



# **Galaxy formation and evolution**

**PAP 318, 5 op, autumn 2022**

BK106, Exactum

**Lecture 6: Correlation functions and the  
spectrum of the initial fluctuations –  
Additional notes, 12/10/2022**



# Lecture 6 additional notes I

- Page 4: Overdensity delta definition:

$$\Delta = \frac{\delta\rho}{\rho_0}, \quad \rho = \rho_0[1 + \Delta(\bar{x})] = \rho_0 \left[ 1 + \frac{\delta\rho}{\rho_0} \right] = \rho_0 + \delta\rho$$

- Page 6: Fourier transform and inverse Fourier transform:

$$\Delta(\bar{r}) = \frac{V}{(2\pi)^3} \int \Delta_{\bar{k}} e^{-i\bar{k}\cdot\bar{r}} d^3 k$$

$$\Delta_{\bar{k}} = \frac{1}{V} \int \Delta(\bar{r}) e^{i\bar{k}\cdot\bar{r}} d^3 x$$



# Lecture 6 additional notes II

- Page 6: Definition and short-hand notation:

$$\langle \Delta^2 \rangle = \frac{1}{V} \int \Delta^2(\vec{r}) d^3x$$

- Page 7: Basics of complex numbers:  $\Delta^*$ =complex conjugate

$$|\Delta(\vec{r})|^2 = |\Delta(\vec{r})\Delta^*(\vec{r})|$$

$$e^{i\vec{k}\cdot\vec{r}} = \cos(\vec{k}\cdot\vec{r}) + i\sin(\vec{k}\cdot\vec{r})$$



# Lecture 6 additional notes III

- Page 8: Two-point correlation function integration:

$$\xi(r) = \frac{V}{(2\pi)^3} \int |\Delta_k|^2 \cos(kr \cos \theta) \frac{1}{2} \sin \theta d\theta k^2 dk$$

$$\xi(r) = \frac{V}{(2\pi)^3} \int |\Delta_k|^2 \frac{\sin kr}{kr} k^2 dk$$

- Page 9: Initial Power spectrum:  $\sin(kr)$  Taylor expansion

$$\sin kr \approx kr - \frac{(kr)^3}{6} + \frac{(kr)^5}{120} + \dots$$



# Lecture 6 additional notes III

- Page 9: Initial Power spectrum:

$$\xi(r) \propto \int_0^{k_{\max}} k^{n+2} dk \propto k^{n+3}$$

$$k_{\max} \approx 1/r, \quad \xi \propto r^{-(n+3)}, \quad \xi(M) \propto M^{-(n+3)/3}$$

- Page 10: Relation between the density fluctuation and the 2-point correlation function:

$$\langle \Delta^2 \rangle^{1/2} \propto \xi^{1/2}$$



# Lecture 6 additional notes IV

- Page 11: Horizon mass in the radiation dominated era

$$M_H \propto \rho_d (ct)^3 \propto a^{-3} (a^2)^3 = a^3, \quad a \propto M_H^{1/3}$$

- Page 18: Bias can also be defined in terms of the  $\sigma_8$  parameter, which is defined to be root-mean-square density fluctuation when smoothed with a top-hat filter of radius  $8h^{-1}$  Mpc.

$$b^2 = \frac{\sigma_8^2(\text{gal})}{\sigma_8^2(\text{mass})}$$