Composition of analytic paraproducts and the radicality property for spaces of symbols of bounded integral operators.

José Ángel Peláez

Departamento de Análisis Matemático, Facultad de Ciencias,

Universidad de Málaga, 29071 Málaga, Spain

Let $\mathcal{H}(\mathbb{D})$ denote the space of analytic functions on the unit disc $\mathbb{D}$ of the complex plane. An N-letter g-word is the composition $L = L_1 \cdots L_N$ of N operators L_j, where each L_j is either of the analytic paraproducts $T_g f(z) = \int_0^z (fg')(\zeta)\, d\zeta$, $S_g f(z) = \int_0^z (f'g)(\zeta)\, d\zeta$ and $M_g f(z) = (fg)(z)$, $f, g \in \mathcal{H}(\mathbb{D})$. The boundedness of a single paraproduct on classical Hardy spaces and standard Bergman spaces A^p_α, $0 < p < \infty$ and $\alpha \geq -1$, is well understood and the bounded 2-letter g-words on these spaces have been recently described in [1]. We shall present new results obtained in [2] for $N \geq 3$, and in particular we shall show that the boundedness of a N-letter g-word on A^p_α only depends on the symbol g, N and the total number of T_g's that it contains.

On the other hand, to obtain the previous results it is shown that the Bloch space $\mathcal{B}$ in the unit disc has the following radicality property: if an analytic function g satisfies that $g^n \in \mathcal{B}$, then $g^m \in \mathcal{B}$, for all $m \leq n$. Since $\mathcal{B}$ coincides with the space $\mathcal{T}(A^p_\alpha)$ of analytic symbols g such that the integral operator $T_g f(z) = \int_0^z f(\zeta) g'(\zeta)\, d\zeta$ is bounded on A^p_α, we pose the following question: *Given $0 < p < \infty$, which are the weights such that $\mathcal{T}(A^p_\omega)$ has the radicality property?* Here A^p_ω is the Bergman space induced by a weight ω. We plan to end this talk sheding some light on this question using the theory of composition of analytic paraproducts.

These results are part of of several joint works together with A. Aleman, Carmé Cascante, Joan Fábrega and Daniel Pascuas.

References

[1] A. Aleman, C. Cascante, J. Fàbrega, D. Pascuas, and J. A. Peláez, *Composition of analytic paraproducts*, J. Math. Pures Appl. 158 (2022), 293–319.

[2] A. Aleman, C. Cascante, J. Fàbrega, D. Pascuas, and J. A. Peláez, *Words of analytic paraproducts on Hardy and weighted Bergman space*, preprint, submitted. http://dx.doi.org/10.48550/arXiv.2311.05972 doi:10.48550/arXiv.2311.05972.