Closed ideals in the quotient algebra of compact-by-approximable operators
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I will describe recent joint work with Henrik Wirzenius (Helsinki) about non-trivial closed ideals of the compact-by-approximable algebra $\mathfrak{A}_X =: \mathcal{K}(X)/\mathcal{A}(X)$ on Banach spaces X. Here $\mathcal{K}(X)$ is the algebra of compact operators on X and $\mathcal{A}(X) =: \overline{\mathcal{F}(X)}$ is the uniform closure of the finite rank operators $\mathcal{F}(X)$, so that $\mathfrak{A}_X \neq \{0\}$ is possible only if X does not have the approximation property.

The results and examples include the following: (i) if X has cotype 2, Y has type 2, $\mathfrak{A}_X \neq \{0\}$ and $\mathfrak{A}_Y \neq \{0\}$, then $\mathfrak{A}_{X \oplus Y}$ has at least 2 (and in some cases even up to 8) closed ideals, (ii) there are closed subspaces $X \subset \ell^p$ for $4 < p < \infty$ and $X \subset c_0$ such that $\mathfrak{A}_X$ contains a non-trivial closed ideal, (iii) there is a Banach space Z such that $\mathfrak{A}_Z$ contains an uncountable lattice of closed ideal having the reverse order structure of the power set $(\mathcal{P}(\mathbb{N}), \subset)$.