Reduced bias and increased variance: a possible trade-off in calibration for nonresponse treatment?

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Calibration for Nonresponse

- Nonresponse causes bias and increased variance of estimators
- If auxiliary information is available, calibration is a possibility in order to ”compensate” for nonresponse

Calibration for nonresponse

- Some important references:
- Brick, J.M. (2013). ”Unit Nonresponse and Weighting Adjustment: A Critical Review” Journal of Official Statistics, 29, 329-353.
- Deville, J.C. and Särndal, C.E. (1992). ”Calibration Estimators in Survey Sampling” Journal of the American Statistical Association 87, 376-382.
- Särndal, C.E. and Lundström, S. (2005) Estimation in Surveys with Nonresponse, UK: Chichester, Wiley.

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- Kott, P.S. (2006). ”Using Calibration Weighting to Adjust for Nonresponse and Coverage Errors” Survey Methodology 32, 133-142.
- Kott, P.S. and Chang, T. (2010). ”Calibration Weighting to Adjust for Nonignorable Unit Nonresponse” Journal of the American Statistical Association 105, 1265-1275.

Calibration for Nonresponse

Two features of an auxiliary variable:

- The *information*, specifying the detailed knowledge about the variable.
- The *role* of the variable in the calibration

Two possible *roles*:

- As a known population total
- As variable values known for all units in the sample

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- Examples of auxiliary *information*:
- **Ex 1** The auxiliary variable value is known individually for all units in the population and there are two possible *roles*:
- As a known population total
- As values used for the sample units only

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- **Ex 2** Population totals are taken from reliable sources outside the survey itself; totals are known (imported), but individual values are unknown at the population level (Canadian Labour Force Survey: demographic modeling)
- **Ex 3** The auxiliary variable is known (observed) for the sample units only: *paradata*, consisting of e.g. information from the data collection process

Calibration for Nonresponse

- Population U of size N
- Sample s of size n_s
- Response r of size n_r

- Target: Population total $\sum_U y_k$
- General form of estimator: $\sum_r w_k\, y_k$,

  where w_k is the calibration weight

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- Two main components of calibration:
- The underlying distance measure (here assumed to be the ”standard”)
- The calibration equation: $\sum_r w_k \, x_k = X$,
  where X is the (linear) calibration constraint.
- Example: $X = \sum_U x_k$, or $X = \sum_s d_k \, x_k$,
  where $d_k = 1/\pi_k$ (the sampling weight)

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- Auxiliary information at two levels:
- Population level: ”star” vector x^*
- Sample level: ”moon” vector x^o

- The star-vector values are assumed known for all $k \in U$
- The moon-vector values are assumed known for $k \in s$

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- Observe that the calibration constraint X can contain either ”star-information”, ”moon-information” or both.
- Calibration can now be performed in one or two steps.
- One step: all auxiliary information is used simultaneously

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Two-steps (two possibilities):

- ”Bottom-up”: First the moon-information is used for a calibration from r to s and then we use the star-info(or both types of info) for a calibration from r to U.
- ”Top-down”: First a calibration from s to U using the star-information and then a calibration from r to s using the moon-info (or both types of info)

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- Questions:
- Calibration in one or two steps?
- Should we ”reuse” information from the first step in the second step? Possible *trade-off*: lower bias against higher variance?
- For the two-step situation: bottom-up or top-down?
- Simplification: using sample instead of population information, what effect has that?

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- Population for simulation study: KYBOK, $N = 832$, clerical municipalities in Sweden 1992.
- y_k is expenditure on administration and maintenance
- Division into four groups according to size, yielding the moon-vector, ${x^o}_k$ consisting of indicator variables

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- ${x^*}_k = (1, x_k)$, where x_k is the square root of Revenue advances

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The simulation study:

- Sample size $n_s = 300$
- Response probability $\theta_k = 1 - \exp(-0.0318 x_k)$: increasing exponential response distribution)
- Here this leads to the average response probability 0.86.

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- $10\ 000$ simulated samples according to simple random sampling
- Each response set created by 300 independent Bernoulli trials, each with probability θ_k of success

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Measures of performance of estimators:

- Empirical first and second moments, yield estimates of:
- Bias
- Variance
- Mean squared error (MSE)

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- As a benchmark the estimator $N\overline{y_r}$ was also considered. As expected the simulation shows this estimator to be inferior to the other choices. (This estimator corresponds to an auxiliary variable which is a constant.)

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- Simulation results:
- The MSE is much lower when calibrating on the star-information at the population level, instead of calibrating at the sample level. The bias though, is smaller for the latter case.
- Comparing bottom-up with top-down, bottom-up estimators yield slightly less biased estimators with similar variance.

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- For the bottom-up approach, using the moon information AGAIN in the second step, leads to a slight decrease in bias and similar variance.
- Direct calibration produces estimates with slighly higher bias than the two-step procedures, but with similar variance.

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- We may now ”manipulate” the KYBOK population, by transforming and adding variables, e.g. by constructing a quantitative ”moon” variable.
- Preliminary results: For some cases the observed tendencies for the estimators using the original KYBOK population move in opposite directions.

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”This is still not the end,…”