Linnunradan rakenne – Kaavakokoelma

- Pallokolmion kaavat:

$$\begin{cases} \sin a \sin B = \sin b \sin A \\ \cos a = \cos b \cos c + \sin b \sin c \cos A \\ \sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \end{cases}$$

- Muunnos-kaavat (epookki 2000):

$$\begin{cases} \cos b \cos(l - 33.9^\circ) = \cos\delta \cos(\alpha - 18^h 51^m) \\ \sin b = \sin\delta \cos 62.9^\circ - \cos\delta \sin 62.9^\circ \sin(\alpha - 18^h 51^m) \\ \cos b \sin(l - 33.9^\circ) = \sin\delta \sin 62.9^\circ + \cos\delta \cos 62.9^\circ \sin(\alpha - 18^h 51^m) \end{cases}$$

- Ominaisliike:

$$\begin{cases} \mu''_\alpha = 15\mu^s_\alpha \cos\delta \\ t_\alpha = K r \mu''_\alpha \\ t_\delta = K r \mu'_\delta \end{cases}$$

Vakio $K = 4.7406$ ja $\sin 1'' = 1/206265$

- Nopeudet kiinteässä koordinaatistossa:

$$\begin{cases} v_r = \dot{x}\cos\alpha\cos\delta + \dot{y}\sin\alpha\cos\delta + \dot{z}\sin\delta \\ t_\alpha = -\dot{x}\sin\alpha + \dot{y}\cos\alpha \\ t_\delta = -\dot{x}\cos\alpha\sin\delta - \dot{y}\sin\alpha\sin\delta + \dot{z}\cos\delta \end{cases}$$

$$\begin{cases} \dot{x} = v_r\cos\alpha\cos\delta - t_\alpha\sin\alpha - t_\delta\cos\alpha\sin\delta \\ \dot{y} = v_r\sin\alpha\cos\delta + t_\alpha\cos\alpha - t_\delta\sin\alpha\sin\delta \\ \dot{z} = v_r\sin\delta + t_\delta\cos\delta \end{cases}$$

- Tähtienvälinen ekstinktio ja värieksesssi:

$A(r) = 1.086\tau(r) = 1.086 Q_{\text{ext}}\pi a^2 \int_0^r N(l)dl$

$m_V = M_V + 5\log\frac{r}{10\ \text{pc}} + A_V$

$(B-V)_{\text{obs}} = (B-V)_0 + E_{B-V}$

$Q = (U-B)_{\text{obs}} - 0.72(B-V)_{\text{obs}}$

- Stellaaristatistiikka ja spektriluokat:

$N(m|l,b) = \int_{-\infty}^m A(m|l,b)dm$

$\frac{dN(m|l,b)}{dm} = A(m|l,b)$

$A(m) = \omega\int_0^\infty D(r)\Phi(m - 5\log r + 5)r^2 dr,$ kun $A(r) = 0$

$$\begin{cases} A(m) = 10^3\cdot 0.2\ln 10\cdot\omega\int_{-\infty}^{\infty} 10^{0.6y} D_y(y)\Phi(m-y)dy \quad ; \quad y = m - M \text{ ja kun } A(r) = 0 \\ A(m) = 10^3\cdot 0.2\ln 10\cdot\omega 10^{0.6m}\int_{-\infty}^{\infty} D_y(m-M)10^{-0.6M}\Phi(M)dM \end{cases}$$

$$\begin{cases} A(m,S) = \omega\int_0^\infty D_S(r)\Phi(M,S)r^2 dr, \text{ kun } A(r) \neq 0 \\ m_V = M_V + 5\log\frac{r}{10\ \text{pc}} + A_V \end{cases}$$

$\Phi(M,S) = \frac{\phi_0}{\sigma} e^{-\frac{1}{2\sigma^2}(M-M_0)^2}$

- Tähtien nopeusjakaumat Linnunradassa:

$\phi(\dot{x},\dot{y},\dot{z}) = \frac{1}{(2\pi)^{3/2}\sigma^3} e^{-\frac{1}{2\sigma^2}[(\dot{x}-\dot{x}_0)^2 + (\dot{y}-\dot{y}_0)^2 + (\dot{z}-\dot{z}_0)^2]}$

$\phi(\xi,\eta,\zeta) = \frac{1}{(2\pi)^{3/2}\Sigma_1\Sigma_2\Sigma_3} e^{-\frac{1}{2}\left[\left(\frac{\xi}{\Sigma_1}\right)^2 + \left(\frac{\eta}{\Sigma_2}\right)^2 + \left(\frac{\zeta}{\Sigma_3}\right)^2\right]}$

- Linnunradan rotaatio:

  $$\begin{cases} v_r = R_0(\omega - \omega_0)\sin l \\ t = R_0(\omega - \omega_0)\cos l - \omega d \end{cases}$$

  $$\begin{cases} v_r = Ad\sin 2l \\ t = Ad\cos 2l + Bd \end{cases}$$

  Voimassa kun $d << R_0$.

- Oortin kaavat:

  $A = \frac{1}{2}\left[\frac{\Theta_0}{R_0} - \left(\frac{d\Theta}{dR}\right)_{R_0}\right]$

  $B = -\frac{1}{2}\left[\frac{\Theta_0}{R_0} + \left(\frac{d\Theta}{dR}\right)_{R_0}\right]$

  $$\begin{cases} \frac{\Theta_0}{R_0} = A - B \\ \left(\frac{d\Theta}{dR}\right)_{R_0} = -(A+B) \end{cases}$$

  $\mu_t = \frac{t_l}{4.74d} = \frac{A}{4.74}\cos 2l + \frac{B}{4.74}$

- Tähtien radat Linnunradassa:

  $v^2 = M\left(\frac{2}{R} - \frac{1}{a}\right) = R_0\Theta_0^2\left(\frac{2}{R} - \frac{1}{a}\right)$

  $\frac{\Pi^2}{\Theta_0^2} + \frac{\Theta^2}{\Theta_0^2} + \frac{\Theta_0^2}{\Theta^2}(1-e^2) - 2 = 0$

  $$\begin{cases} \xi = H\sin\kappa(t-t_0) \quad \ddot{\xi} = -H\kappa^2\sin\kappa(t-t_0) \\ -\kappa^2 = 4B(A-B) \quad \kappa = 2\sqrt{-B(A-B)} \approx 36.7 \text{ kms}^{-1}\text{kpc}^{-1} \end{cases}$$

  $\sigma_\Pi/\sigma_\Theta = \sqrt{(A-B)/B} = 1.47$

  $F_{z'} = -4\pi G\rho_0 z$ harmoninen voima

  $\ddot{z} = F_z = -Cz; \quad z = Q\sin\lambda t; \quad \dot{z} = Q\lambda\cos\lambda t = Z_0\cos\lambda t$

- Linnunradan massatiheys:

  $\nabla^2\Phi = 4\pi G\rho(x,y,z)$

  $\frac{\partial F_r}{\partial r} + \frac{F_r}{r} + \frac{\partial F_z}{\partial z} = -4\pi G\rho(x,y,z)$

  $\frac{1}{2}w^2(z) - \int_0^z F_{z'}dz' = \frac{1}{2}w_0^2 = E$

  $\frac{D_{w0}(z)}{D_{w0}(0)} = \frac{w_0}{\sqrt{(w_0^2 + 2\int_0^z F_{z'}dz')}}$