Helsinki Analysis Seminar, February 9th 2015
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Abstract

NAYATANI'S METRIC TENSORS AND CONFORMAL MEASURES

Let Γ be a non-elementary Kleinian group acting on the unit sphere $\mathbb{S}^n$, $n \geq 3$, with the limit set $L(\Gamma) \subset \mathbb{S}^n$ and the non-empty set of discontinuity $\Omega(\Gamma) = \mathbb{S}^n \setminus L(\Gamma)$.

If $O \subset \Omega(\Gamma)$ is a non-empty, open, connected and Γ-invariant set such that no non-trivial element in Γ has a fixed point in O, then the quotient space O/Γ is a typical example of a locally conformally flat Riemannian manifold.

Let $s > 0$ and let μ be an s-conformal measure of Γ, i.e. a positive and finite Borel measure supported by $L(\Gamma)$ such that

$$\mu(\gamma A) = \int_A |\gamma'|^s d\mu$$

for every μ-measurable set A and every $\gamma \in \Gamma$. Using μ, we can define a metric tensor g^μ on $\Omega(\Gamma)$ by setting that

$$g_x^\mu = \left(\int_{L(\Gamma)} \left(\frac{2}{|x-y|^2} \right)^s d\mu(y) \right)^{2/s} g_x^e$$

for every $x \in \Omega(\Gamma)$, where g^e is the standard euclidean metric tensor of $\mathbb{S}^n$. The tensor g^μ is Γ-invariant, which means that it can be projected onto manifolds of the form O/Γ.

The tensor g^μ was introduced in [N] in the case where μ is a Patterson-Sullivan measure of Γ, i.e. a δ_Γ-conformal measure of Γ obtained from a canonical construction invented by Patterson and generalized by Sullivan, where δ_Γ is the exponent of convergence of Γ. Indeed, it is a prevailing feature of the literature concerning g^μ that μ is assumed to be a Patterson-Sullivan measure.

In this talk, we discuss our paper [A] where μ is allowed to be any conformal measure of Γ. It turns out that this change in the point of view makes the theory appear in a more natural light and we are able to prove many generalizations of previously known results as well as completely new results.

Our main results concern primarily the sign of the scalar curvature of g^μ as well as the maximality of the isometry group of the projection of g^μ onto a manifold of the form O/Γ. We also obtain a strikingly simple new proof for the following result which was originally proved by Izeki in [I]. Suppose that $\Omega(\Gamma)/\Gamma$ has a non-empty compact component and that $\delta_\Gamma \leq (n-2)/2$. Then $\Omega(\Gamma)$ is connected and Γ is convex cocompact.

References

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[I] Izeki H. : "Convex-cocompactness of Kleinian groups and conformally flat manifolds with positive scalar curvature", Proc. Amer. Math. Soc. 130 no. 12, 2002, 3731-3740.

[N] Nayatani S. : "Patterson-Sullivan measure and conformally flat metrics", Math. Z. 225, 1997, 115-131.