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Abstract

EXCEPTIONAL SETS FOR SUBHARMONIC, PLURISUBHARMONIC AND CONVEX FUNCTIONS

P. Blanchet has given the following result [Bl95]:

Let D be a domain in $\mathbb{R}^n$ (resp. in $\mathbb{C}^n$) and let S be a hypersurface of class $\mathcal{C}^1$ which divides D into two subdomains D_1 and D_2. Let $u \in \mathcal{C}^0(D) \cap \mathcal{C}^2(D_1 \cup D_2)$ be subharmonic (resp. plurisubharmonic (convex)) in D_1 and D_2. If $u|D_1 = u_1 \in \mathcal{C}^1(D_1 \cup S)$, $u|D_2 = u_2 \in \mathcal{C}^1(D_2 \cup S)$ and

$$\frac{\partial u_i}{\partial \overline{n}^k} \geq \frac{\partial u_k}{\partial \overline{n}^k}$$

on S with $i, k = 1, 2$ and $i \neq k$, then u is subharmonic in D (resp. plurisubharmonic (convex)).

Above $\overline{n}^k = (\overline{n}_1^k, \dots, \overline{n}_n^k)$ is the unit normal exterior to D_k, and $u_k \in \mathcal{C}^1(D_k \cup S)$, $k = 1, 2$, means that there exist n functions v_k^j, $j = 1, \dots, n$, continuous on $D_k \cup S$, such that

$$v_k^j(x) = \frac{\partial u_k}{\partial x_j}(x)$$

for all $x \in D_k$, $k = 1, 2$ and $j = 1, \dots, n$.

We point out that, as exceptional sets, instead of $\mathcal{C}^1$ hypersurfaces S considered by Blanchet, one can consider rather general exceptional sets E in D of finite $(n-1)$-dimensional Hausdorff measure (respectively of finite $(2n-1)$-dimensional Hausdorff measure). In our result, however, the additional conditions for the function u are slightly different than those used by Blanchet, see [Ri04], [Ri06].

References

[Bl95] P. Blanchet: On removable singularities of subharmonic and plurisubharmonic functions, Complex Variables, **26** (1995), 311–322.

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[Ri06] J. Riihentaus: Subharmonic functions, mean value inequality, boundary behavior, nonintegrability and exceptional sets, arXiv:math/0312508v3 [math.AP] 1 Nov 2006.