Measuring Triebel-Lizorkin fractional smoothness on domains in terms of first-order differences

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Abstract. We give equivalent characterizations for a fractional Triebel-Lizorkin space $F^s_{p,q}(\Omega)$ in terms of first-order differences in a uniform domain Ω. The characterization is valid for any positive, non-integer real smoothness $s \in \mathbb{R}_+ \setminus \mathbb{N}$ and indices $1 \leq p < \infty$, $1 \leq q \leq \infty$ as long as the fractional part $\{s\}$ is greater than $d/p - d/q$.

Namely, $\|f\|_{F^s_{p,q}(\Omega)}$ is comparable to

$$\|f\|_{W^{\lfloor s \rfloor,p}(\Omega)} + \sum_{|\alpha|=\lfloor s \rfloor} \left(\int_\Omega \left(\int_{B(x,c\delta(x))\cap\Omega} \frac{|D^\alpha f(x) - D^\alpha f(y)|^q}{|x-y|^{\{s\}q+d}} \, dy \right)^{\frac{p}{q}} dx \right)^{\frac{1}{p}},$$

where $\delta(x)$ is the distance of x to the boundary of Ω.

Time permitting, we can discuss about the use of these norms to prove T1-type theorems for smooth CZO restricted to domains.