GEODESICS ON HYPERBOLIC SURFACES AND KNOT COMPLEMENTS

Abstract

Due to the Hyperbolization Theorem, we know precisely when does the interior of a compact orientable 3-manifold whose boundary components are tori admit a complete hyperbolic metric of finite volume. Moreover, by the Mostow's Rigidity Theorem this geometric structure is unique. However, finding effective and computable connections between the geometry and topology is a challenging problem. Most of the results on this talk fit into the theme of making the geometrization program more concrete and effective.

To every oriented closed geodesic γ on a hyperbolic surface Σ has associated a canonical lift $\hat{\gamma}$ on the unit tangent bundle $T^1\Sigma$ of Σ, and we can see $\hat{\gamma}$ as a knot in the 3-manifold $T^1\Sigma$. The knot complement given in this way $M_{\hat{\gamma}} = T^1\Sigma \setminus \hat{\gamma}$ has a hyperbolic structure.

The objective of this talk is to estimate the volume of $M_{\hat{\gamma}}$. For every hyperbolic surface Σ we give a sequence $(\gamma_n)_n$ of geodesics on Σ, such that the manifolds $M_{\hat{\gamma}_n}$ are not homeomorphic with each other and the sequence of the corresponding volumes is bounded. We also give a lower bound of the volume of $M_{\hat{\gamma}}$ by an explicit real number which discribes a relation between γ and a pants decomposition of Σ. This give us a method to construct a sequence $(\eta_n)_n$ of geodesics where the volume of $M_{\hat{\eta}_n}$ is bounded from below in terms of the length of η_n.

For the particular case of the modular surface, we obtain estimations for the volume of $M_{\hat{\gamma}}$ in terms of the period of the continuous fraction expansion of γ.