Anomalies in Non-Linear Rayleigh Quotients

Peter Lindqvist Norwegian University of Science and Technology

The non-linear eigenvalue problem

$$\nabla \cdot (|\nabla u|^{p-2} \nabla u) + \lambda |u|^{p-2} u = 0, \quad u \in W_0^{1,p}(\Omega),$$

comes from the problem of minimizing the Rayleigh Quotient

$$\lambda(p) = \inf_{\phi} \frac{\int_{\Omega} |\nabla \phi|^p \, dx}{\int_{\Omega} |\phi|^p \, dx}$$

taken over all test functions $\phi \in C_0^{\infty}(\Omega)$. As a function of the parameter p, it exhibits a puzzling behaviour in *very* irregular, yet bounded, domains Ω in \mathbb{R}^N . I shall discuss an old counter example of mine showing that the situation

$$\lim_{s \to p-} \lambda(s) < \lambda(p), \qquad 1 < p < N,$$

is possible. A generalized Cantor Set, the Wiener Criterion, and the Kellogg Property are involved in the construction.

The phenomenon is that the "obvious" implication

$$u \in W^{1,p}(\Omega)$$
 and $u \in W^{1,s}_0(\Omega)$ for all s

is *false*.