PLANAR SOBOLEV EXTENSION DOMAINS

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We try to explain the state of art for the search of geometric characterizations for those bounded planar simply connected domains for which each Sobolev function is the restriction of a Sobolev function of the entire plane to the domain in question. For a person not familiar with Sobolev spaces, the following equivalent statement of this extension/restriction problem is perhaps accessible: given a bounded C^1-function u defined on a bounded domain G, when can we find a function v defined on entire $\mathbb{R}^2$ so that $v = u$ on G, v is C^1 on $\mathbb{R}^2 \setminus \overline{G}$, v is absolutely continuous on almost all lines parallel to the coordinate axes and so that

$$\int_{\mathbb{R}^2} |v|^p + |\nabla v|^p \leq C \int_G |u|^p + |\nabla u|^p ?$$

Here absolute continuity on almost all lines requires that for almost every $y \in \mathbb{R}$, the function $f(x) = v(x, y)$ is absolutely continuous on each closed interval $I \subset \mathbb{R}$, and that the similar conclusion holds for almost every $x \in \mathbb{R}$, for the function $g(y) = v(x, y)$. Above "almost every y" or "almost every x" refer to the Lebesgue measure on the real line.