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Bayesian Kernel Density Estimation applied to sensitive Geo-Coded Data of Berlin

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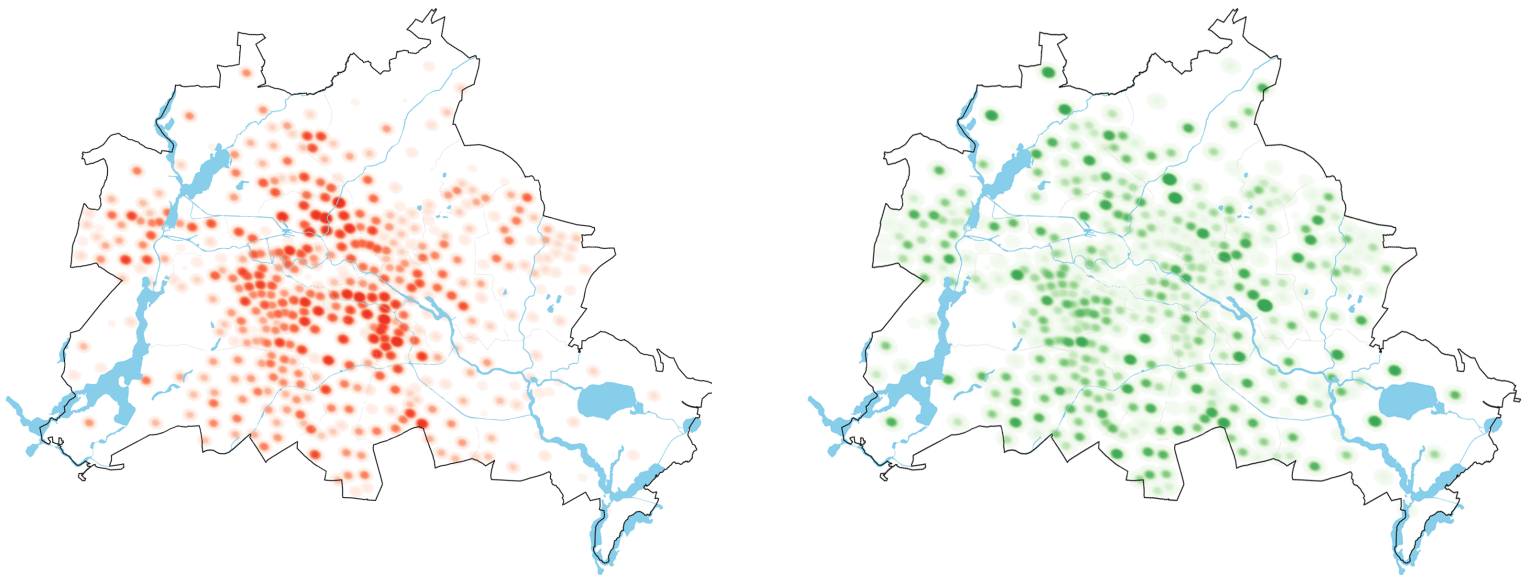
Motivation - Density estimates in Berlin

Ethnic minority background in Berlin (left map) and **aged people** in Berlin (right map) based on the publicly available data.



$\Longrightarrow$ Fundamental density **structure is not preserved**

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Motivation

- Modern systems of **official statistics require** the timely estimation of area-specific densities of sub-populations.
- Estimates should be based on **precise geo-coded information** - **hardly available** due to confidentiality constraints.
- A version of the Berlin register data is publicly available including aggregates for the 447 urban planning areas (LOR) - Fundamental density **structure is not preserved**.
- **Research question**: Can we derive precise density estimates of sub-groups by using data that has been subjected to disclosure control via aggregation or rounding of the geographic coordinates?

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Multivariate kernel density estimation

For a sample of bivariate data $X = (X_1, ..., X_n)$ from a density f the **kernel density** estimate is **defined** as

$$\hat{f}_X(x) = \frac{1}{n|H|^{\frac{1}{2}}} \sum_{i=1}^{n} K\left(H^{-\frac{1}{2}}(x - X_i)\right)$$

- $K()$ is a two dimensional kernel function and H is a bandwidth matrix.
- Through the anonymisation process only the rounded values W_i are available.

$\Longrightarrow$ Does the anonymisation process effect the density estimates?

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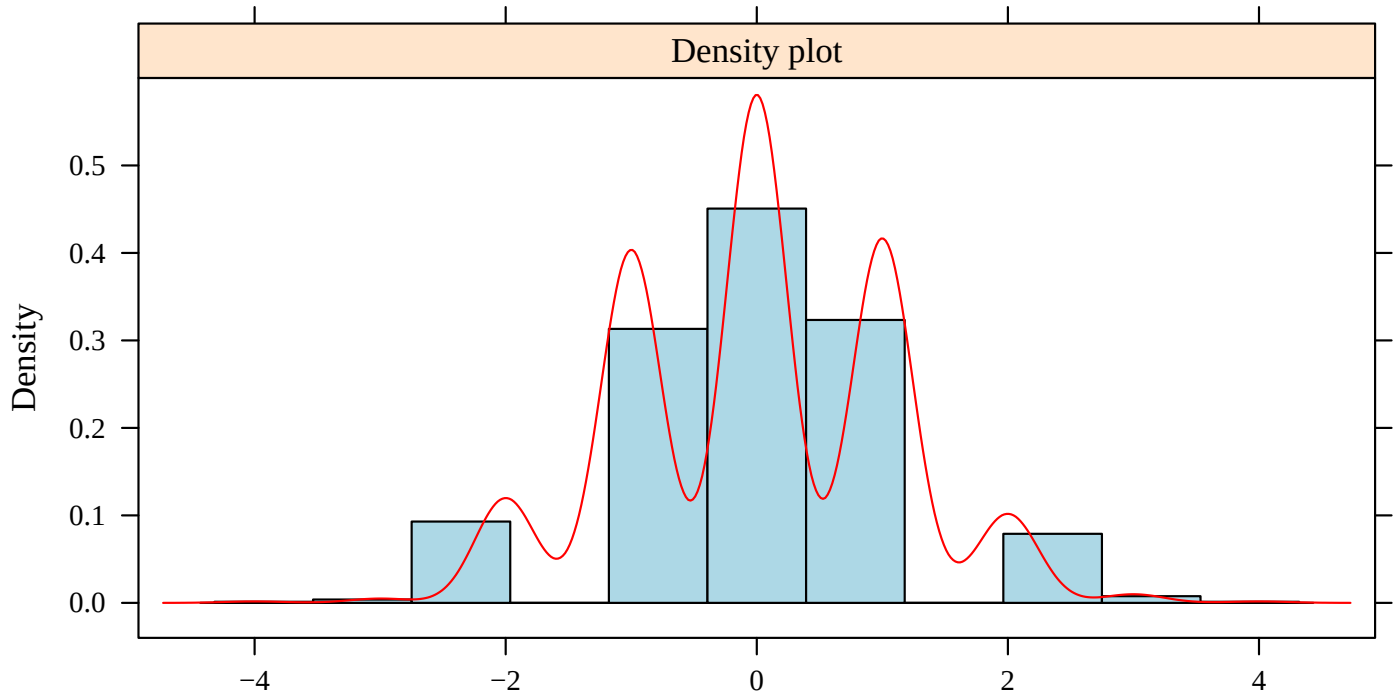
Effect of the anonymisation process

Univariate case:



- Kernel density estimator applied on data generated under normality (values rounded to the next integer).

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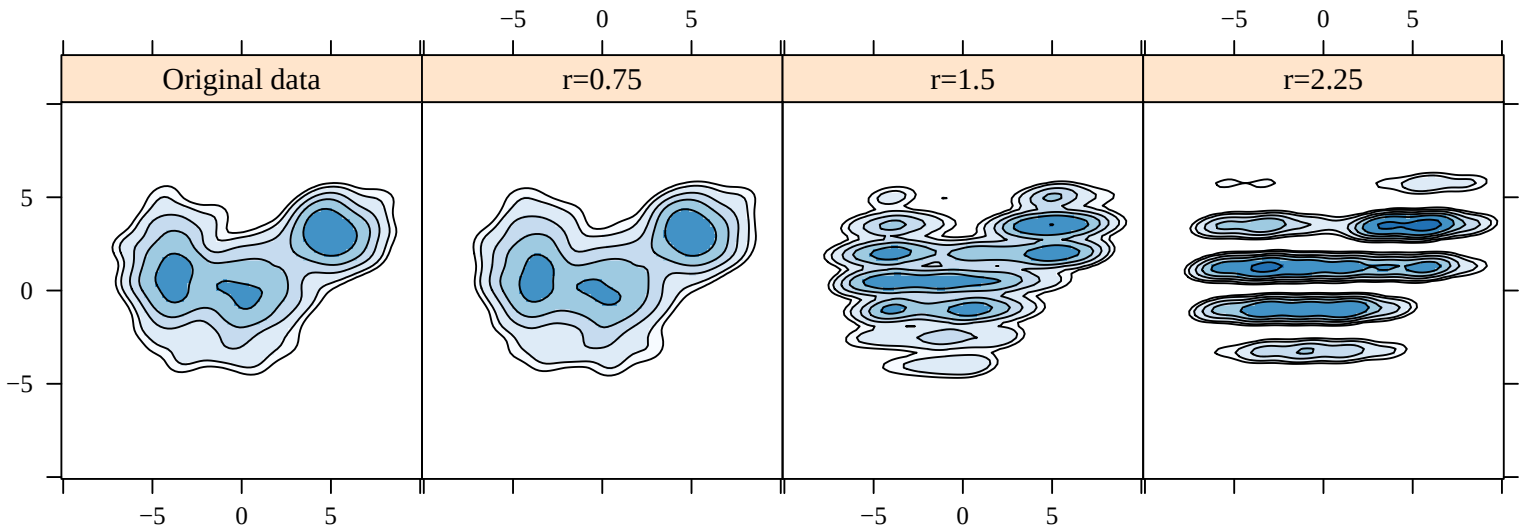
Effect of the anonymisation process

Multivariate case:



- Multivariate kernel density estimator applied on data (mixture of three uncorrelated bivariate normal distributions).

Idea:
Regard the anonymisation process as a **Berkson measurement error** on the true values X_i (Berkson, 1950).

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Rounding and kernel density estimation

Berkson measurement error model (Berkson, 1950):

- True, unknown, values $X_i = (X_{i1}, X_{i2})$ given the rounded values $W_i = (W_{i1}, W_{i2})$ are distributed in a rectangle with W_i in its center,

$$[W_{i1} - \frac{1}{2}\mathrm{r}, W_{i1} + \frac{1}{2}\mathrm{r}] \times [W_{i2} - \frac{1}{2}\mathrm{r}, W_{i2} + \frac{1}{2}\mathrm{r}],$$

  r denotes the rounding parameter.

- Can be translated into a Berkson measurement error model with uniformly distributed measurement error $U_i = (U_{i1}, U_{i2})$, $U_{i1}, U_{i2} \sim Unif(-\frac{1}{2}r, \frac{1}{2}r)$ and U_{i1}, U_{i2} independent of W_{i1} and W_{i2} such that,

$$X_{i1} = W_{i1} + U_{i1}, \ \ i = 1, 2, .., n$$

$$X_{i2} = W_{i2} + U_{i2}, \ \ i = 1, 2, .., n.$$

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Bayesian measurement error model

- Treat the unknown true values X_i as latent variables
- W_i only depends on X_i, the Likelihood can be split in two parts (measurement error model and observation model).

Posterior distribution by using a hierarchical model:

$$\pi(X, H|W) \propto \underbrace{\pi(W|X) \times \pi(X|H)}_{\text{Likelihood}} \times \underbrace{\pi(H)}_{\text{Prior}}$$

where $\pi(W|X)$ (measurement error model) is the indicator function and $\pi(X|H)$ (observation model) is defined by the leave-one-out kernel density estimator (Zhang et al., 2006)

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Computational details

Iterative MCMC-type algorithm:

1. Get a pilot estimate of f_X by setting H to $\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$, where I is a sufficiently *large* value such that no rounding spikes occur.
2. **Evaluate the density** estimate $\hat{f}_X$ on an **equally-spaced fine grid** $G = \tilde{x_1} \times \tilde{x_2}$.
3. X_i **is repeatedly drawn** from the square of side length r around W_i according to the current density estimate $\hat{f}_X$.
4. **Estimate the bandwidth matrix** H by the multivariate plug-in estimator of Wand and Jones (1994) and recompute $\hat{f}_X$.
5. Repeat steps 2-4 B (burn-in iterations) + N (additional iterations) times.
6. Discard the burn-in samples and get **final estimate of** f_X **by averaging** over the remaining samples.

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Empirical Studies - Aims

Model-based simulations:

- Investigate the ability of the proposed methodology to account for measurement error, under different scenarios for the intensity of the measurement error process.

Application to Berlin Register of Residents:

- Evaluate the performance the density estimators under a more realistic setting.
- Discuss the results of the density estimates in the context of two applications.

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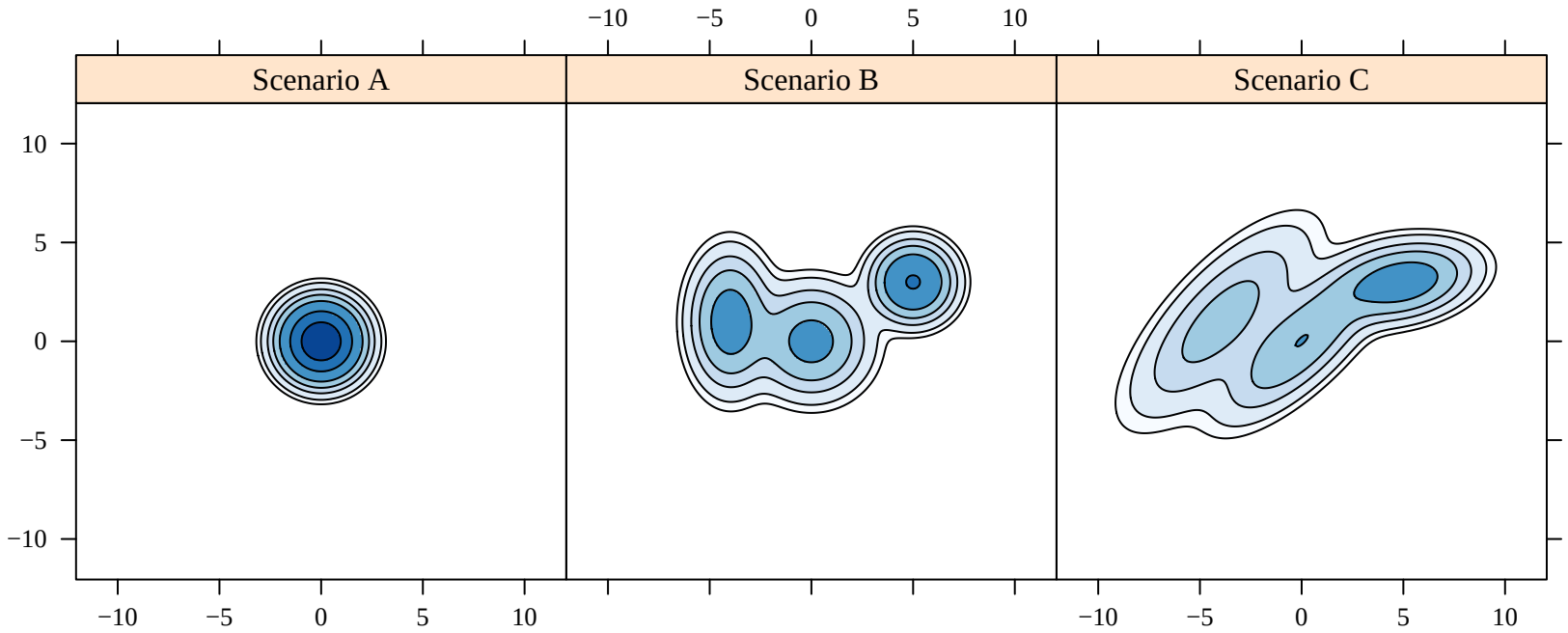
Simulation setup



Contour plots of the simulated data under the three simulation scenarios.

- For each scenario we generate a dataset S_0 of size $n = 500$ from the corresponding distribution.
- The dataset S_0 includes the exact x- and y-coordinates.

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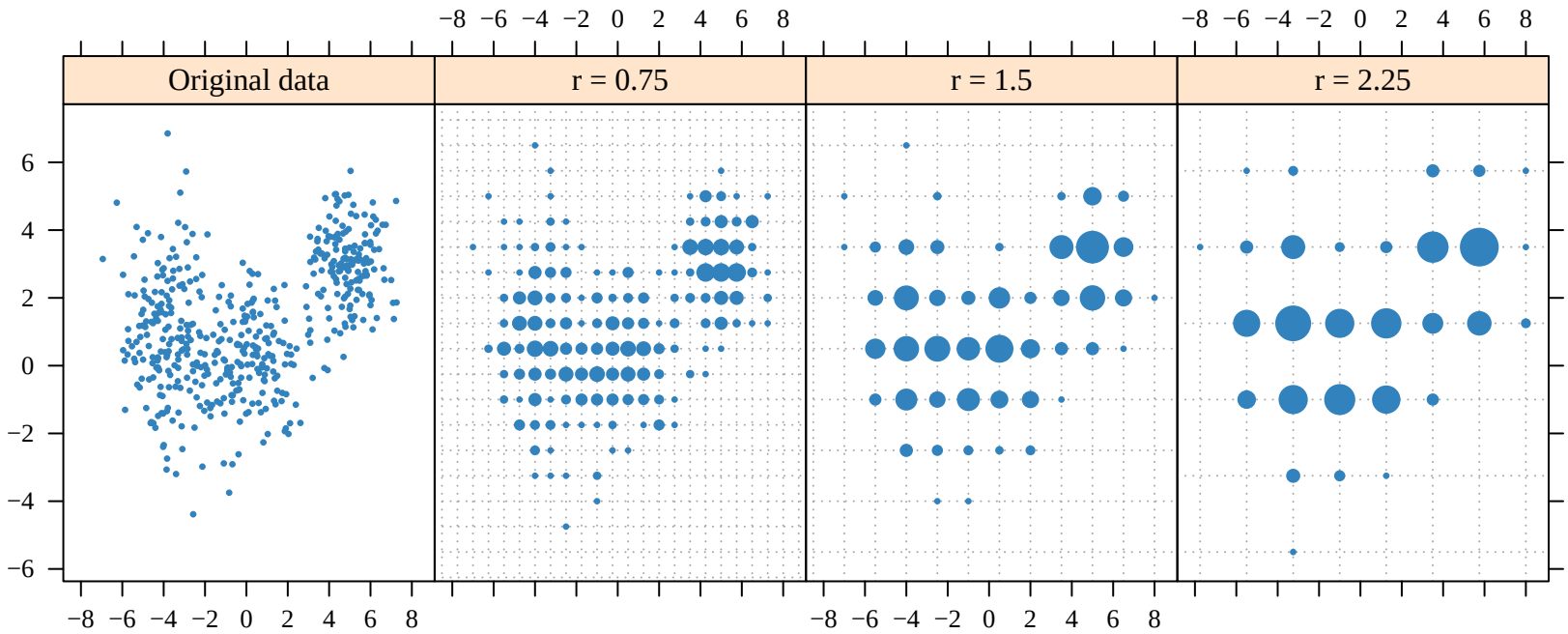
Simulation setup



Scenario B: Rounding procedure for a specific dataset.

- Define a grid for the x- and y-coordinates ranging from -10 to 10 with gridwidth according to rounding values r=0.75, 1.5 and 2.25.
- The size of the points represents the number of points at a specific rounding tick.

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Simulation setup

We estimate the density with two methods:

- *Naive*: a standard kernel density estimator that ignores measurement error.
- MCMC: the proposed Bayesian kernel density estimator.

The performance of the density estimates $\hat{f}$ is assessed by the root integrated mean squared error (RMISE):

$$\text{RMISE}(\hat{f}) = \sqrt{E\left(\int (f(x) - \hat{f}(x))^2 dx\right)} \approx \sqrt{\frac{1}{m}\sum_{j=1}^{m}(f(g_j) - \hat{f}(g_j))^2 \delta_g^2},$$

where f denotes the underlying true density, m is the number of grid points g_j and δ_g is the gridwidth.

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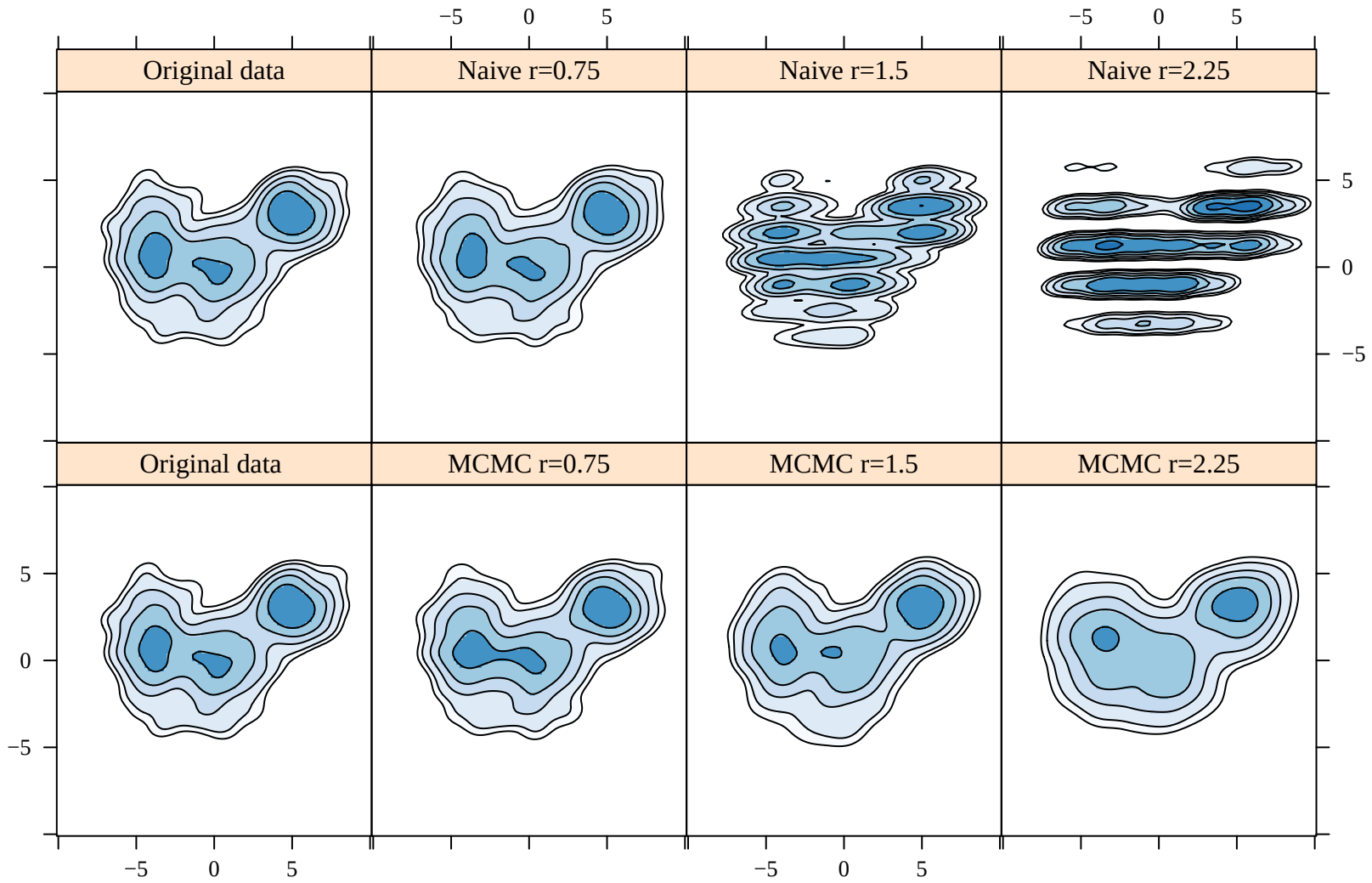
Simulation results: Scenario B



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Simulation results

Mean RMISE for different grid sizes (r) and scenarios based on 500 Monte-Carlo replications.

	r= 0	r= 0.75		r= 1.5		r= 2.25	
	Original	*Naive*	MCMC	*Naive*	MCMC	*Naive*	MCMC
Scenario A	0.205	0.238	0.239	3.952	0.242	4.917	0.568
Scenario B	0.162	0.172	0.170	0.380	0.183	0.679	0.256
Scenario C	0.119	0.125	0.121	0.147	0.131	0.351	0.152

- For the scenarios with small rounding errors the *Naive* and the MCMC density estimators perform similarly.
- For the scenarios with more severe measurement error (r=1.5 and r=2.25) the MCMC density estimator clearly outperforms the *Naive* estimator.

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Application: Data Sources

- The data contains all **308,754 Berlin household addresses** on the 31st of December 2012 with the **exact geo-coded coordinates** subject to different degrees of rounding errors.
- Registration at the local residents' office is compulsory in Germany and is carried out by the federal state authorities.
- One of the scenarios we explore is **rounding by using grids of size 2000 meters by 2000 meters** that approximately correspond to the LOR geography.
- The original data includes the total number of residents at their principal residence and the number of persons according to **some key demographic characteristics**:
  - Ethnic background (Ethnic)
  - Age (Age over 60).

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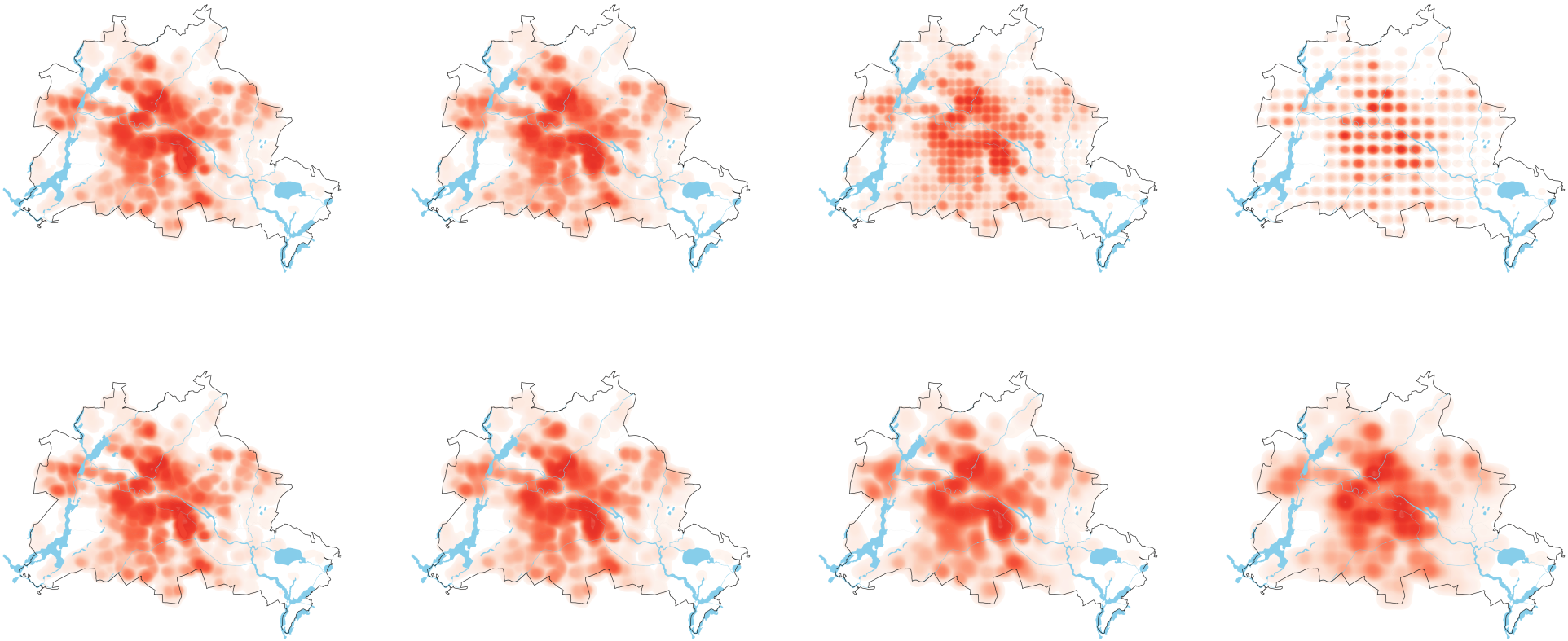
Density of population: Ethnic minority background

Naive (top panel) and MCMC estimators (bottom panel) with rounding step sizes of 0 (left), 500, 1250 and 2000 m (right).



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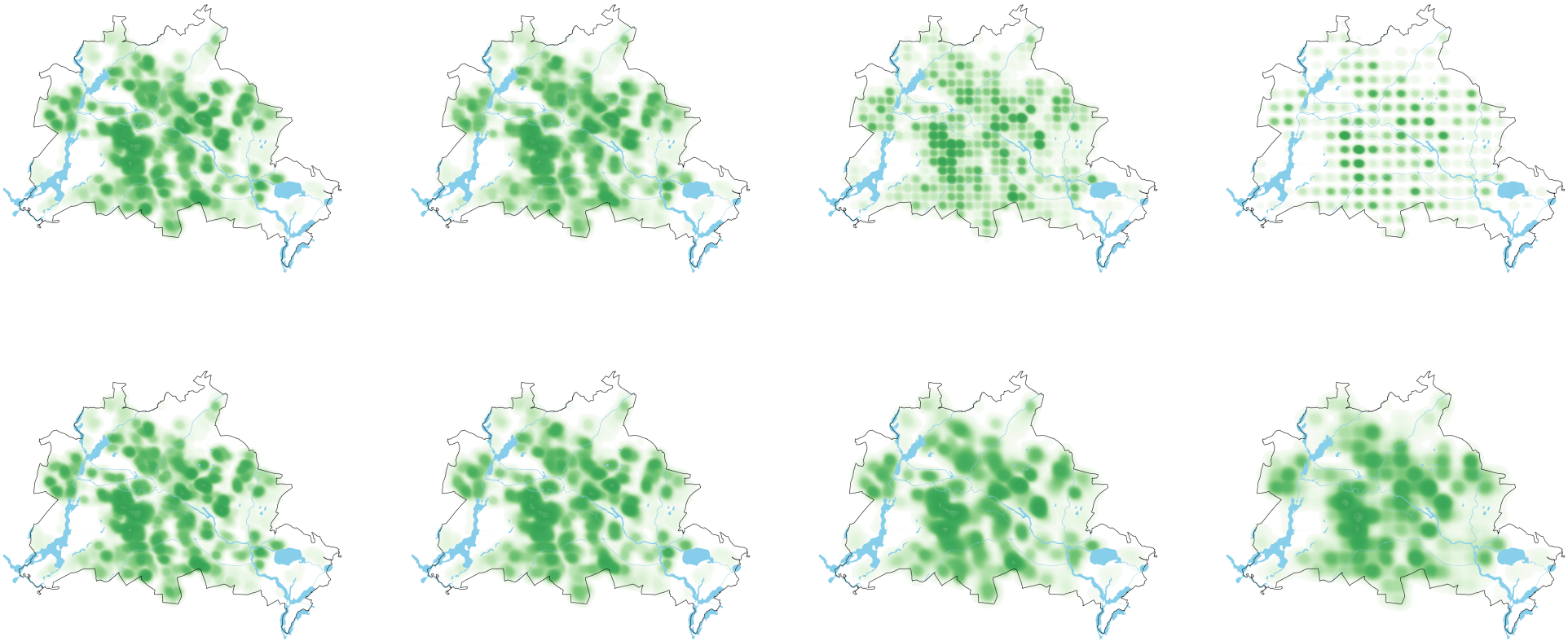
Density of population: Aged 60 and above

Naive (top panel) and MCMC estimators (bottom panel) with rounding step sizes of 0 (left), 500, 1250 and 2000 m (right).



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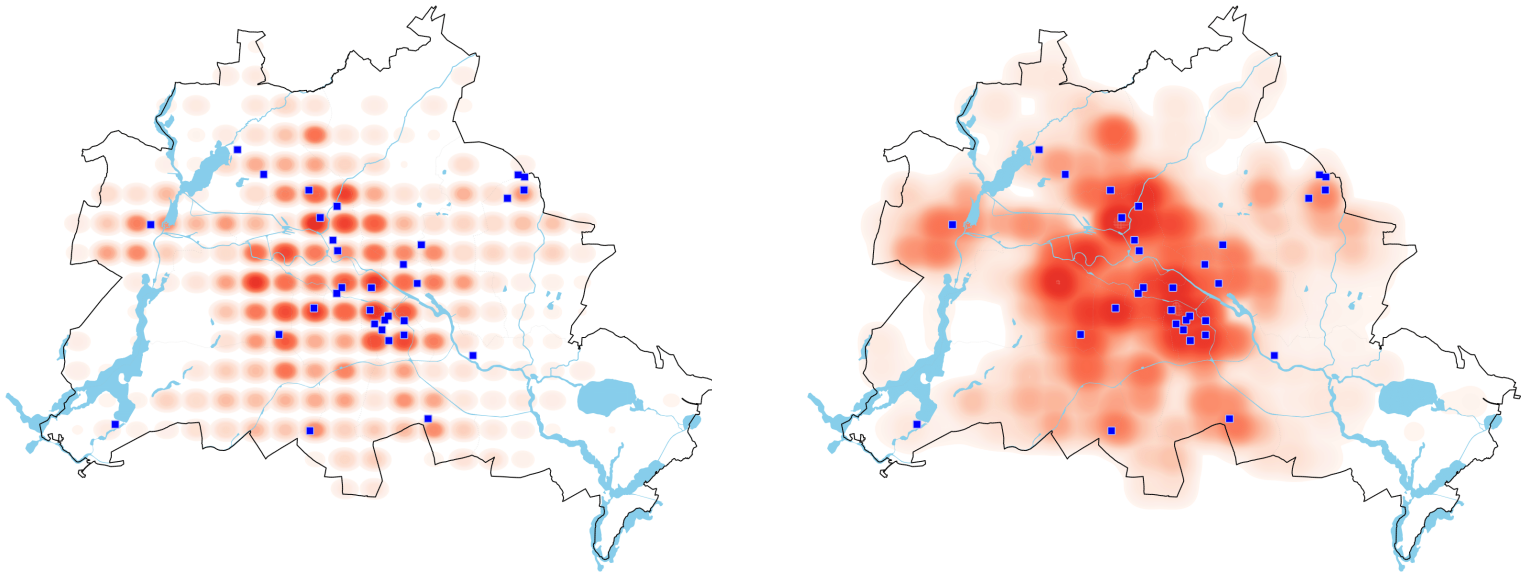
Advisory services for ethnic minorities



Ethnic background for rounding step size of 2000 m. Blue points indicate migrant advisory centres in Berlin.

- **Spatial distribution** of advisory centres **covers** ethnic minorities in the centre and north of Berlin quite well.
- **Some hotspots** with a high density of ethnic minority population but without any advisory service centres.

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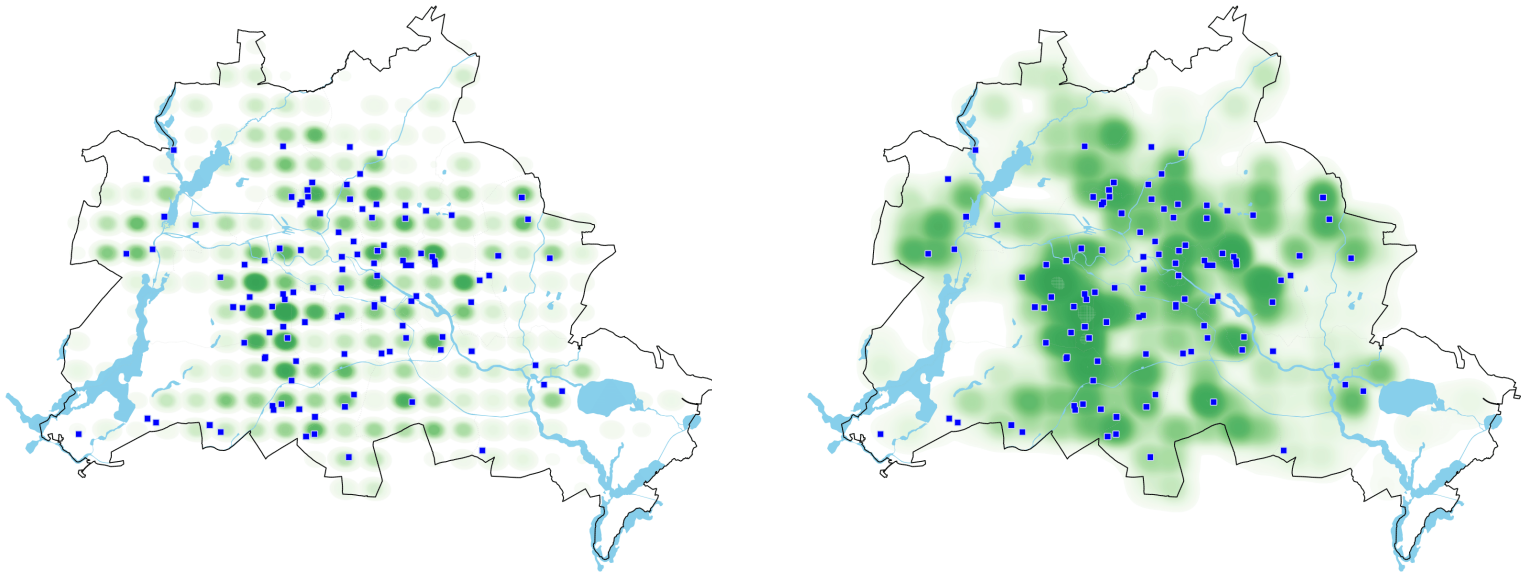
Care for the elderly



Age above 60 for rounding step size of 2000 m. Blue points indicate retirement houses in Berlin.

- The **supply of retirement houses is good** in the center of Berlin.
- **Locations for future expansion** of retirement houses and other support structures can be identified, like some areas in the north (Reineckendorf) or in the south-east (Gropiusstadt).

Summary

- The proposed MCMC method **can offer considerably deeper insights**, compared to a Naive estimator, to data analysts about the density of target populations.
- The **structure preserving property** of the proposed MCMC method is particularly attractive when working with data that has been **subjected to disclosure control** via aggregation or rounding of the geographic coordinates.
- The use of the proposed methodology is facilitated by the **availability of a computationally efficient algorithm**.
- **Local authorities may prefer** the density estimates produced by the proposed estimator for making informed decisions.

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Thank you very much for your attention.

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