Galaxy formation and evolution – Problem set 5. Autumn 2022

The answers should be returned by ***Monday (21.11) 4pm (16.00) in Moodle****, link through the official course homepage. The answers to the problem set will be discussed on Wednesday (23.11) at 12.15-14.00 in Room A128, Chemicum.*

1. Let E be the energy and J the angular momentum of a system.

   (a) Show that $GE^\beta J^\gamma$ cannot be dimensionless.

   (b) Find values of α, β and γ for which $GM^\alpha E^\beta J^\gamma$ is dimensionless.

   (c) Calculate this dimensionless spin parameter (λ) for the Milky Way-like galaxy assuming that it is in virial equilibrium and that the virial radius is $r_{\rm vir} = 160h^{-1}$ kpc and the disk scale radius is $r_D = 3.5$ kpc. Assume that the virial mass of the Milky Way is $M_{\rm vir} = 9.5 \times 10^{11} h^{-1} M_\odot$, where $h = 0.71$.

2. Cooling flows in galaxy clusters. Suppose we model the hot gas in a galaxy cluster as a singular isothermal sphere, i.e. the number density scales with radius as $n(r) \propto r^{-2}$ out to a radius of $r_{\rm max} = 1$ Mpc. The temperature can be assumed to be constant at $T = 10^8$ K. The total mass in baryons (assumed to be fully ionised hydrogen) is $M = 10^{14} M_\odot$.

   (a) Estimate the radius r_c at which the bremsstrahlung cooling time of the gas is of the order of the Hubble time. For more on bremsstrahlung see pages 368 and 758 in the Mo, van den Bosch and White textbook.

   (b) Show that the mass flow rate $\dot{M}$ per unit time is given by:

   $$\dot{M}(r) = \frac{2\mu m_h L(<r)}{5k_b T},$$

   where μ is the mean molecular weight of the particles in the plasma, T is the temperature of the plasma and $L(<r)$ is the observed X-ray cooling luminosity. Using this formula estimate the mass flow rate for galaxy clusters and discuss what feedback process could potentially offset the cooling flow by heating the gas.

3. Galaxy potentials. Spherical potentials of stellar clusters and sometimes galaxies can be described by a *Plummer sphere* for which the potential is given as:

   $$\Phi_P(r) = -\frac{GC}{\sqrt{r^2 + a_p^2}},$$

   where a_p and C are parameters describing the size of the sphere.

   (a) Show that this form approaches the potential of a point mass when $r >> a_P$ and use this fact to find the total mass of the system in terms of C.

   (b) Find the density profile of the system using the Poisson equation: $\nabla^2 \Phi(\mathbf{r}) = 4\pi G \rho(\mathbf{r})$ in spherical coordinates.

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(c) The dark matter halo is sometimes approximated with the following density distribution:

$$\rho(r) = \frac{\rho_0 {a_d}^2}{r^2 + {a_d}^2},$$

where ρ_0 and a_d are constants. Show that if the disk of stars is placed in circular orbits within the potential created by this density profile, the circular velocity of these stars is given by:

$$v(r) = v_0[1 - (a_d/r)\arctan(r/a_d)]^{1/2}.$$

Evaluate v_0 in terms of ρ_0, a_d and G and show either by plotting or some other means that $v(r)$ goes to a constant when r is much larger than a_d.

4. Download and read the paper: *"Forming Realistic Late-type Spirals in a ΛCDM Universe: The Eris Simulation"* by Guedes, Callegari, Madau & Mayer 2011, ApJ, 742, 76 using the link on the course homepage. Based on the paper answer the questions below:

   (a) Describe the simulation setup. What is the numerical resolution of the simulation? How many particles are used, what is the mass per particle? What is meant by gravitational softening? What is the spatial resolution of the simulation? What is the smallest structures that can be resolved and how does this compare to the sizes of observed molecular clouds?

   (b) Which parameters characterise the star formation and feedback recipes? How is the star formation and supernova feedback treated in the simulation? In particular, what happens to the gas that is heated by supernova explosions? Why is the density threshold for star formation a critical parameter in determining the structure of the final galaxy?

   (c) Figures 1-3 in the paper. What is the peak velocity of the rotation curve and what is the shape of the curve at large radii, how does this compare with the observed rotation curve of the Milky Way? How does the visual appearance of the simulated galaxy, its surface brightness profile and its HI content compare with observations? In particular what is the mass fraction of the bulge, i.e. the bulge-to-disk ratio for this galaxy?

5. (a) What does the two panels in Fig 4. show? How well does the simulated galaxy agree with the relations plotted in Fig 4.? Can you explain what is the main reason for the good agreement between the simulation and the observations?

   (b) How is the gas distributed in the simulated galaxy? What amount of the gas is in the hot, warm and cold gas components? How does this compare to the corresponding values of observed galaxies and how in particular can we constrain the amount of hot gas in the Milky Way observationally?

   (c) In addition to the Eris simulation the authors also run a simulation called ErisLT. What is the main difference between these two simulations and how does this difference manifest in the simulated properties of the galaxies? Why is the Eris simulation more realistic than the Eris LT simulation. Finally, what is the main conclusion of Fig 7.?