Galaxy formation and evolution
PAP 318, 5 op, autumn 2022

BK106, Exactum

Lecture 3: Cosmology and the evolution of perturbations – Additional notes, 21/09/2022

Lecture 3 additional notes I

- Page 6: Hubble constant

$$l = a(t)\chi(r) \Rightarrow \dot{l} = \dot{a}(t)\chi(r) + a(t)\dot{\chi}(r) = \dot{a}(t)\chi(r)$$

- Page 7: Redshift
  Often $a(t_0)$, i.e the scale factor at the present-time is set to $a(t_0)=1$.

- Page 10: Luminosity distance

$$F = \frac{\omega L a_e^2}{4\pi(a_0 r_e)^2 \omega a_0^2} = \frac{L a_0^2}{4\pi(a_0 r_e)^2 (1+z)^2 a_0^2}$$

Lecture 3 additional notes II

- Page 12-15: General relativity and the Friedmann equations
  The metric tensor $g_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$ $\mathrm{g}_{\mu\nu}\mathrm{g}^{\mu\nu} = 4$

- Page 12-15: Further details on how the derivation is done see for example: http://diposit.ub.edu/dspace/bitstream/2445/59759/1/TFG-Arnau-Romeu-Joan.pdf

- Page 20: Age of the Universe:

$$\frac{\dot{a}}{a} = H_0 E(z) \Rightarrow dt = \frac{da}{aH_0 E(z)}$$

$$a = \frac{1}{1+z} \Rightarrow da = \frac{-1}{(1+z)^2} dz \Rightarrow dt = -\frac{dz}{H_0(1+z)E(z)}$$

Lecture 3 additional notes III

- Page 17: For a closed model K=+1 the solution for the Friedmann equation can be found in a parametric form:

$$\frac{a}{a_0} = \frac{1}{2}\frac{\Omega_{m,0}}{(\Omega_{m,0}-1)}(1-\cos\vartheta); \qquad H_0 t = \frac{1}{2}\frac{\Omega_{m,0}}{(\Omega_{m,0}-1)^{3/2}}(\vartheta - \sin\vartheta)$$

- The angle $0 \leq \vartheta \leq 2\pi$. This model reaches its maximum size a_{max} at $t=t_{max}$ when $\vartheta = \pi$, after which it collapses:

$$\frac{a_{\max}}{a_0} = \frac{\Omega_{m,0}}{\Omega_{m,0}-1}; \quad H_0 t_{\max} = \frac{\pi}{2}\frac{\Omega_{m,0}}{(\Omega_{m,0}-1)^{3/2}}$$

- At early times $a \propto t^{2/3}$, because the curvature term K is then still small.

Lecture 3 additional notes IV

- Page 17: For an open model K=-1 the solution for the Friedmann equation can also be found in a parametric form:

$$\frac{a}{a_0} = \frac{1}{2}\frac{\Omega_{m,0}}{(1-\Omega_{m,0})}(\cosh\vartheta - 1); \quad H_0 t = \frac{1}{2}\frac{\Omega_{m,0}}{(1-\Omega_{m,0})^{3/2}}(\sinh\vartheta - \vartheta)$$

- The angle $0 \leq \vartheta \leq \infty$. At early times $a \propto t^{2/3}$, because the curvature term K is again now small.
- At late times: $\vartheta >> 1 \;\; \sinh\vartheta = \cosh\vartheta \quad a \propto t$
- The Universe expands freely and forever.

Lecture 3 additional notes V

- Page 21: Distances in the Universe

$$\tau(t) = \int_0^t \frac{cdt'}{a(t')} \qquad \chi(r) = \tau(t_0) - \tau(t) = c\int_{a(t)}^{a_0} \frac{da}{a\dot{a}}$$

$$\dot{a} = aH_0E(z) \Rightarrow \chi(r) = c\int_0^z -\frac{dz}{(1+z)^2}\frac{1}{a^2H_0E(z)}$$

- Page 25-29: For further details and intermediate steps, see for example: pages 1-5 https://www.uio.no/studier/emner/matnat/astro/AST4320/h14/beskjeder/combinednotes.pdf