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GALAXY FORMATION AND EVOLUTION

PAP 318, 5 op, autumn 2022
B119, Exactum

Lecture 11: Galaxy interactions and transformations

TODAY WE WILL COVER

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1. **Galaxy interactions and timescales**
2. **High-speed encounters and the impulse approximation.**
3. **Tidal stripping and the formation of tidal arms.**
4. **Dynamical friction: Intuitive picture and mathematical formulation.**
5. **Orbital decay: Theory and applications**

The lecture notes correspond to:

- **MBW: pages 544-561 (§12.1-12.3)**

11.1 INTERACTIONS

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- Consider a perturber *P* at a distance *R* from a body *S*, in an orbit with impact parameter *b*
- Consider a stellar particle *q* inside *S* at a distance *r* wrt *S* and *R-r* wrt to *P*
- The gravitational force per unit mass (aka gravitational field/acceleration) experienced by *q* will not be uniform due to *P*, hence

$$\mathbf{F}_{\rm tid}(\mathbf{r}) = -\nabla\Phi_{\rm P}(|\mathbf{R}-\mathbf{r}|) + \nabla\Phi_{\rm P}(\mathbf{R})$$

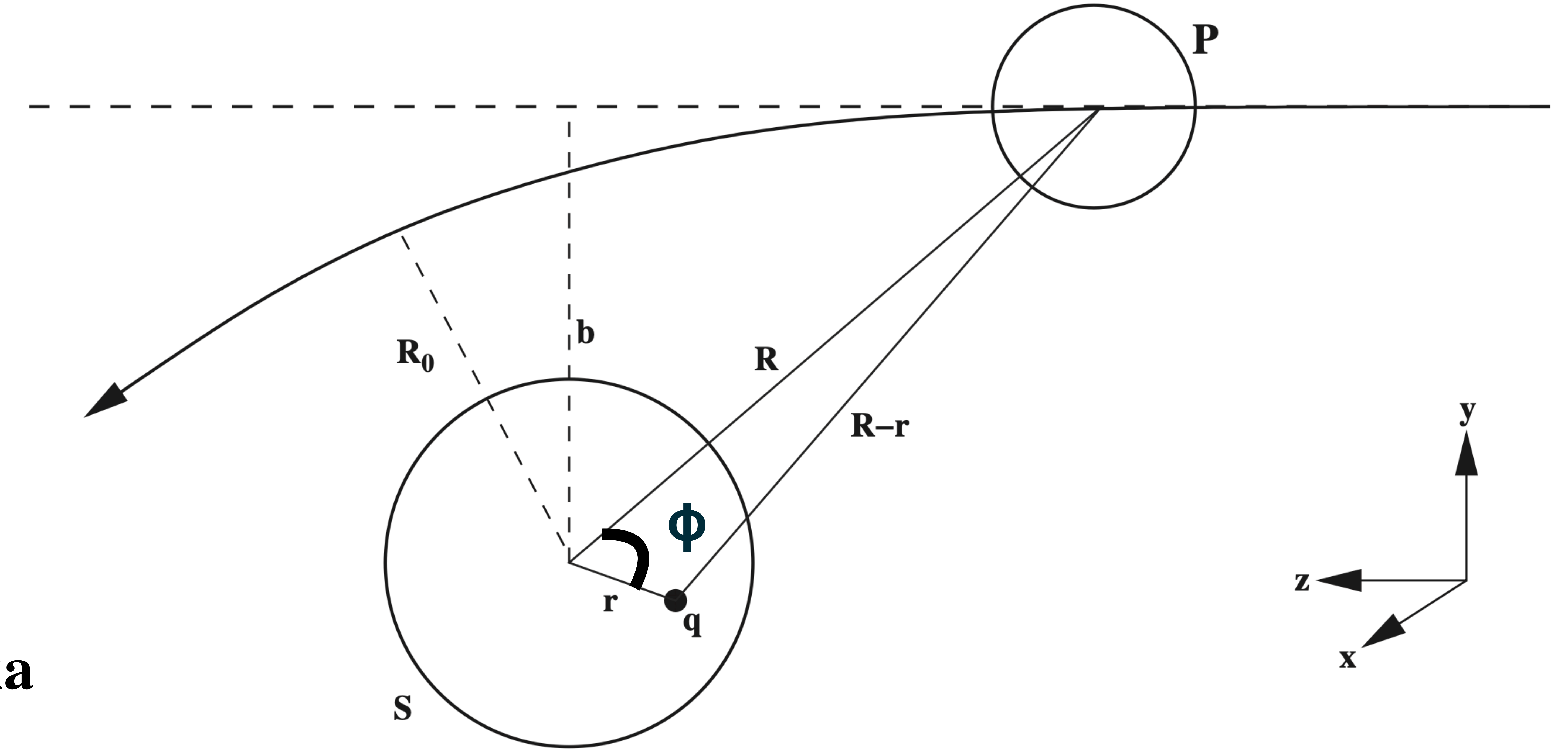

Fig. 12.1, MBW

11.1.1 TIMESCALES

- The interaction between S and P causes tidal distortions, which in turn may cause a back-reaction on their orbit
- The tidal interaction timescale can be defined as $t_{tide} \sim R_{sys}/\sigma$, where R_{sys} and σ are the size and velocity dispersion of the system that experiences the tides.
- The encounter timescale can be defined as $t_{enc} \sim R/V$, where the radius $R = \max[R_0, R_S, R_P]$. R_0 is the minimum distance of the encounter and R_S and R_P are the characteristic radii of S and P. V is the encounter velocity at the minimum distance, $R = R_0$.

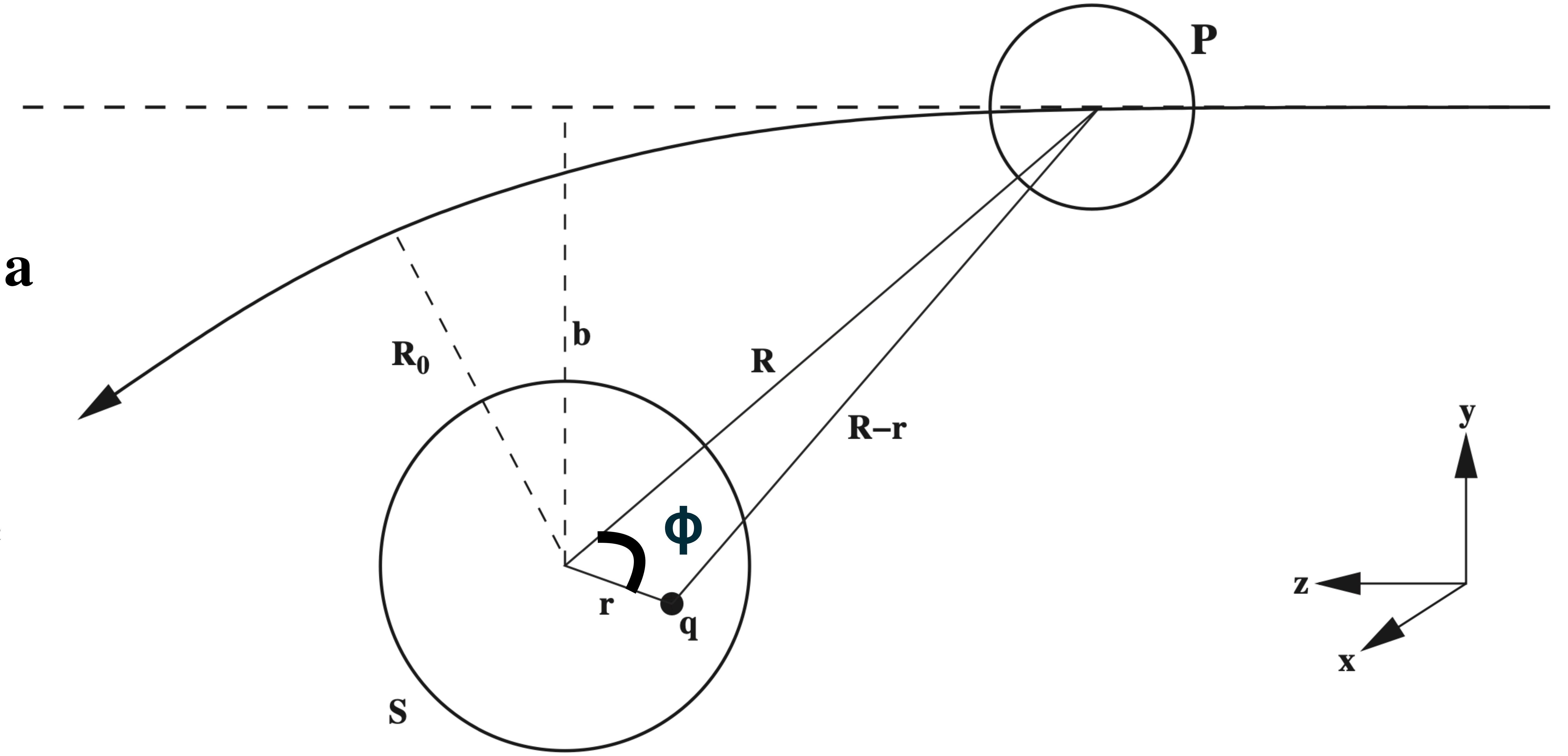

Fig. 12.1, MBW

11.1.2 TIMESCALES

- **If $t_{enc} >> t_{tide}$ we are in the adiabatic limit, i.e. the system has sufficient time to respond to tidal deformations. Deformation during the approach and departure cancel each other and there is no net transfer of energy.**
- **However, $t_{enc} >> t_{tide}$ implies $V << (R_{max}/R_{sys})\sigma$, which is not possible since $R_{max} \geq R_{sys}$ by construction and $V \sim \sigma$, because the same gravitational field is responsible for both velocities.**
- **Thus, $t_{enc} >> t_{tide}$ never occurs for collisionless (i.e. no gas) galactic systems. Which means that an interaction between two collisionless systems will result in an increase of their internal energy.**

11.1.3 TIMESCALES

- **In actual cases $t_{enc}<t_{tide}$, the response of the system lags behind the instantaneous tidal force, causing a back reaction on the orbit.**
- **The net effect is the transfer of orbital energy to internal energy of both *S* and *P*, i.e. the velocity dispersions, σ of both *S* and *P* increases.**
- **Under certain conditions, if enough orbital energy is transferred, the two bodies can become gravitationally bound to each other. This process is called gravitational capture.**
- **If the transfer of orbital energy continues, the capture will ultimately result in the merger of the two objects.**
- **When the internal energy gain is large, some particles may however become unbound resulting in mass loss.**

11.2 HIGH-SPEED ENCOUNTERS

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- **In general numerical simulations are required to investigate the outcome of a gravitational encounter. However, in the special case of a high-speed encounter ($V \gg \sigma$) e.g. in cluster environments, the change in the internal energy can be obtained analytically using the impulse approximation.**

- **In the impulse approximation, we can assume that the particle q is stationary wrt the centre of S (i.e. its potential energy remains unchanged). Thus, during the encounter, q only experiences an increase of its specific energy due to a velocity change Δv**

$$\Delta E = \frac{1}{2}(\mathbf{v}+\Delta\mathbf{v})^2 - \frac{1}{2}\mathbf{v}^2 = \mathbf{v}\cdot\Delta\mathbf{v} + \frac{1}{2}|\Delta\mathbf{v}|^2$$

11.2.1 HIGH-SPEED ENCOUNTERS

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- **We are interested in calculating ΔE_S, obtained by integrating ΔE over the entire system S. Because of symmetry the first term in ΔE vanishes (the average of v is zero), we get**

$$\Delta E_S = \frac{1}{2}\int |\Delta \mathbf{v}(\mathbf{r})|^2 \rho(\mathbf{r})\, d^3\mathbf{r}$$

- **Under the large-v and distant-encounter approximations, the perturber P can be considered to be a point mass. Hence, the potential due to the perturber at r is**

$$\Phi_P(\mathbf{r}) = -\frac{GM_P}{|\mathbf{r}-\mathbf{R}|}$$

- ***where***

$$|\mathbf{r}-\mathbf{R}| = \sqrt{R^2 - 2rR\cos\phi + r^2}$$

11.2.2 HIGH-SPEED ENCOUNTERS

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- which can be expanded using the $(1+x)^{-1/2}$ Taylor series

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{15}{48}x^3 + \ldots$$

- Hence,

$$\Phi_P(\mathbf{r}) = -\frac{GM_P}{R} - \frac{GM_P r}{R^2}\cos\phi - \frac{GM_P r^2}{R^3}\left(\frac{3}{2}\cos^2\phi - \frac{1}{2}\right) + \mathscr{O}\left[(r/R)^3\right]$$

- where the first term is a constant term yielding no tidal force
- the second term describes how the centre of mass of *S* changes under a uniform acceleration
- the third term describes the tidal force per unit mass, which is proportional to $\sim R^{-3}$
- and we drop any terms with order higher than 3, following the tidal approximation

11.2.3 HIGH-SPEED ENCOUNTERS

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- **Calculating the gradient of the potential and dropping the second term (constant acceleration), we get the tidal force per unit mass**

$$\vec{F}_{\rm tid}(\vec{r}) = \nabla\Phi_P = d\vec{v}/dt$$

- **Integrating the tidal force over time gives the cumulative change in the velocity**

$$\Delta\mathbf{v} = \frac{2GM_{\rm P}}{v_p b^2}(-x, y, 0)$$

- **which once substituted into the expression of the specific energy change (slide 8) gives**

$$\Delta E_{\rm S} = \frac{1}{2}\int |\Delta\vec{u}|^2 \rho(r)\mathrm{d}^3\vec{r} = \frac{2G^2M_{\rm P}^2}{u_{\rm P}^2 b^4}\int \rho(r)(x^2+y^2)\mathrm{d}^3\vec{r} = \frac{2G^2M_{\rm P}^2}{u_{\rm P}^2 b^4} M_{\rm S} < x^2 + y^2 >$$

11.2.4 HIGH-SPEED ENCOUNTERS

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- **And by assuming spherical symmetry, i.e.**

$$< x^2 + y^2 >= \frac{2}{3} < x^2 + y^2 + z^2 >= \frac{2 < r^2 >}{3}$$

- **we get (in the impulse approximation regime)**

$$\Delta E_S = \frac{4}{3} G^2 M_S \left(\frac{M_P}{v_P} \right)^2 \frac{\langle r^2 \rangle}{b^4}$$

- **The change in energy scales as $\propto b^{-4}$, which means that closer encounters have a much larger impact. If ΔE_S exceeds the binding energy of *S*, the system will be tidally disrupted.**
- **The above formula is accurate even for relatively slow encounters with $v_P \sim \sigma_S$ as long as the impact parameter $b \geq 5 \max(R_P, R_S)$. For smaller impact parameters one needs to account for the detailed internal mass distribution of *P*.**

11.3 IMPULSIVE HEATING

- In all of the above we followed the impulse approximation which assumes that the encounter only changes the kinetic energy of S and leaves the potential energy constant.
- After the encounter, S is no longer in virial equilibrium and consequently it must undergo relaxation to re-establish virial equilibrium. If K_S is the original pre-encounter kinetic energy, we have the following
  - Virial equilibrium: $E_S=-K_S$.
  - After encounter: $E_S \rightarrow E_S+\Delta E_S$.
  - All new energy is kinetic: $K_S \rightarrow K_S+\Delta E_S$.
  - After relaxation: $K_S=-(E_S+\Delta E_S)=-E_S-\Delta E_S$
- Hence, relaxation decreases the kinetic energy by $2\Delta E_S$

11.3.1 IMPULSIVE HEATING

- The $2\Delta E_S$ energy decrease is transferred from kinetic to potential energy, which becomes less negative. Hence, during tidal shocks (i.e. encounters for which both the tidal and impulsive approximation are valid) the system expands and becomes less bound
- A prime example of galaxies being systems with negative heat capacity. Adding energy/heat to them results in (kinematically) colder systems

11.4 TIDAL STRIPPING

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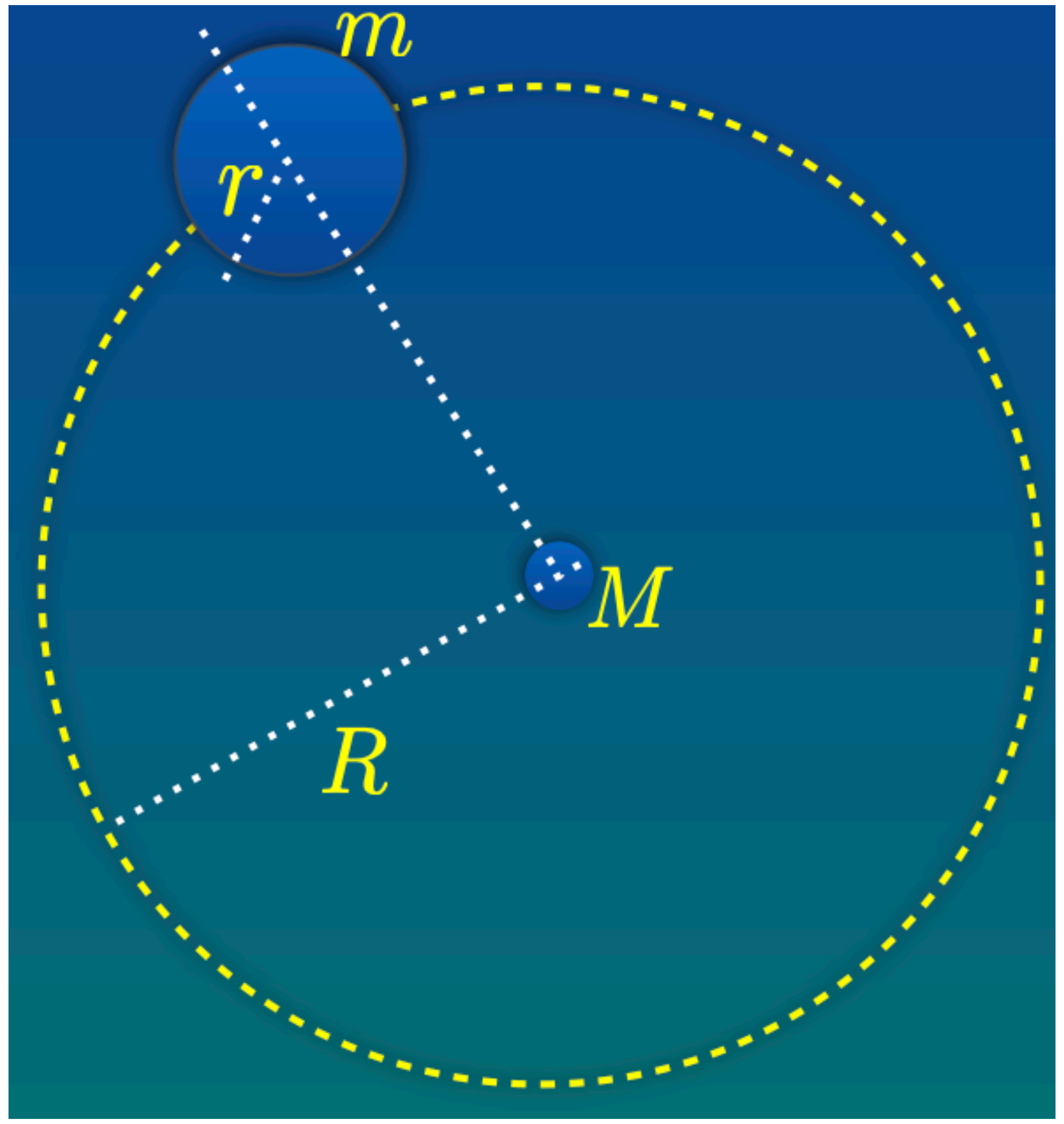
- **The expansion of a systems due to tidal shocks can lead to mass loss. But even in the general non-impulsive case, tidal forces can strip matter (=tidal stripping) from the outskirts of a system**

- **Consider a mass *m*, with radius *r*, orbiting a point mass *M* on a circular orbit of radius *R*.**

- **Calculating the tidal gravitational acceleration at the edge of *m* closest to the central mass *M***

$$\vec{g}_{\rm tid}(r) = \frac{GM}{R^2} - \frac{GM}{(R-r)^2} \simeq \frac{2GMr}{R^3} \qquad (r << R)$$



Credit: vdB

11.4.1 TIDAL STRIPPING

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- **If the tidal acceleration exceeds the binding force per unit mass (*Gm*/*r*²), the material at distance *r* from the centre of *m* will be stripped. This defines the tidal radius**

$$r_t = \left(\frac{m}{2M}\right)^{1/3} R$$

- **Taking into account the centrifugal force associated with the circular motion results in a more accurate tidal radius**

$$r_t = \left(\frac{m/M}{3 + m/M}\right)^{1/3} R$$

11.4.2 TIDAL STRIPPING

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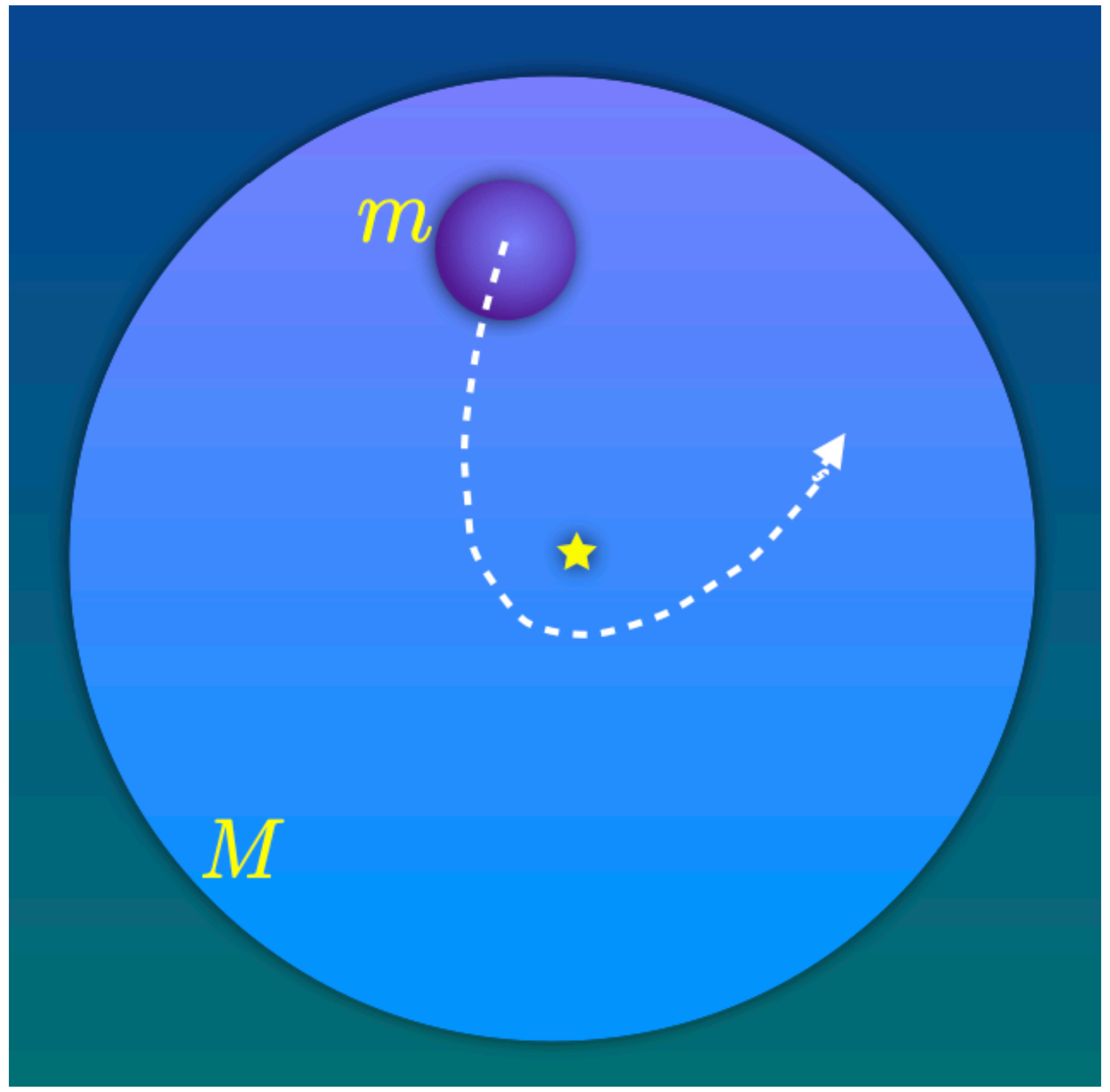
- **In a more *realistic* case the object *m* is on an eccentric orbit within an extended mass *M*. The tidal "radius" in this case is**

$$r_t = \left[\frac{m(r_t)/M(R_0)}{2 + \frac{\Omega^2 R_0^3}{GM(R_0)} - \frac{d\ln M}{d\ln R}\Big|_{R_0}} \right]^{1/3} R_0$$



Credit: vdB

11.5 DYNAMICAL FRICTION

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- **When a massive object *M* moves through a medium of collisionless particles, it experiences a drag force, called dynamical friction.**
- **Dynamical friction transfers the orbital energy of orbiting satellite galaxies and dark matter subhaloes to the dark matter particles and stars that make up the host halo, causing the satellite (subhalo) to "sink" to the centre of the potential well, where it can ultimately merge with the central galaxy.**

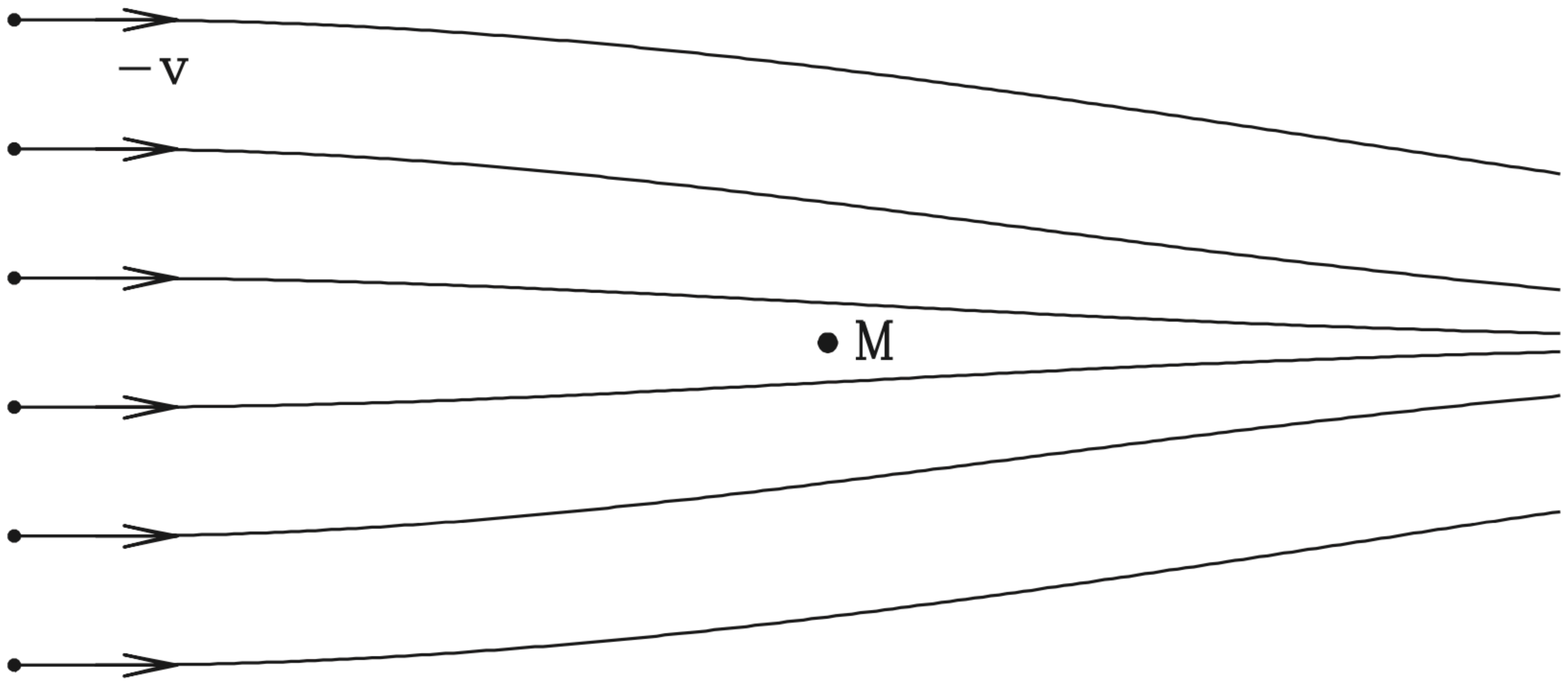

Fig. 12.3, MBW

11.5.1 DYNAMICAL FRICTION: EQUIPARTITION

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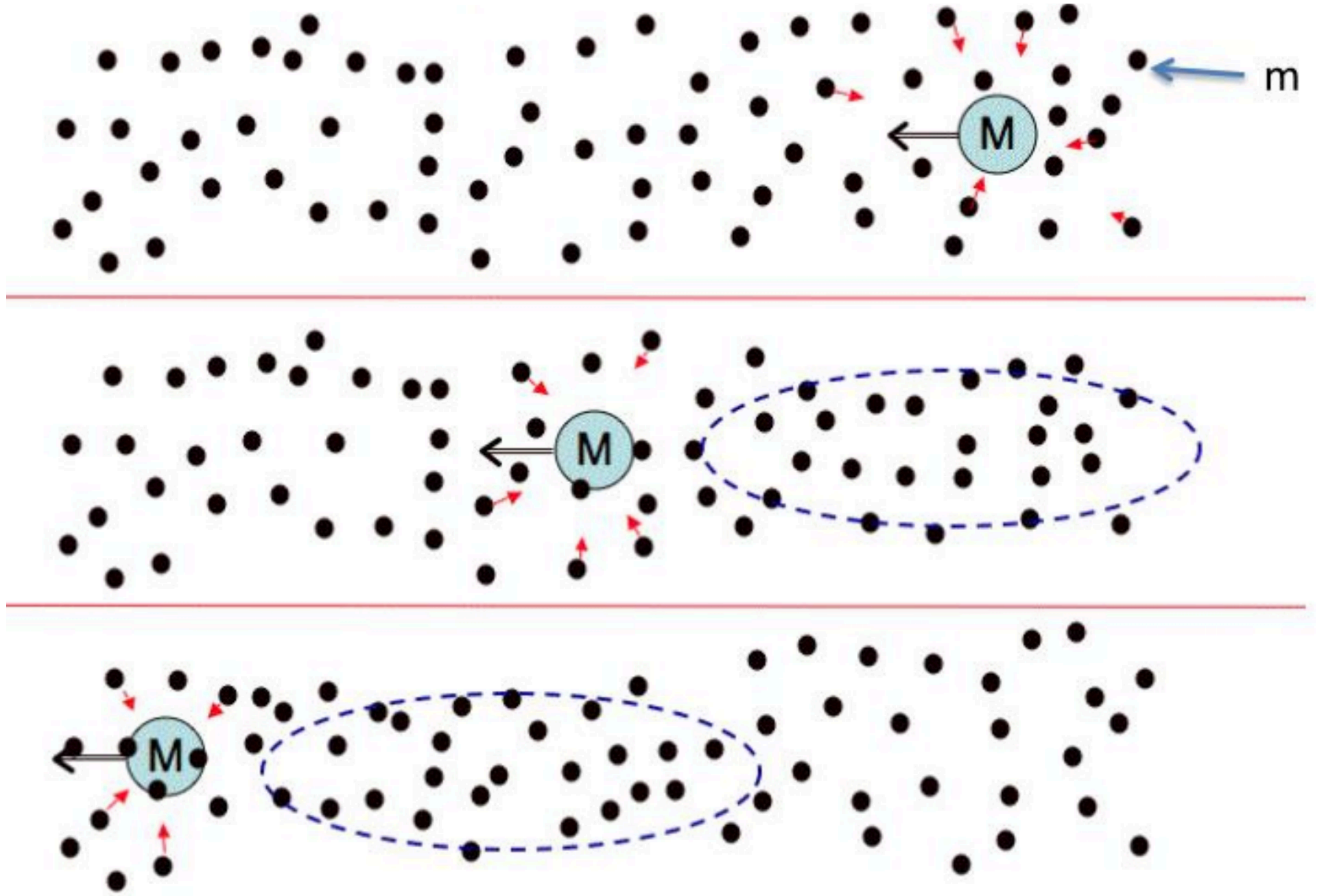
- **Two-body interactions move a system towards equipartition (i.e. the mean kinetic energy per particle is locally the same)**

$$m_1 \langle v_1^2 \rangle = m_2 \langle v_2^2 \rangle = m_3 \langle v_3^2 \rangle = \text{etc}$$

- **since in the previous slide we assumed $M >> m_i$ and we expect particles at the same radius to have similar orbital velocities, we can infer that M has much higher KE than a typical field particle m_i.**
- **Hence, M will tend to lose both energy and momentum to field particles**

11.5.2 DYNAMICAL FRICTION: GRAVITATIONAL WAKE

- **The moving subject mass perturbs the distribution of the field particles creating a trailing density enhancement ("wake"). The gravitational force of this wake on *M* slows it down.**



Credit: Alice Quillen

11.5.3 DYNAMICAL FRICTION: DERIVATION

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- **The analytical expression for the dynamical friction force is**

$$\vec{F}_{\rm df} = M_S \frac{d\vec{v}_S}{dt} = -4\pi \left(\frac{GM_S}{v_S}\right)^2 \ln\Lambda \rho(< v_S) \frac{\vec{v}_S}{v_S}$$

- **where $\varrho(<v_S)$ is the local density of the field particles with speeds less than v_S, and $\ln\Lambda$ is the Coulomb logarithm, which can be approximated as**

$$\ln\Lambda \approx \ln\left(\frac{b_{\rm max}}{b_{90}}\right) \quad b_{\rm max} \sim R \quad b_{90} \sim \frac{G(M_S + m)}{v_\infty^2}$$

- **The dynamical friction force is proportional to M_S^2 and independent of the individual masses of the field particles. For small velocities v_S, $F_{df} \propto v_s$ and for large velocities v_S, $F_{df} \propto v_s^{-2}$, unlike hydrodynamical friction which always increases with the velocity.**

11.5.4 DYNAMICAL FRICTION: DERIVATION

- Chandrasekhar's expression for the dynamical friction is based on the following three assumptions:
  1. The subject mass and the field particles are point masses.
  2. The self-gravity of the field particles can be ignored.
  3. The distribution of the field particles is infinite, homogenous and isotropic.
- Chandrasekhar's dynamical friction is considered as the sum of uncorrelated two-body interactions between a field particle and the subject mass. However, this ignores the collective effects due to self-gravity of the field particles.
- In reality dynamical friction is a global phenomenon, which is evident from the fact that the subject mass experiences dynamical friction even if it orbits beyond the outer edge of the finite host system. The proper way to treat dynamical friction would be instead using linear response theory.

11.5.5 DYNAMICAL FRICTION: ORBITAL DECAY

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- **We can use an idealised case to study the effects of dynamical friction. Consider a test particle on a circular orbit in a spherical, singular, isothermal halo consisting of field particles following a Maxwell velocity distribution with velocity dispersion $\sigma^2 = V^2_C/2$. Then the density profile is**

$$\rho(r) = V_c^2/(4\pi G r^2)$$

- **The DF force experienced by the test mass can be written as**

$$F_{\rm df} = -0.428 \frac{G M_S^2}{r^2} \ln \Lambda \frac{\vec{v}_S}{v_S}$$

- **since the test mass is on a circular orbit, we can calculate the rate at with it loses orbital angular momentum due to the DF force as**

$$\frac{dL_S}{dt} = r \frac{dv_S}{dt} = r \frac{F_{\rm df}}{M_S} = -0.428 \frac{G M_S}{r} \ln \Lambda$$

11.5.6 DYNAMICAL FRICTION: ORBITAL DECAY

- **Since we assumed that the circular velocity in the halo is constant (no dependence in *r*), the test mass will continue to orbit at V_C but its orbit will shrink at a rate**

$$v_S \frac{dr}{dt} = -0.428 \frac{GM_S}{r} \ln\Lambda \Rightarrow r\frac{dr}{dt} = -0.428 \frac{GM_S}{V_c} \ln\Lambda$$

- **which when integrated gives the time it will take the test mass's orbital radius to be zero**

$$t_{\rm df} = \frac{1.17}{\ln\Lambda} \frac{r_{\rm i}^2 V_{\rm c}}{GM_{\rm S}} = \frac{1.17}{\ln\Lambda} \left(\frac{r_{\rm i}}{r_{\rm h}}\right)^2 \left(\frac{M_{\rm h}}{M_{\rm S}}\right) \frac{r_{\rm h}}{V_{\rm c}}$$

11.5.7 DYNAMICAL FRICTION: ORBITAL DECAY

- **To get a feeling of how effective DF can be, we can compare the derived timescale with the Hubble time (i.e. an estimate of the age of the Universe).**
- **Assuming that $r_H/V_C \sim 1/[10H(z)]=0.1t_H$ and that $\ln\Lambda \sim \ln(M_h/M_S)$, we get**

$$t_{\rm df} \simeq 0.117 \frac{(M_h/M_s)}{\ln(M_h/M_s)} t_H$$

- **which implies that for $M_h/M_s>15$ dynamical friction will take longer than the age of the Universe to decay a circular orbit from the edge to the centre of the assumed halo**

11.5.8 DYNAMICAL FRICTION: MASS LOSS

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- **In the previous slides we assumed that the mass of the test particle is constant. However, if we generalise the example and don't consider the test particle to be point mass, the tidal forces from the host system will remove some of its mass. This will affect the dynamical friction time we derived.**

- **Rough analytical estimates indicate that the new dynamical friction time will be**

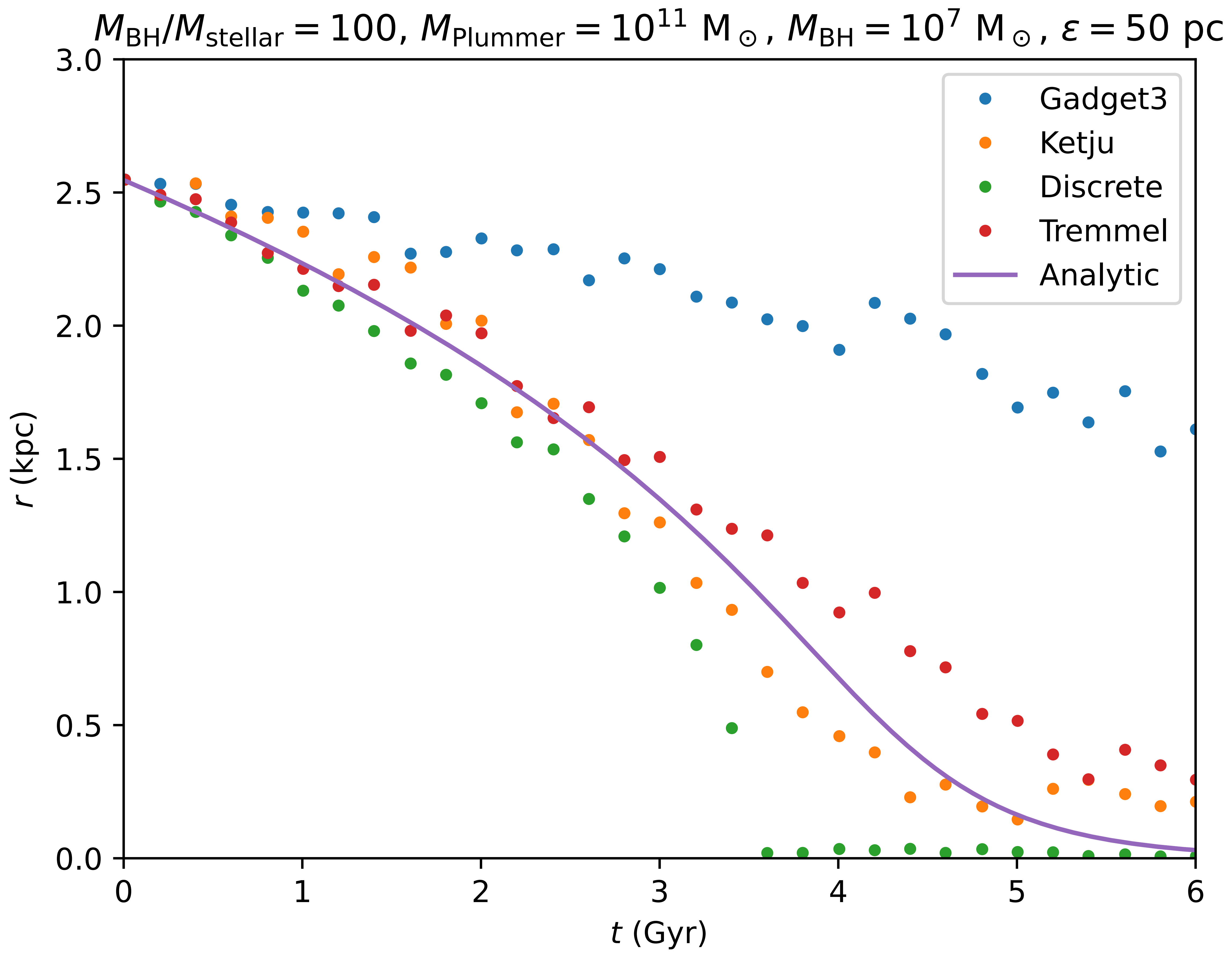
$$\tilde{t}_{\rm df} = \frac{3.30}{\ln\Lambda}\left(\frac{r_{\rm i}}{r_{\rm h}}\right)\left(\frac{V_{\rm h}}{V_{\rm S}}\right)^3\frac{r_{\rm h}}{V_{\rm h}}$$

- **which is ~2.8 larger than t_{df}**

- **Accurate modelling of tidal stripping, tidal heating and dynamical friction is important for predicting the disruption and merger rates of satellite galaxies.**

11.5.9 DYNAMICAL FRICTION: APPLICATIONS

- **Runs with five different prescriptions: GADGET (1511.00695), KETJU, discrete (2208.12275), Tremmel+15 (1501.07609), analytic from Chandrasekhar (1943)**
- **Orbital decay of an SMBH of mass 10^7 solar masses and with stellar particles of 10^5 solar masses**



Credit: Atte Keitaanranta

WHAT HAVE WE LEARNT?

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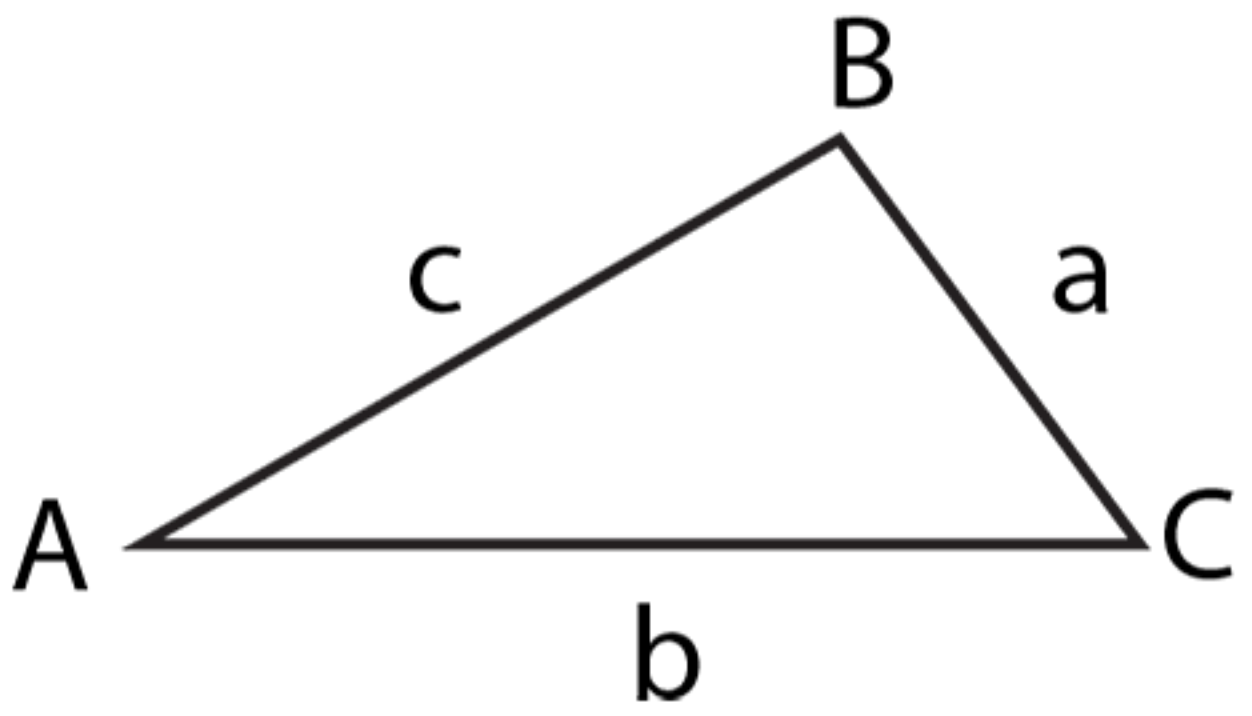
- In actual interactions the encounter timescale is always shorter than the tidal timescale: The response of the system lags behind the instantaneous tidal force, causing a back reaction on the orbit.
- In high-speed encounters the impulse approximation can be used as the particle only experiences a velocity change Δv, but its potential energy remains unchanged.
- Tidal forces are proportional to R^{-3} and if the tidal acceleration exceeds the binding force per unit mass (Gm/r^2), the material at distance r from the centre of m will be stripped.
- The dynamical friction force is proportional to M^2 and is most effective for slowly moving objects ($\propto v$), for large velocities the effect weakens as ($\propto v^{-2}$).

EXTRA SLIDES

- **Law of cosines: relation between lengths and angles of triangles (generalised Pythagorean theorem)**

$$|\vec{r} - \vec{R}| = \sqrt{R^2 - 2rR\cos\phi + r^2}$$

$$a^2=b^2+c^2 - 2bc\cos A$$
$$b^2=a^2+c^2 - 2ac\cos B$$
$$c^2=a^2+b^2 - 2ab\cos C$$



Credit: Calcworkshop.com

EXTRA SLIDES

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- **Tidal disruption events: when an object (e.g. a star) gets too close to another object (e.g. a black hole) that gets pulled apart due to tidal forces**

Credit: ESO, https://www.eso.org/public/videos/eso2018b/