Galaxy formation and evolution
PAP 318, 5 op, autumn 2022

BK106, Exactum

Lecture 5: Baryonic and dark matter models of galaxy formation – Additional notes, 05/10/2022

Lecture 5 additional notes I

- Page 3: Radiation-matter equality:
- Black-body radiation density:

$$\rho_r = \frac{4\sigma}{c}T^4 = \frac{4\sigma}{c}T_0(1+z)^4$$

- Matter density:

$$\rho_m = \Omega_0 \rho_c (1+z)^3 c^2$$

- Photon to baryon ratio, when Ω_b=0.05 and h=0.7, assuming that the baryons are made primarily of protons and that all the photons are in the CMB:

$$\frac{N_\gamma}{N_B} = \frac{3.7\times 10^7}{\Omega_b h^2} = \frac{3.7\times 10^7}{0.05\times 0.7^2} = 1.5\times 10^9$$

Lecture 5 additional notes II

- Page 5: The sound speed

$$c_S^2 = \frac{(\partial p/\partial T)_r}{(\partial \rho/\partial T)_r + (\partial \rho/\partial T)_m} = \frac{\frac{4}{3}\rho_r c^2}{4\rho_r + 3\rho_m}$$

- Page 8: Adiabatic fluctuations in the radiation-dominated era
- Sound speed in the radiation dominated era:

$$c_S = \frac{c}{\sqrt{3}} \qquad \lambda_J = c_S \left(\frac{3\pi}{8G\rho}\right)^{1/2}$$

- Page 8: The Jeans mass, λ_J the Jeans length is the diameter for the spherical Jeans mass:

$$M_J = \frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3 \rho = \frac{\pi \lambda_J^3}{6}\rho$$

- Page 8: Jeans mass scaling with scale factor

$$\lambda_J \propto \rho^{-1/2}, \quad \rho \propto a^{-4}, \quad \lambda_J \propto a^2, \quad M_J \propto \lambda_J^3 \rho_b \propto a^6 a^{-3} \propto a^3$$

Lecture 5 additional notes III

- Page 9: Adiabatic fluctuations in the matter-dominated era
- Baryonic-mass within the particle horizon, scaling with scale factor

$$M_{b,\mathrm{hor}} \propto r_H^3 \rho_b, \quad r_H \propto a^{3/2}, \quad \rho_b \propto a^{-3}, \quad M_{b,\mathrm{hor}} \propto a^{3/2}$$

- Page 9: Jeans mass scaling with scale factor

$$M_J \propto \lambda_J^3 \rho, \quad \lambda \propto c_S \rho^{-1/2} \propto a^{-1/2} a^{3/2} \propto a, \quad M_J \propto a^3 a^{-3} \propto a^0$$

- Page 10: Silk Dampening. Diffusion constant D:

$$D = \frac{1}{3} \lambda c$$

Lecture 5 additional notes IV

- Page 11: Scaling of Silk dampening mass with scale factor
- Radiation dominated era:

$$N_e \propto (1+z)^3, \quad t \propto (1+z)^{-2}, \quad r_D \propto [(1+z)^{-2}(1+z)^{-3}]^{1/2} \propto (1+z)^{-5/2}$$

$$M_S \propto r_D^3 \rho_b \propto (1+z)^{-15/2}(1+z)^3 \propto (1+z)^{-9/2}$$

- Matter dominated era:

$$N_e \propto (1+z)^3, \quad t \propto (1+z)^{-3/2}, \quad r_D \propto [(1+z)^{-3/2}(1+z)^{-3}]^{1/2} \propto (1+z)^{-9/4}$$

$$M_S \propto r_D^3 \rho_b \propto (1+z)^{-27/4}(1+z)^3 \propto (1+z)^{-15/4}$$

Lecture 5 additional notes V

- Page 22-23: Instabilities in the presence of dark matter

$$\ddot{\Delta}_B + 2\left(\frac{\dot{a}}{a}\right)\dot{\Delta}_B = 4\pi G\rho_D B a$$

- Page 23: The background is the critical Ω_0=1 model.

$$a^{3/2}\frac{d}{da}\left(a^{-1/2}\frac{d\Delta}{da}\right) + 2\frac{d\Delta}{da} = \frac{3}{2}B$$

Lecture 5 additional notes VI

- Page 23: The solution Δ=B(a-a_0) satisfies this equation:

$$a^{3/2}\frac{d}{da}\left(a^{-1/2}B\right)+2B=\frac{3}{2}B \Rightarrow a^{3/2}\left(-\frac{1}{2}a^{-3/2}B\right)+2B=\frac{3}{2}B$$

- Page 23: Growth of the baryon perturbations:

$$\Delta_d = Ba, \quad \Delta_B = B(a-a_0), \quad \Delta_B = Ba\left(1-\frac{a_0}{a}\right)$$

$$\frac{a_0}{a}=\frac{1+z}{1+z_0}, \quad \Delta_B = \Delta_D\left(1-\frac{z}{z_0}\right)$$