Galaxy formation and evolution – Problem set 6. Autumn 2022

The answers should be returned by ***Monday (5.12) 4pm (16.00) in Moodle****, link through the official course homepage. The answers to the problem set will be discussed on Wednesday (7.12) at 12.15-14.00 in Room A128, Chemicum.*

1. Consider a class of quasars for which the flux as a function of the frequency ν is given by $F_\nu \propto \nu^{-0.5}$. Perform the calculations in a) and b) first for a matter-dominated Universe with $\Omega_{m,0} = 1$, $\Omega_{\Lambda,0} = 0$ and $H_0 = 70$ km/s/Mpc. *Hint: The calculation of cosmological volumes is discussed in MBW on pages 121-124.*

   (a) What is the absolute B-band magnitude of a quasar that is observed to have an apparent B-band magnitude $m_B = 19.5$ and a redshift of $z = 2$?

   (b) There are two quasars per square degree of the sky with redshifts between $z = 1.75$ and $z = 2.25$ having B magnitudes brighter than $m_B = 19.5$. Estimate the comoving number density of quasars around redshift $z \simeq 2$.

   (c) Repeat the calculations in a) and b) using the currently favoured Planck cosmology: $\Omega_{m,0} = 0.313$, $\Omega_{\Lambda,0} = 0.687$ and $H_0 = 67.3$ km/s/Mpc. How different are the results?

2. The derivation of the dynamical friction formula (Lecture notes 11) assumed that the subject system is a point mass, but in many cases of interest the subject system is an extended body, such as a star cluster or a satellite galaxy, characterized by a half-mass radius r_h. If the point of closest approach of the field star to the centre of the subject body is $\lesssim r_h$ then the deflection of the field-star orbit, and its contribution to the drag force, will be smaller than if the subject body were a point of the same total mass.

   (a) Argue that the total drag force is largely unaffected by the non-zero size of the subject body if $r_h \lesssim b_{90}$, where $b_{90} \sim G(M_S + m)/v_\infty^2$ as defined in the lecture notes (Lecture 11.17).

   (b) If $r_h \gtrsim b_{90}$, argue that encounters with impact parameter $\lesssim r_h$ make a negligible contribution to the total drag force. Show that in this case the argument of the Coloumb logarithm should be given by $\Lambda \simeq b_{\max}/r_h$.

   (c) Combine these conclusions to argue that the correct value of the argument of the Coulomb logarithm Λ for a subject body of half-mass r_h is approximately:

   $$\Lambda = \frac{b_{\max}}{\max(r_h, GM/v_{\rm typ}^2)}$$

   and that the fractional error in $\ln \Lambda$ that arises from using this expression is of the order $(\ln \Lambda)^{-1}$.

3. Major mergers of elliptical galaxies. Observations and evolutionary models for elliptical galaxies indicate that they can grow by mergers with other small and large ellipticals. Using the virial theorem we will here estimate the size, the velocity dispersion, and the mean density of an elliptical galaxy as it accretes stars. *Hint: The paper "Minor Mergers and the Size Evolution of Elliptical galaxies" Naab et al., 2009, ApJL, 699, 178 will be useful for solving this problem.*

   (a) Assuming the merging systems have isotropic velocity dispersions and no net rotation, give expressions for the total kinetic and gravitational energy (and the total energy) of a galaxy as a function of its mass M, stellar mean square velocity $\langle v^2 \rangle$, and the effective gravitational radius r_g. What is the energy of the merged galaxy?

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(b) Let $\eta = M_a/M_i$ be the ratio of the accreted mass M_a to the initial mass M_i, so that the final galaxy has a mass $M_f = M_i + M_a = (1+\eta)M_i$, and let $\epsilon = \langle v_a^2 \rangle / \langle v_i^2 \rangle$ be the ratio of the initial average square speeds to the average square speeds of the accreted material. How do the velocities, sizes and densities change when two equal-mass systems merge ($\eta = 1$)? Compute the final-to-initial ratios of these three quantities.

(c) What happens if you have accretion of smaller systems (minor merging) with $\langle v_a^2 \rangle << \langle v_i^2 \rangle$, i.e. $\eta = 1$ remains the same, but $\epsilon \to 0$.

4. Download and read the paper: *"A substantial population of low-mass stars in luminous elliptical galaxies"* by van Dokkum & Conroy, 2010, Nature, 468, 940 using the link on the course homepage. Based on the paper answer the questions below:

   (a) What is the main result of the paper and what type of observations are used to derive it? Why is the initial mass function (IMF) of the stars such a crucial parameter in galaxy studies?

   (b) What does Fig 1. in the paper show? Why can different IMFs be distinguished using spectroscopy? What particular spectral lines are used in this paper to constrain the IMFs of elliptical galaxies?

   (c) What does Fig 2. in the paper show? What are the different coloured lines in the left-hand panel and what is meant by "bottom-light" and "bottom-heavy" IMFs? If the IMF is not universal, what are the implications for galaxy stellar mass-measurements in the local Universe and especially at high redshifts?

5. Download and read the paper: *"A luminous quasar at a redshift of $z = 7.085$"* by Mortlock et al. 2011, Nature, 474, 616 using the link on the course homepage. Based on the paper answer the questions below:

   (a) What is special about the quasar found in this paper? What is its redshift, when was the observed light emitted and what is the estimated mass of the supermassive black hole in the centre of the galaxy and how is the mass measured? From Fig 1. in the paper, what spectral features identifies the object as a quasar?

   (b) What does Fig 2. of the paper show? The authors state that the striking aspect of this quasar is the almost complete lack of observed flux blueward of the Lyα line. What are the implications of this measurement?

   (c) What does Figs 3. and 4. show? How is this quasar different to the two high-z quasars found by the SDSS survey earlier? Finally, why is the observation of a very massive black hole in the very early Universe a potential challenge to theoretical models of black hole formation, i.e. why is it difficult to form supermassive black holes in the very early Universe?