A Hitchhiker's Guide to Limit Operators

(and their applications)

Raffael Hagger

Leibniz Universität Hannover

raffael.hagger@math.uni-hannover.de

I will give a friendly introduction to limit operator theory, starting with the Fredholm theory of infinite matrices acting on the sequence spaces $\ell^p(\mathbb{Z})$. Even though the use of limit operators can be dated back to at least 1927 (Favard), major breakthroughs even for $\ell^2(\mathbb{Z})$ have been achieved only recently. In their groundbreaking work Lindner and Seidel [1] finally proved a conjecture that has been open for almost 30 years. I will present (parts of) their results and some related work [2]. Inspired by the results of Lindner and Seidel, many generalizations to other function spaces have been obtained, e.g. to Bergman [3] and Fock spaces [4]. I will talk about how classical results for Toeplitz operators can be obtained directly via limit operator theory, how these results can be generalized and some open problems which may be tackled using this approach.

[1] M. Lindner, M. Seidel: *An affirmative answer to a core issue on limit operators*, J. Funct. Anal. 267 (2014), 901-917.

[2] R. Hagger, M. Lindner, M. Seidel: *Essential Pseudospectra and Essential Norms of Band-Dominated Operators*, J. Math. Anal. Appl. 437, No. 1 (2016), 255-291.

[3] R. Hagger: *Limit Operators, Compactness and Essential Spectra on Bounded Symmetric Domains*, J. Math. Anal. Appl. 470 (2019), 470-499.

[4] R. Fulsche, R. Hagger: *Fredholmness of Toeplitz Operators on the Fock Space*, Complex Anal. Oper. Theory (2018).