RIEMANN HYPOTHESIS AND MATHEMATICAL PHYSICS

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In both analytic number theory (the Riemann Hypothesis) and mathematical physics (Ising models and Euclidean field theories) the following complex analysis issue arises. For ρ a finite positive measure on the real line $\mathbb{R}$, let $H(z;\rho,\lambda)$ denote the Fourier transform of $\exp(\lambda u^2)d\rho(u)$, i.e., the integral over $\mathbb{R}$ of $\exp(izu+\lambda u^2)d\rho(u)$ extended from real to complex z, for those λ (including all $\lambda < 0$) where this is possible. The issue is to determine for various ρ's those λ's for which all zeros of H in the complex plane are real. We will discuss some old and new theorems about this issue.