

On typicality of functional observations  
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With the rapid increase in measurement precision and storage capacity, we have seen a tremendous jump in the dimensionality of data. One of the common methodologies used when dealing with such high dimensional data is to assume that the observed units are random functions (from some generating process) instead of random vectors.

The concept of statistical depth was originally introduced as a way to provide a center-outward ordering from a depth-based multivariate median. Several different depth functions for functional observations have been presented in the literature. Most of these approaches, however, are solely interested in the -pointwise- centrality of the functions as a measure of (global) centrality. As a result, they are missing some important features inherent to functional data such as variation in shape, roughness or range. Thus, due to the richness of functional data, we opt to talk about typicality rather than centrality of an observation.

We provide a new concept of depth for functional data. It is based on a new multivariate Pareto depth applied after mapping the functional observations to a vector of statistics of interest. These quantities allow incorporating the inherent features of the distribution, such as shape or roughness. In particular, in contrast to most existing functional depths, the method is not limited to centrality only. Properties of the depths are explored and the benefits of a flexible choice of features are illustrated on several examples. In particular, its excellent classification capacity is demonstrated on a real data example.