Helsinki Analysis Seminar, 2014-03-03
Tuomo Äkkinen, University of Jyväskylä
Abstract

MAPPINGS OF FINITE DISTORTION: RADIAL LIMITS AND BOUNDARY BEHAVIOR

Fatou's Theorem states that a bounded analytic function $f\colon \mathbb{D} \to \mathbb{C}$ has radial limits at almost every point in $\partial\mathbb{D}$. It is not known what is the correct analog of Fatou's theorem in higher dimension. We give conditions for mappings $f\colon B^n(0,1) \to \mathbb{R}^n$ that guarantee existence of radial limits at almost every point in $\partial B^n(0,1)$. Moreover, we get a dimension estimate for the set

$$E_f = \{\xi \in S^{n-1}(0,1) : f \text{ does not have radial limit at } \xi\}.$$

For mappings of finite distortion these conditions are given in terms of integrability of the Jacobian determinant J_f on $B^n(0,r)$ for $r < 1$. We will also discuss the behavior of mappings of finite distortion on lower dimensional sets that approach $\partial B^n(0,1)$ in a certain cusp-like manner.

Chang and Marshall proved the following regularity result for radial extensions of analytic functions of the unit disk: There exists a constant $C < \infty$ such that

$$\int_0^{2\pi} \exp\left(|f(e^{i\theta})|^2\right) \frac{d\theta}{2\pi} \leq C$$

whenever $f\colon \mathbb{D} \to \mathbb{C}$ is an analytic function with $f(0) = 0$ and

$$\int_{\mathbb{D}} |f'(z)|^2 \, dz \leq 2\pi.$$

We prove an analog of this result for mappings of finite distortion with exponentially integrable distortion.