

UNIFORMISATION AND DESCRIPTION OF A ONCE PUNCTURED ANNULUS

The Uniformisation Theorem shows that the universal covering space \tilde{X} of a hyperbolic Riemann surface X is homeomorphic, by a conformal map \mathfrak{m} , to the unit disk \mathbb{D} , or, equivalently, the upper half-plane \mathbb{H} , and then the fundamental group $\Pi_1(X)$ has a representation as a group G of conformal homeomorphisms of $\mathfrak{m}(\tilde{X})$. Hempel and Smith studied the hyperbolic Riemann surface model of the twice punctured disk $\mathbb{D} \setminus \{p_1, p_2\}$ in 1980s. They estimated the hyperbolic density on it near one puncture and considered the coalescing of the two punctures. Later on Beardon gave five different ways to uniformize $\mathbb{D} \setminus \{p_1, p_2\}$ in 2012. In this talk, we extend his work to the once punctured annulus $A := \{z : 1/R < |z| < R\} \setminus \{a\}$, $R > 1$, $1/R < a < R$. We provide several parameter pairs to uniformize and characterize A . These parameter pairs are divided into two classes, which describe the the hyperbolic and complex structures of A , respectively. We give explicit formulas about the relation between the two structures of A . Several degenerating cases are also treated. In these cases, A is degenerating to the thrice punctured Riemann sphere, a twice punctured disk, the once punctured unit disk, or an annulus. This presentation is based on [Zh].

References

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