Galaxy formation and evolution
PAP 318, 5 op, autumn 2022

BK106, Exactum

- **Lecture 4: Jeans' instabilities and horizons in an expanding Universe – Additional notes, 28/09/2022**

Lecture 4 additional notes I

- Page 3: Static case -> Simple harmonic oscillator

$$\frac{d^2\Delta}{dt^2} = \Delta(4\pi G\rho_0 - k^2 c_s^2)$$

- Page 3: Solutions are of the form:

$$\Delta \propto e^{i(\vec{k}_c \cdot \vec{r} - \omega t)}$$

- Page 3: Dispersion relation:

$$(-\omega)^2 i^2 = 4\pi G\rho_0 - k^2 c_s^2$$

Lecture 4 additional notes II

- Page 6: Expanding medium

$$a = \left(\frac{3}{2}H_0 t\right)^{2/3}, \quad \dot{a} = \left(\frac{3}{2}H_0 t\right)^{2/3} \cdot \frac{2}{3} t^{-1/3}$$

$$4\pi G\rho = 4\pi G \cdot \frac{3H_0^2}{8\pi G} a^{-3} = \frac{4 \cdot 3H_0^2}{8} \cdot \left(\frac{3}{2}H_0 t\right)^{-2} = \frac{2}{3t^2}$$

- Page 9: Taylor series for cos θ and sin θ.

$$\cos\theta \approx 1 - \frac{1}{2}\theta^2 + \frac{1}{24}\theta^4$$

$$\sin\theta \approx \theta - \frac{1}{6}\theta^3 + \frac{1}{120}\theta^5$$

Lecture 4 additional notes III

- Page 12: The general solution

$$\dot{a}^2 = \frac{\Omega_0 H_0^2}{a} + \Omega_\Lambda a^2 H_0^2 - \kappa c^2$$

- Page 12 In ΛCDM κ=0, if Ω_m<1 -> κ=-1.
- Lambda starts to dominate:

$$z \approx \Omega_m^{-1/3} - 1 \approx 0.5, \quad \text{for } \Omega_\mathrm{m} = 0.3$$

- Page 14: Rotational velocities: $\frac{d\vec{u}_\perp}{dt} + 2\left(\frac{\dot{a}}{a}\right)\vec{u}_\perp = 0$
  Solution:

$$u_\perp \propto a^{-2}, \Rightarrow -2\dot{a}a^{-3} + 2\frac{\dot{a}}{a}a^{-2} = 0$$

Lecture 4 additional notes IV

- Page 16: Potential motions

$$\delta v_{\parallel} \propto a^{1/2}\Delta_0 \propto t^{1/3}, \text{ since } \mathrm{a} \propto \mathrm{t}^{2/3}$$

- Page 21: Horizons and the horizon problem: EdS model:

$$a = \left(\frac{3}{2}H_0 t\right)^{2/3}, \; \dot{a} = \left(\frac{3}{2}H_0 t\right)^{2/3} \cdot \frac{2}{3}t^{-1/3}, \; t = \frac{2}{3}H_0^{-1}a^{3/2}$$

$$r_H = a\int_0^t \frac{cdt}{a} = a\int_0^t \frac{ct^{-2/3}}{(\frac{3}{2}H_0)^{2/3}} = 3ct$$

Lecture 4 additional notes V

- Page 25: The gravitational potential on superhorizon scales:
- Matter era: $\Delta=\Delta_0 a$ and $L=L_0 a$

$$\delta\phi = L_0^2 a^2 \cdot 4\pi G a^{-3} \rho_0 \Delta_0 a = 4\pi G \rho_0 \Delta_0 L_0^2$$

- Radiation era: $\Delta=\Delta_0 a^2$ and $L=L_0 a$

$$\delta\phi = L_0^2 a^2 \cdot 8\pi G a^{-4} \rho_0 \Delta_0 a^2 = 8\pi G \rho_0 \Delta_0 L_0^2$$