays B €10
    - if it comes up tails, B pays A €1
- Would you prefer to be A or B?

Utility

- Economists call the amount you expect to win your utility
- It does not have to be money, but has to be quantified, e.g.
  - fitness
  - happiness
- Economists talk about trying to maximise utility
  - the connection to maximising fitness is clear

Probability and Utility

- We can now define probability in terms of utility
- Play a game, each player puts in a stake and the winner takes all
- e.g. bet on Heads or Tails from a coin toss
- Pr(heads) = 0.25, stake for heads is €1
- What stake for Tails would I be prepared to pay?

Breaking Even

- Mathematically, we can calculate out expected utility:
- Let U be the utility for someone betting Tails
  - x be the stake for tails
- $E(U) = -0.25x + 0.75 \times 1$
  - I either loose x or win 1
- To break even, x should be 0.75/0.25 = 3
  - 0.75/0.25 is the odds of Tails

What Probability Is

- Note that if the stake we are prepared to pay is the odds, then the expected utility is 0
  - this is true for both sides
- Hence, we should be happy to take either side of the bet
- This gambling scheme can be used to define our probabilities
  - if the odds are O, the probability is O/(1+O)
- In general, we are “playing the universe”

Coherence

- Having defined a probability, a full theory for combining probabilities can be developed
- We should still be able to take both sides of a bet in any probability statement
  - such a system is called coherent
- Standard probability theory is the only coherent system
- A system that is not coherent is called a Dutch book
  - one side has a positive utility

Repeated betting

- Note that in this system, our expected utility is 0, but in a single bet we will either win or loose
- But if we repeat our bets, then the wins and losses will even out
  - the law of large numbers
- So, the frequentist concept of probability is a property of our system

Observations

- From our new perspective, probabilities are personal
  - Joe Biden and Sarah Palin would give different probabilities for Obama winning the election
- By its nature, it is subjective – different people will have different opinions
  - hence, this is called a subjectivist probability

Observations

- We can use evidence to learn about our probabilities
  - if more polls put Obama ahead, we are more likely to think he will win
- Probability theory can be used to give us a formal way of using this evidence
  - so that we can make rational decision
  - this is where we need Bayes' theorem...