



UNIVERSITY OF HELSINKI
FACULTY OF SCIENCE

GALAXY FORMATION AND EVOLUTION

**PAP 318, 5 op, autumn 2022
B119, Exactum**

Lecture 10: Formation of disc galaxies



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TODAY WE WILL COVER

- 1. Basic observational results of disc galaxies.**
- 2. Scaling relations and the Tully-Fisher relation of disc galaxies.**
- 3. Formation of disc galaxies. Basic processes.**
- 4. Transport of angular momentum and the angular momentum problem.**
- 5. The standard model of disc galaxy formation. Adiabatic contraction.**

The lecture notes correspond to:

- MBW: pages 495-520 (§11.1-11.4)**



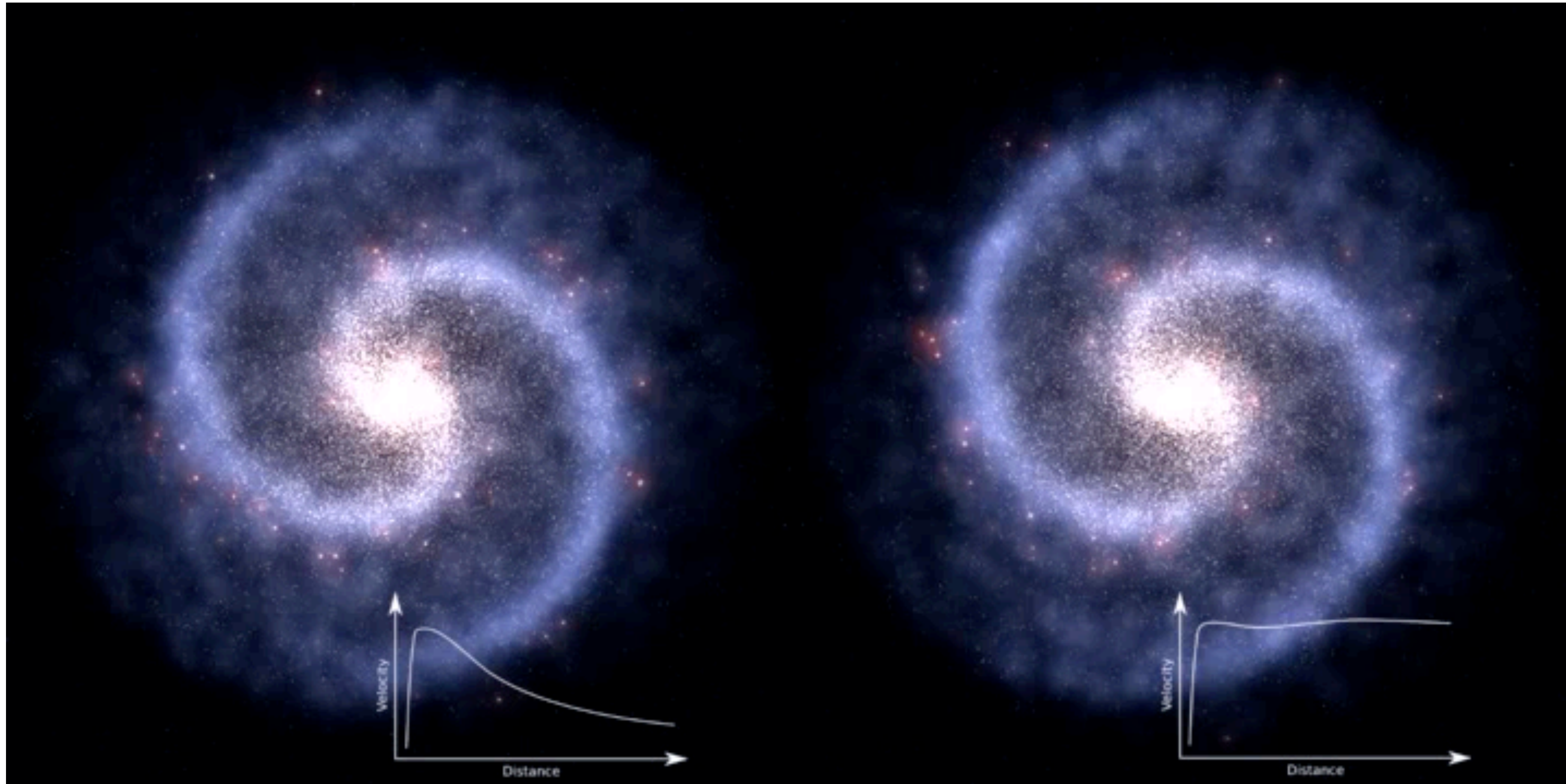
10.1 OBSERVATIONS

- **A disc galaxy (usually) consists of a flat/disky component made of stars and gas (atomic+molecular), spiral arms and (in 2/3 of the local spirals) a central bar**
- **Successful models of galaxy formation should be able to reproduce the observed scaling relations**
 - **Disc galaxies have flat rotation curves.**
 - **The surface brightness profiles of disc galaxies are close to exponential.**
 - **Brighter discs are, on average, larger, redder, rotate faster, and have a smaller gas mass fraction.**
 - **The outer parts of discs are generally bluer, and have lower metallicity than the inner parts**



NGC 5457, HST

10.1.1 OBSERVATIONS: ROTATION CURVES



https://upload.wikimedia.org/wikipedia/commons/3/33/Galaxy_rotation_under_the_influence_of_dark_matter.ogv



10.1.2 OBSERVATIONS: ROTATION CURVES

- Disc galaxies have flat rotation curves, which provides very strong evidence for dark matter.

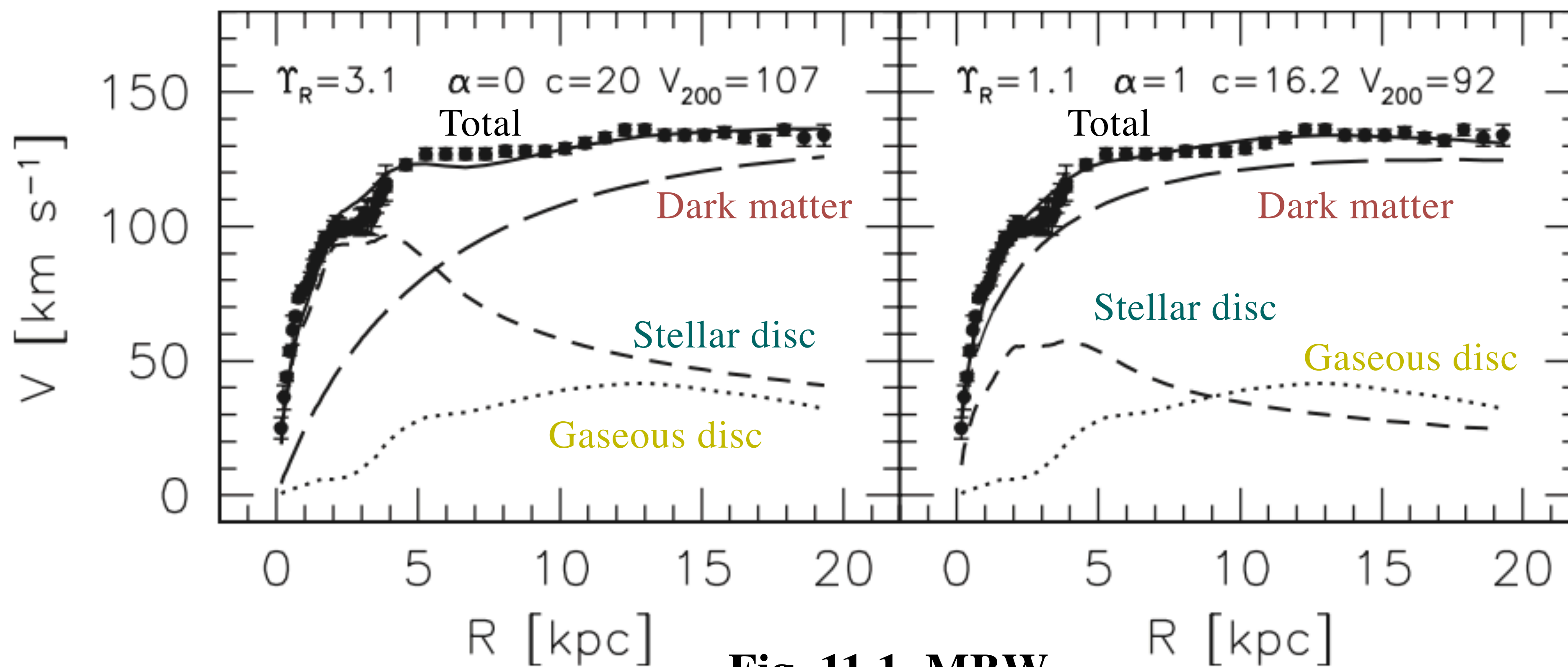


Fig. 11.1, MBW

10.1.3 OBSERVATIONS: SURFACE BRIGHTNESS

- Surface brightness profiles of disc galaxies are close to exponential.
- Deviations from the exponential at small radii are attributed to the bulge and/or bar component, and at large radii to density thresholds for star formation, radial migration and/or the maximum angular momentum of the in-falling gas

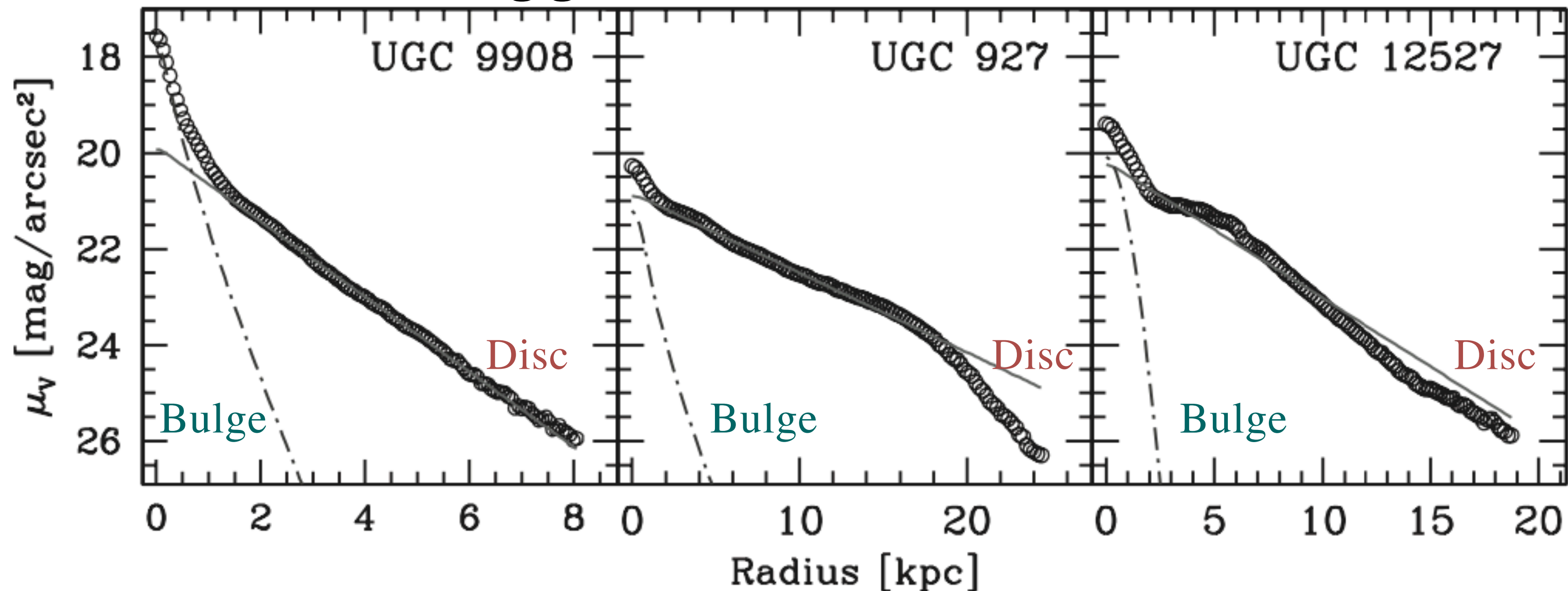


Fig. 2.19, MBW

10.1.4 OBSERVATIONS: RELATIONS

- **Brighter discs are larger and redder, have higher central surface brightness values, have smaller gas fractions.**

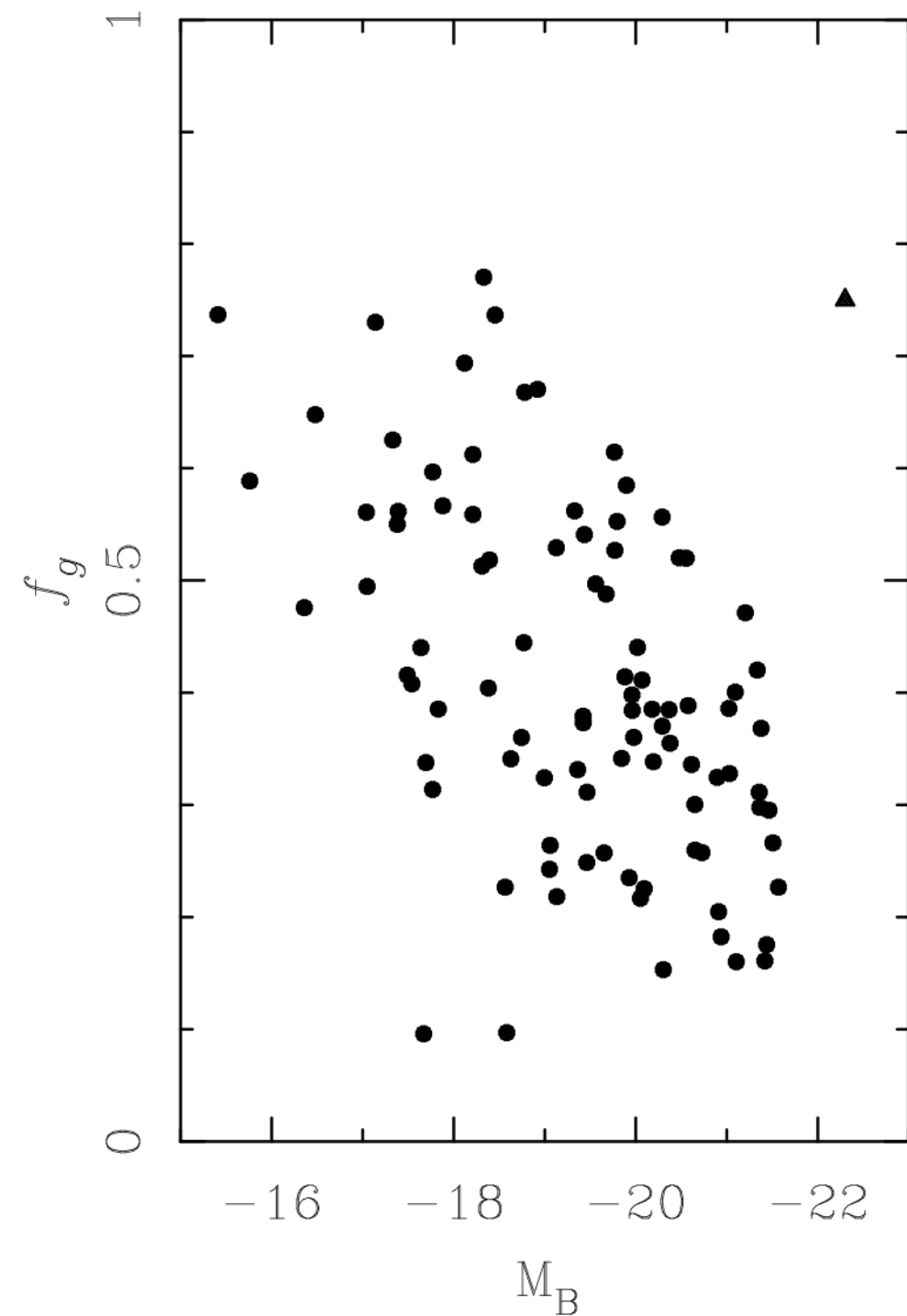


Fig. 7, 9612070

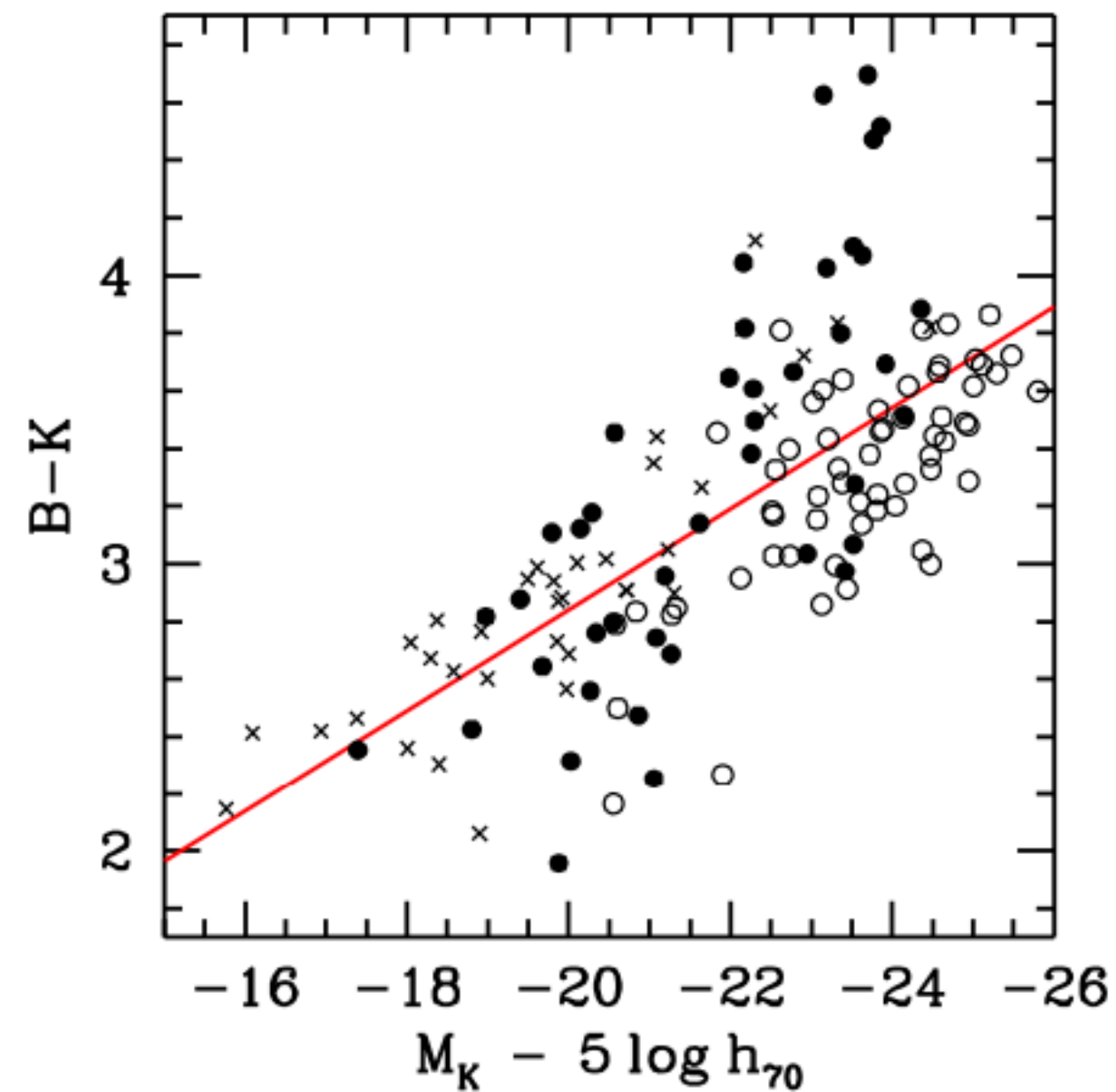


Fig. 11, 9912004

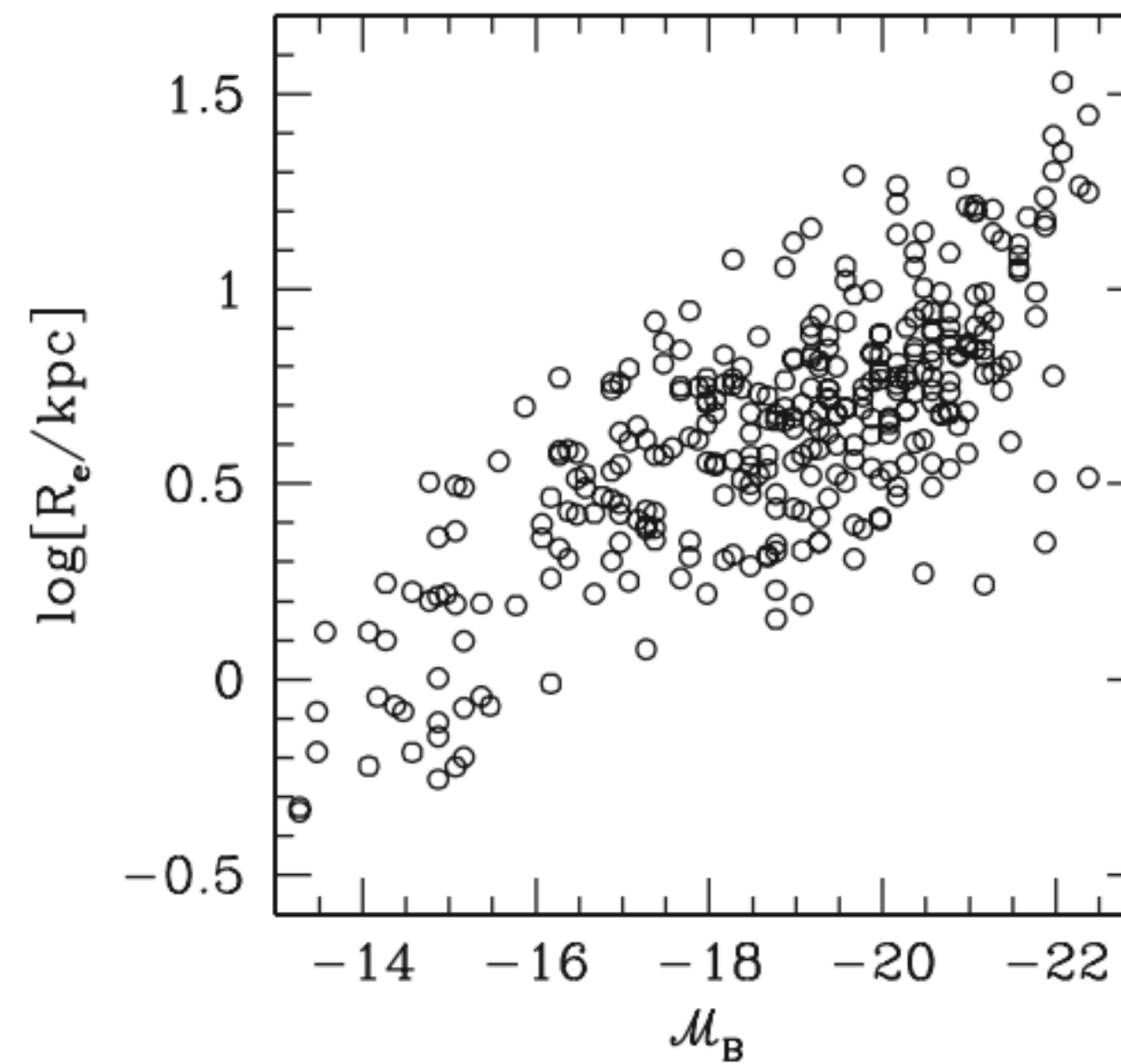
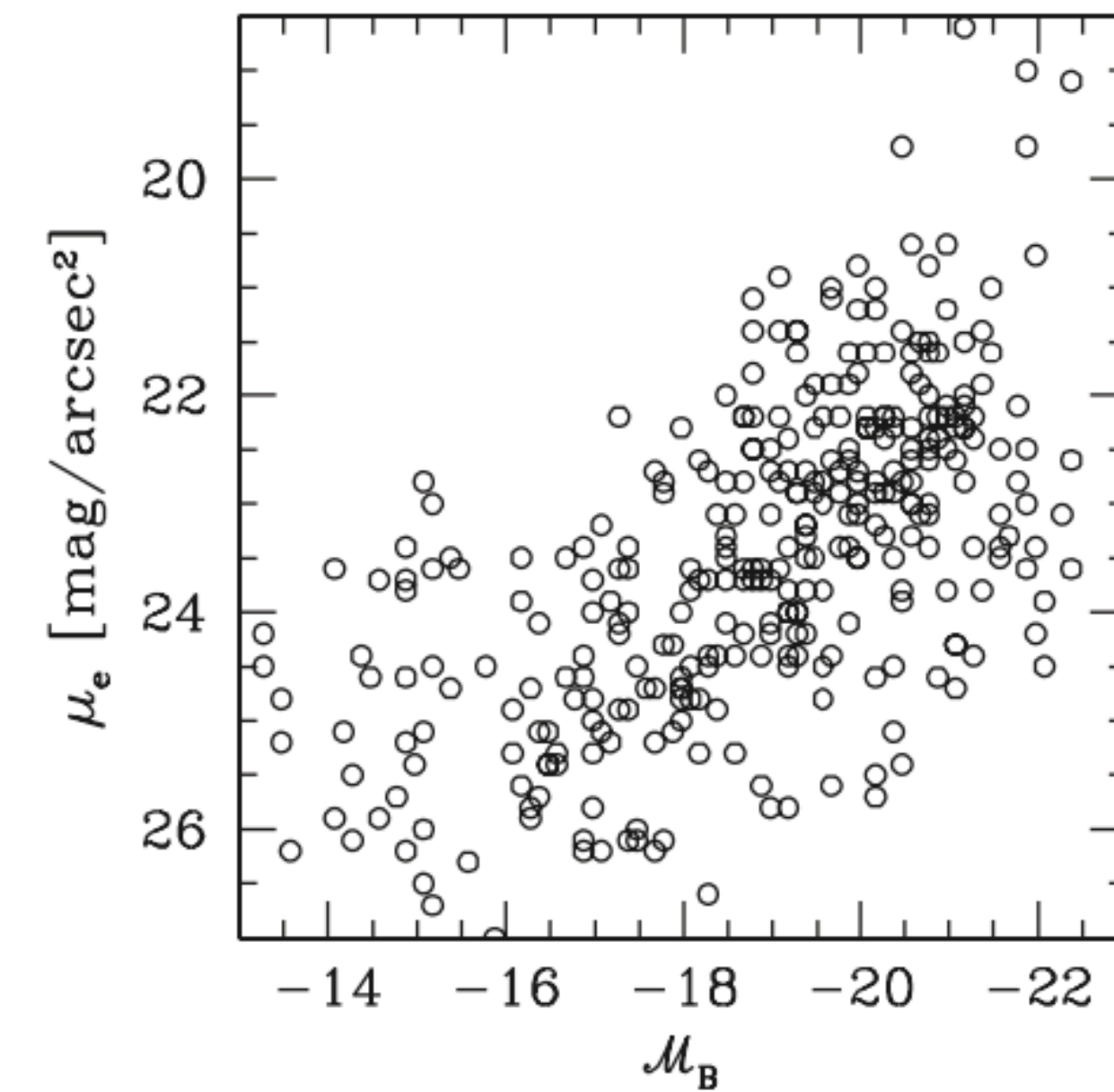


Fig. 2.20, MBW





10.1.5 OBSERVATIONS: TULLY-FISHER

- Brighter discs also rotate faster as manifested in the Tully-Fisher relation

$$V \propto L_I^{0.30}$$

- Theoretically and numerically it is very challenging to reproduce the TF zero point.

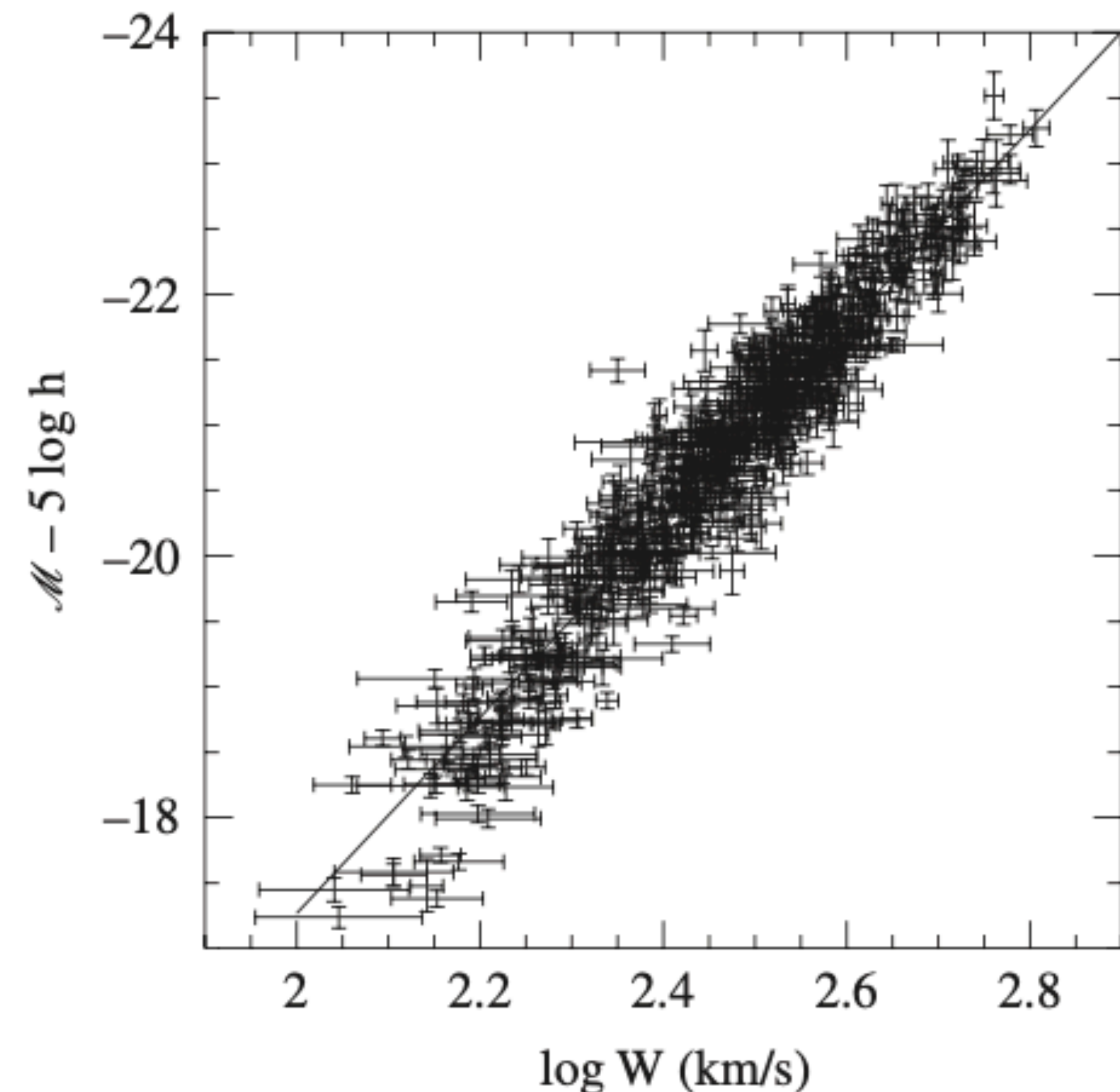
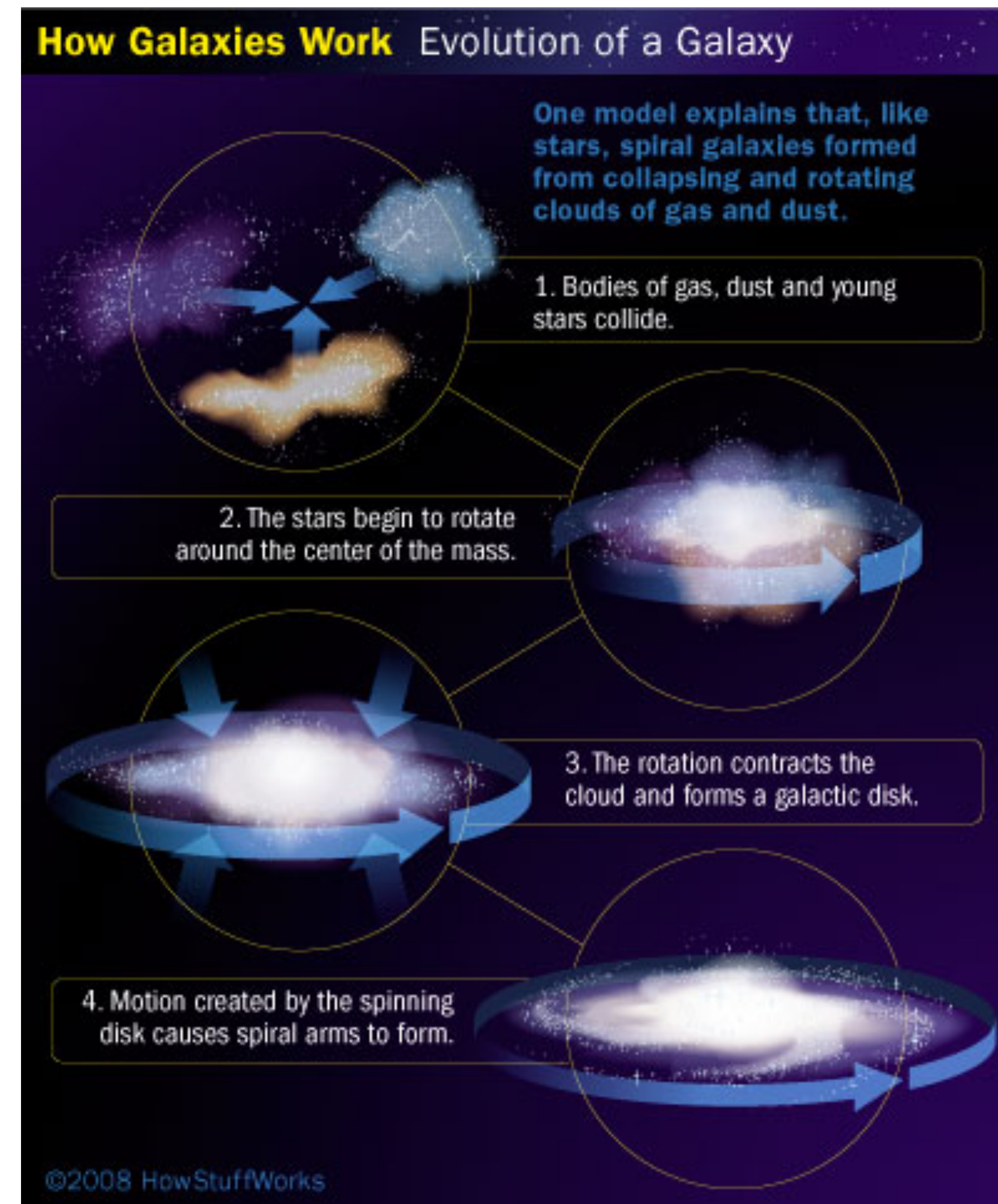


Fig. 2.22, MBW



10.2 FORMATION OF DISCS

- Hot, shock-heated gas (with non-zero *angular momentum*) inside a dark matter halo cools radiatively.
- As gas cools, its pressure decreases causing the gas to contract.
- Assuming that the emission of photons is isotropic, the angular momentum during cooling is conserved.
- The gas sphere contracts, spins up and flattens.
- The surface density of the disc increases until it reaches the critical threshold for star formation.
- A disc galaxy composed of stars and gas forms.





10.2.1 BASIC PROCESSES

- For a given angular momentum J , the state of lowest energy and hence the state preferred by nature, is the one in which all mass except an infinitesimal fraction δM collapses into a point mass, while δM is on a Keplerian orbit with

$$J = \delta M (GM R)^{1/2}$$

- This is clearly very different from realistic exponential discs.
- The reason for this discrepancy is that although the lowest energy state is preferred, its realisation requires efficient transport of angular momentum from inside out.
- Several mechanisms can transport angular momentum:
 - Secular evolution (gas viscosity or resonant scattering of stars&gas).
 - Hierarchical formation (dynamical friction & ram pressure).



10.2.2 BASIC PROCESSES

- **In the standard picture of disc galaxy formation developed by Fall & Efstathiou (1980) disc galaxies are in centrifugal equilibrium and therefore their structure is governed by their specific angular momentum distribution.**
- **The main assumptions in this standard picture are:**
 - **The angular momentum originates from cosmological *torques* between neighbouring dark matter haloes.**
 - **Baryons and dark matter acquire initially identical specific angular momentum distributions.**
 - **Baryons conserve their specific angular momentum while cooling and settling in the dark matter haloes.**



10.2.3 ANGULAR MOMENTUM PROBLEM

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- The fact that galaxies deviate from the minimum energy state indicates that angular momentum transport is not that efficient.
- However, in numerical simulations gas cooling causes much of the gas to condense into dense clumps.
- This clumpy gas is delivered to the disc via dynamical friction transferring orbital angular momentum from gas to dark matter -> simulated discs end-up too small.
- This problem is known as the “angular momentum catastrophe” and strong supernova feedback is required to prevent overcooling and loss of angular momentum.

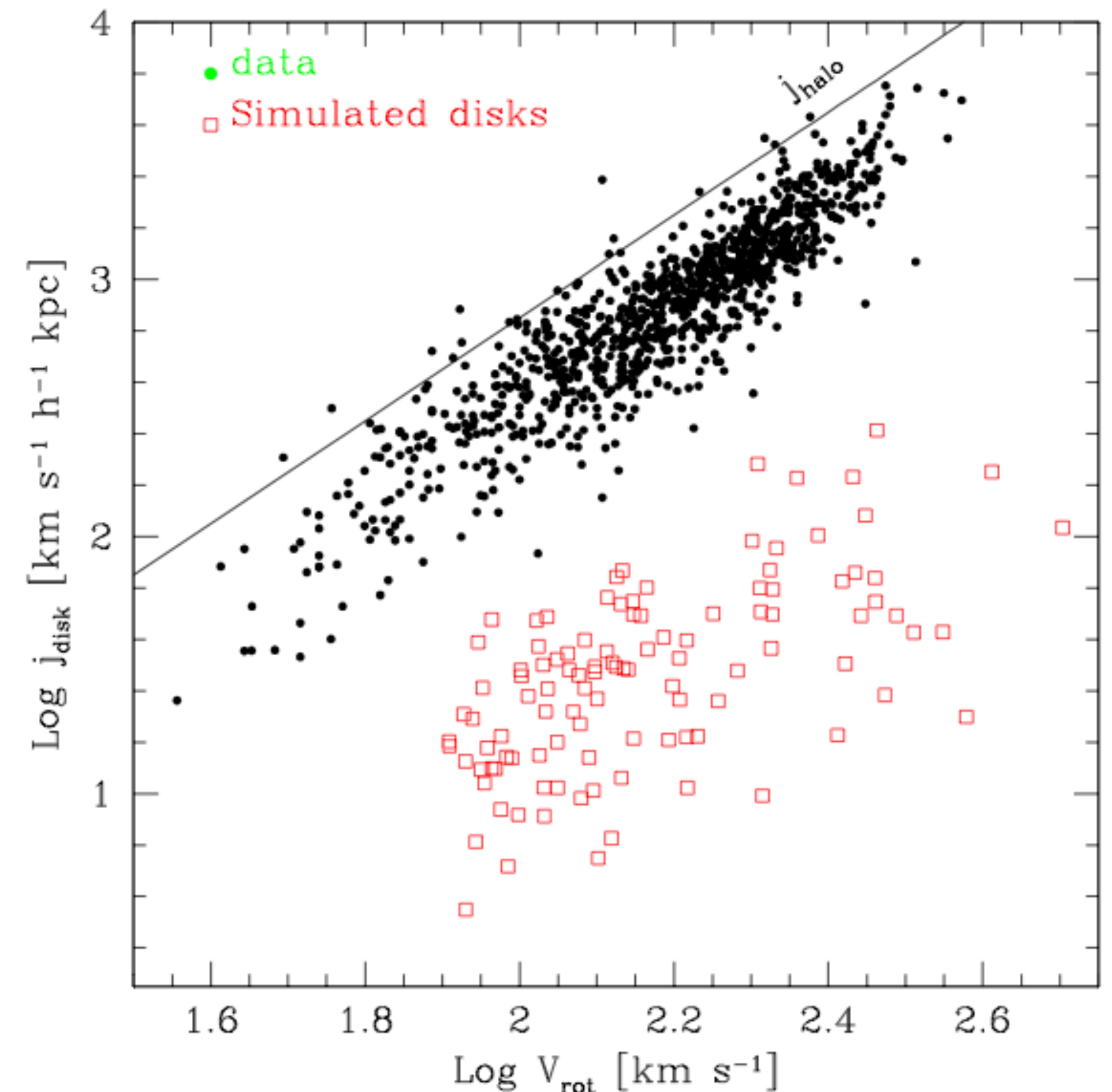


Fig. 4, 9808076



10.3 DISC FORMATION: MASS

- **As an idealised approximation, we can ignore the self-gravity of the disc and assume that the dark matter is a singular, isothermal sphere**

$$\rho(r) = V_{\text{vir}}^2 / (4\pi Gr^2)$$

- **Next we assume that a mass fraction m_d of the total virial mass settles in the disc. The total virial mass can be calculated from the virial velocity together with a cosmological factor that accounts for the evolution of the virial density with redshift**

$$M_d = m_d M_{\text{vir}} \simeq 1.3 \times 10^{11} h^{-1} M_{\odot} \left(\frac{m_d}{0.05} \right) \left(\frac{v_{\text{vir}}}{200 \text{ km/s}} \right)^3 D^{-1}(z)$$

$$D(z) = \left[\frac{\Delta_{\text{vir}}(z)}{100} \right]^{1/2} \left[\frac{H(z)}{H_0} \right]$$



10.3.1 DISC FORMATION: ANGULAR MOMENTUM

- The disc angular momentum for an infinitesimally thin exponential disc

$$J_d = 2\pi \int_0^\infty \Sigma(R) V_c(R) R^2 dR = 2M_d R_d V_{\text{vir}}$$

- Now we can define the parameter j_d as $J_d = j_d J_{\text{vir}}$, where J_{vir} is the angular momentum of the dark matter halo.
- The disc angular momentum can be related to the dimensionless spin parameter

$$\lambda = \frac{J_{\text{vir}} |E|^{1/2}}{GM_{\text{vir}}^{5/2}} = \frac{1}{j_d} \frac{J_d |E|^{1/2}}{GM_{\text{vir}}^{5/2}}$$



10.3.2 DISC FORMATION: SCALE RADIUS

- Using the total energy and the virial theorem

$$E = -K = -\frac{M_{\text{vir}} V_{\text{vir}}^2}{2}$$

- Combining with

$$J_d = \sqrt{2} j_d \lambda M_{\text{vir}} R_{\text{vir}} V_{\text{vir}} \quad \& \quad J_d = 2 M_d R_d V_{\text{vir}}$$

- we can also derive a disc scale radius

$$R_d = \frac{1}{\sqrt{2}} \lambda \left(\frac{j_d}{m_d} \right) R_{\text{vir}}$$

- Combining with the cosmological expression for the virial radius

$$R_d \simeq 10 h^{-1} \text{ kpc} \left(\frac{j_d}{m_d} \right) \left(\frac{\lambda}{0.05} \right) \left(\frac{V_{\text{vir}}}{200 \text{ km/s}} \right) D^{-1}(z)$$

- Using the MW values: $V_{\text{vir}}=220$ km/s, $j_d=m_d$, $M_d=5 \times 10^{10} M_{\odot}$ and $R_d=3.5$ kpc we get $\lambda \sim 0.01$ $m_d \sim 0.01$



10.3.3 DISC FORMATION: SELF-GRAVITY

- Assuming that the disc has 1) self-gravity and 2) is situated in a dark matter halo with an NFW profile we have to do the following modifications

$$R_d = \frac{1}{\sqrt{2}} \lambda \left(\frac{j_d}{m_d} \right) R_{\text{vir}} F_R^{-1} F_E^{-1/2}$$

- F_E (Eq. 7.158) is defined as

$$E = - \frac{M_{\text{vir}} V_{\text{vir}}^2}{2} F_E$$

- and F_R as

$$F_R = \frac{1}{2} \int_0^{R_{\text{vir}}/R_d} u^2 e^{-u} \frac{V_c(uR_d)}{V_{\text{vir}}} du$$



10.3.4 DISC FORMATION: SELF-GRAVITY

- Due to self-gravity the circular velocity contains a contribution from the disc and from the halo

$$V_c^2(R) = V_{c,d}^2(R) + V_{c,h}^2(R) = V_{c,d}^2(R) + \frac{GM_{h,ac}(R)}{R}$$

- which now gives $\lambda \sim 0.05$ in better agreement with observations (Mo, Mao & White 1998)



10.3.5 DISC FORMATION: ADIABATIC CONTRACTION

- **Many theoretical models assume that $V_{\text{rot}} = V_{\text{vir}}$ or $V_{\text{rot}} = V_{\text{max}}$. However, $V_{\text{rot}} / V_{\text{vir}} \sim 1.4-1.8$ if the effect of self-gravity and adiabatic contraction are taken into account**
- **When baryons cool and concentrate in the centres of dark matter haloes, they modify the halo structure by their gravitational action.**
- **If the growth of the disc is gradual enough to change the potential of the system slowly compared to the dynamical time of the dark matter particles in the centre of the halo, the system adjusts itself adiabatically (reversible) and the final state is independent of the path taken.**
- **This can be used to calculate a modified mass profile and more accurately estimate the circular velocity curve of the halo**



10.3.6 DISC FORMATION: ADIABATIC CONTRACTION

- The three galaxies have same mass but different λ . Since smaller spin parameters result in more compact discs, $V_{\text{rot}} / V_{\text{vir}}$ is a strong function of λ !

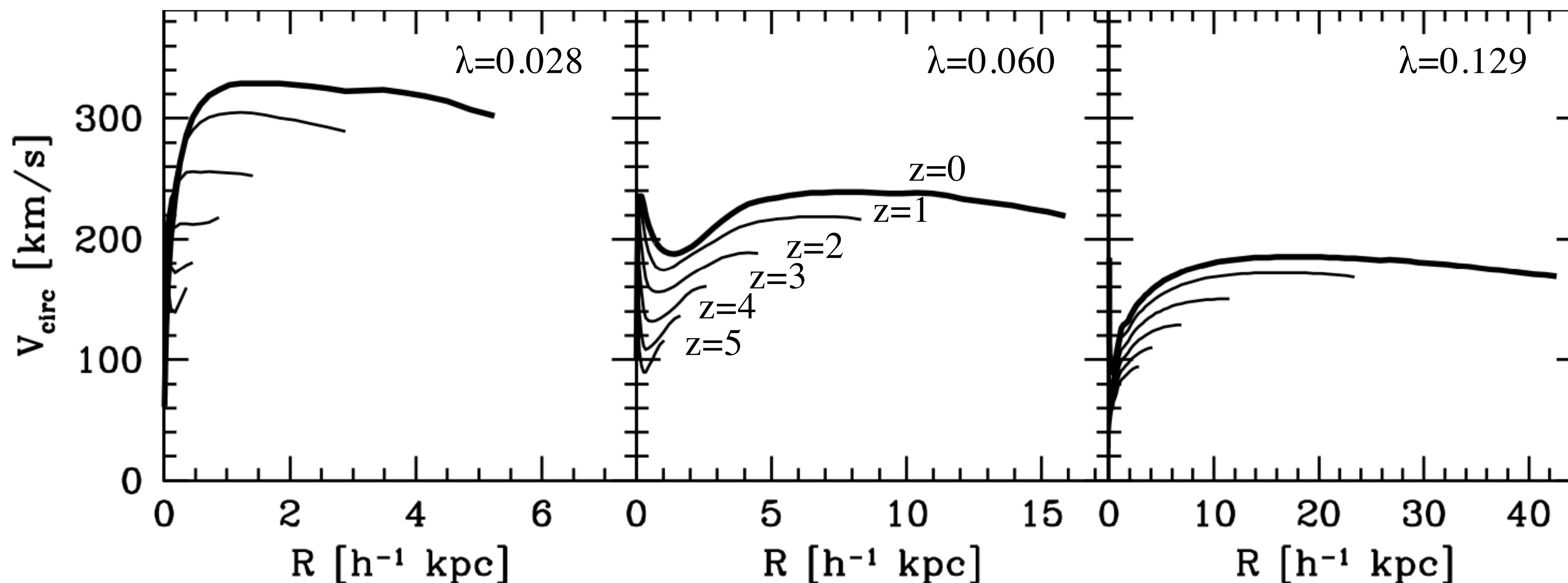


Fig. 4, 0112566



10.4 HOW REALISTIC WERE THE PREDICTIONS?

- The standard formation picture presented above can fit the observed disc scaling relations and the Tully-Fisher relation. However, this only works for unrealistically low stellar mass-to-light ratios and for disc models without gas.
- In reality the stellar mass-to-light ratio of disc galaxies increase with luminosity (as the figure on the right shows) and this should be taken into account in the models.

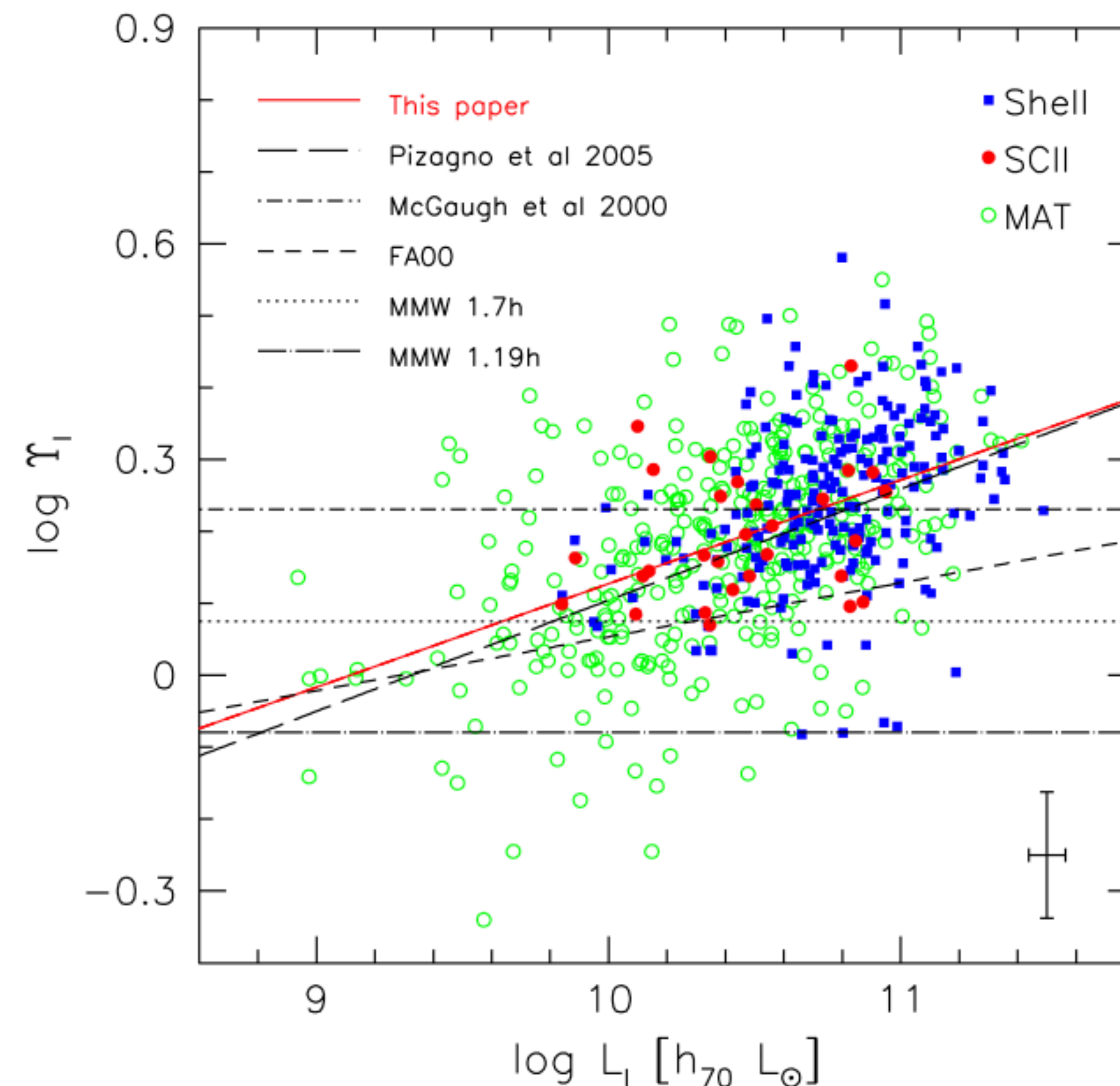


Fig. 5, 0604553



10.4.1 HOW REALISTIC WERE THE PREDICTIONS?

- Including the correction for the stellar M/L ratio (second column), allowing only disc matter with $\Sigma(R) > \Sigma_{\text{crit}}(R)$ to form stars (third column) and adjusting the disc-mass to halo-relation (α_m) (fourth column) improves the fits to the observed scaling relations.

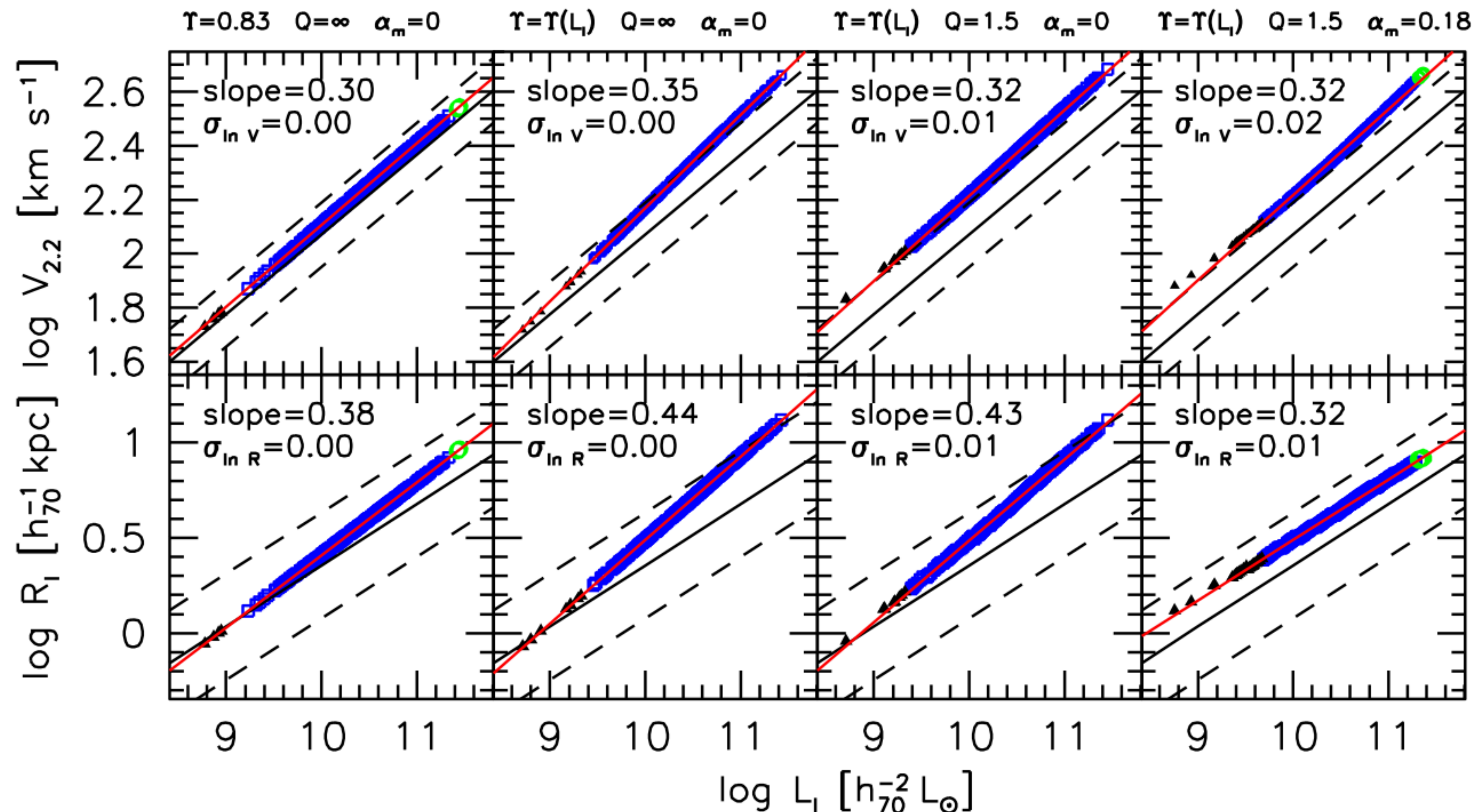


Fig. 6, 0604553



10.4.2 HOW REALISTIC WERE THE PREDICTIONS?

- **The slope of the Tully-Fisher relation can be well fitted in this way, but the zero-point of the TF relation is still too high and also the discs are too large.**
- **Solutions to this problem include:**
 - **Lower stellar mass-to-light ratios:** The M/L ratio can be lowered by increasing the relative weight of more luminous stars, but typically the model would require a large IMF shift, which is directly violated by observations.
 - **Lower halo concentrations:** Lowering the halo concentrations by a factor of two would reduce the velocities and produce a lower TF zero-point. Halo concentrations are determined by cosmology and thus cannot be changed too much.
 - **Modify the adiabatic contraction model:** Require halo expansion, i.e. from impulsive supernova feedback violating the adiabatic assumption and thus lowering the central velocities.



10.5 THE ORIGIN OF EXPONENTIAL DISCS

- Assuming there is no angular momentum loss or redistribution during the disc formation process, the disc surface density profile is a direct reflection of the specific angular momentum distribution of the proto-galaxy: $\Sigma_d(R) \leftrightarrow M_{\text{bar}}(j_{\text{bar}}) \leftrightarrow M_{\text{DM}}(j_{\text{DM}})$

- If the picture of specific angular momentum conservation is correct then the following equation should be valid

$$\frac{M_d(r)}{M_d} = \frac{M_h(< \mathcal{J})}{M_{\text{vir}}} \quad \mathcal{J} = RV_c(R)$$

- This means that for a given dark matter density and angular momentum profile one can predict the disc surface mass $\Sigma_d(R)$.



10.5.1 THE ORIGIN OF EXPONENTIAL DISCS

- Unfortunately the disc surface density profiles predicted in this way look nothing like exponential discs. Observed galaxies lack both high and low specific angular momentum material compared to the predictions.

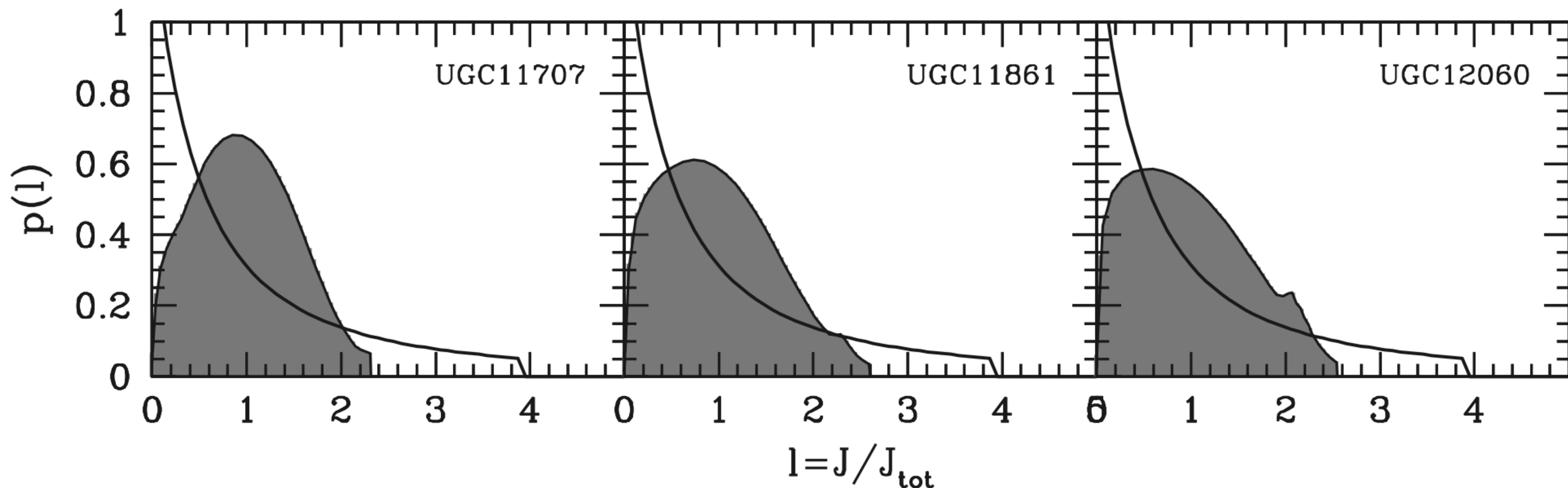


Fig. 11.3

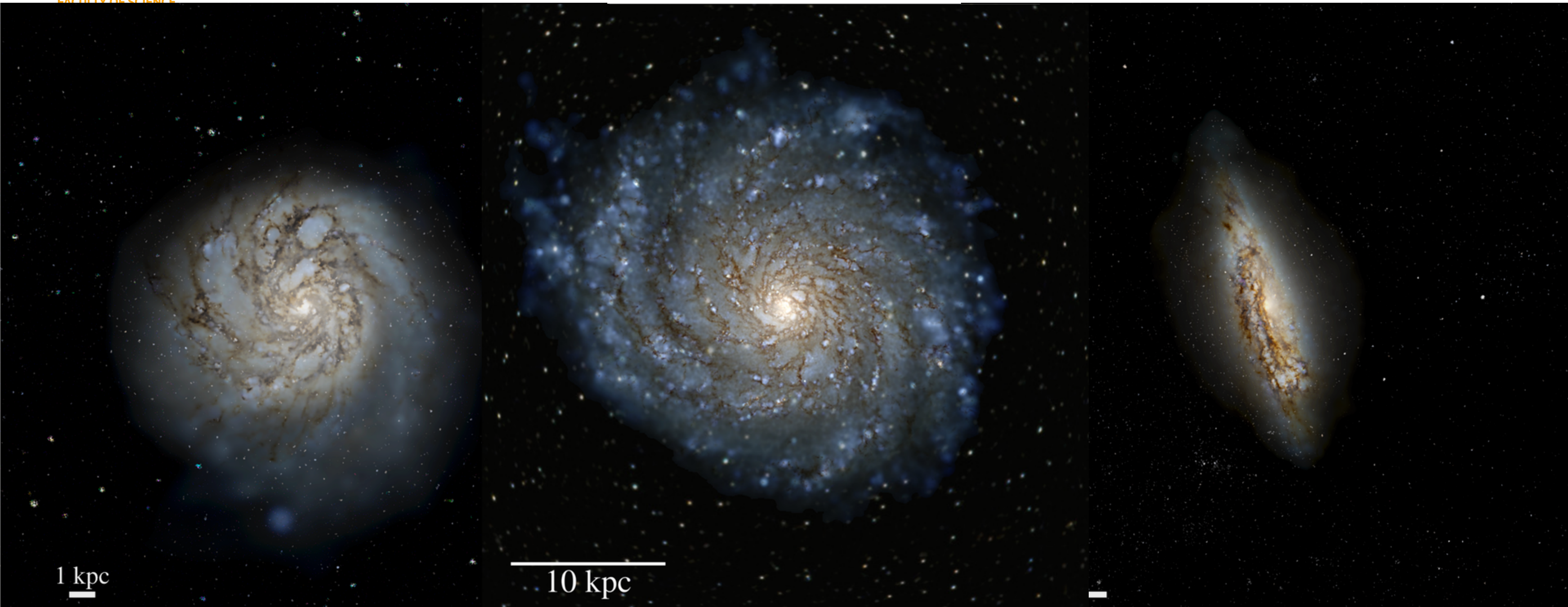
10.5.2 THE ORIGIN OF EXPONENTIAL DISCS

- **The standard picture of disc galaxy formation faces more problems**
 - **A small fraction of the available baryons end up in a disc component**
 - **The highest angular momentum material never ends up in the disc**
 - **The lower angular momentum material is preferentially ejected or prevented to join by feedback**



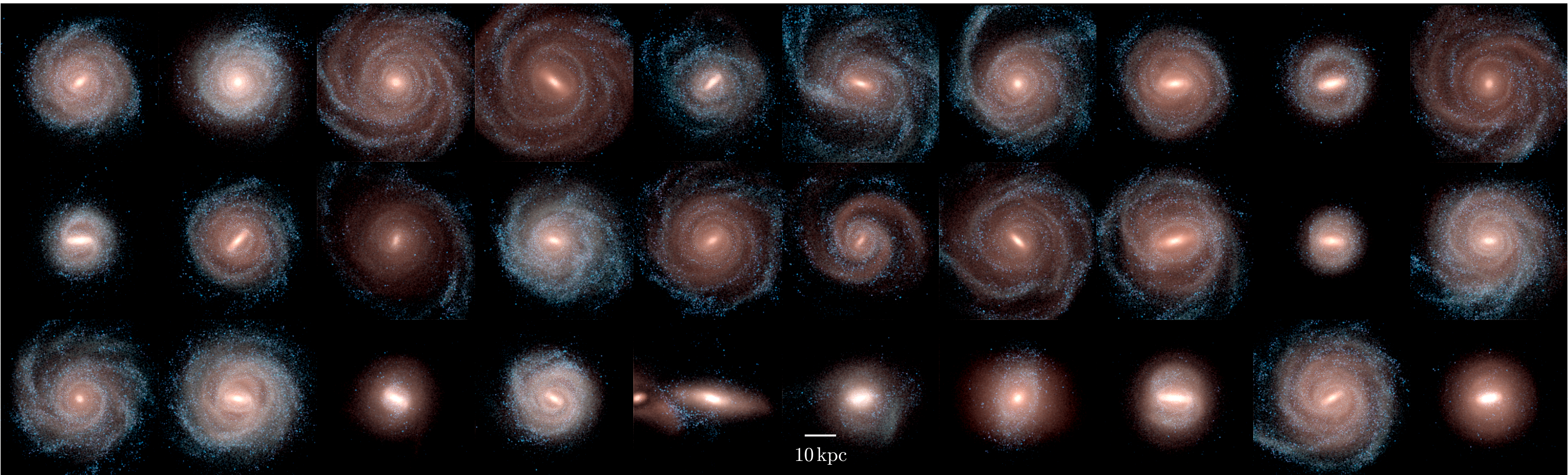
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10.6 WHERE DO WE STAND NOW?



FIRE simulation

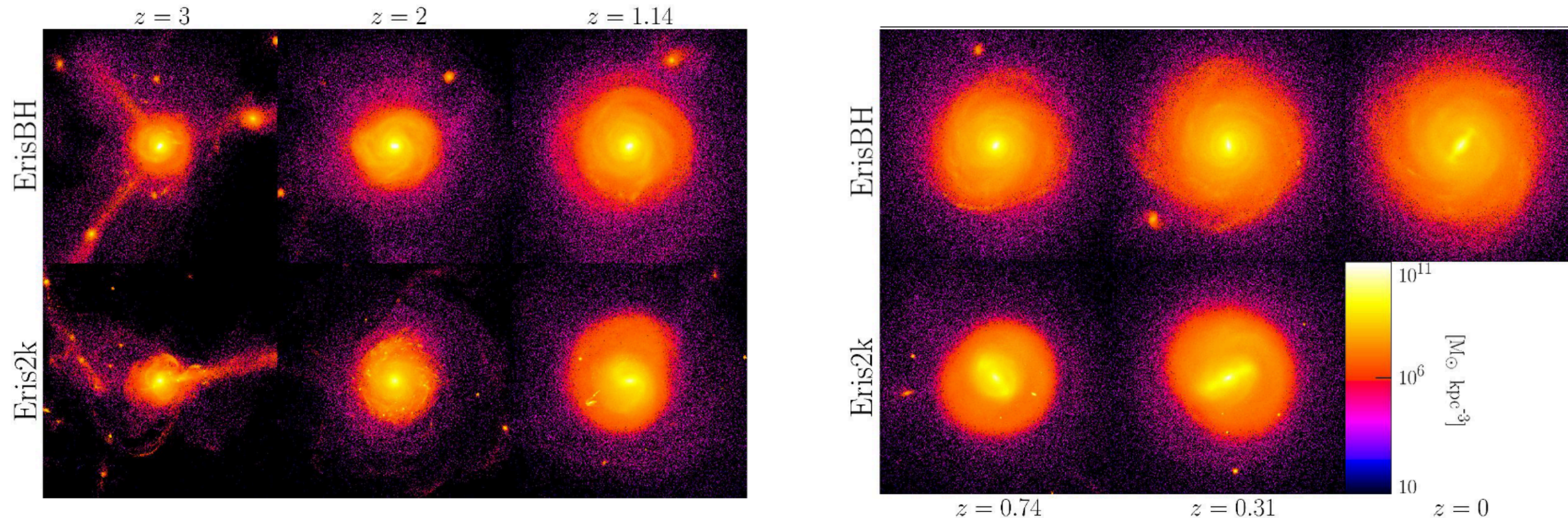
10.6.1 WHERE DO WE STAND NOW?



Auriga simulation



10.6.2 WHERE DO WE STAND NOW?

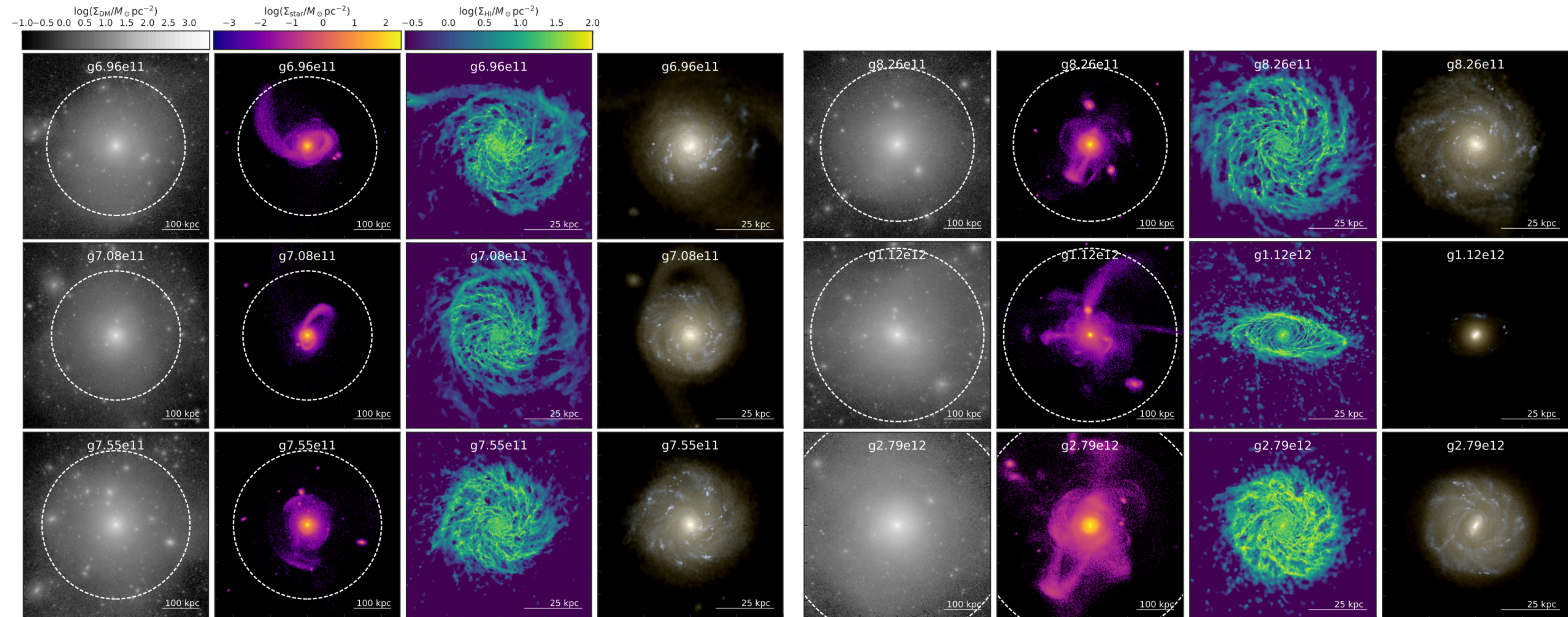


Eris simulation



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10.6.3 WHERE DO WE STAND NOW?



NIHAO simulation

10.6.4 WHERE DO WE STAND NOW?

<http://www.tapir.caltech.edu/~phopkins/Site/animations/gallery-of-simulated-galaxi/>

<https://wwwmpa.mpa-garching.mpg.de/auriga/movies.html>

<https://arxiv.org/pdf/1810.07701.pdf>

<https://arxiv.org/pdf/1909.05864.pdf>



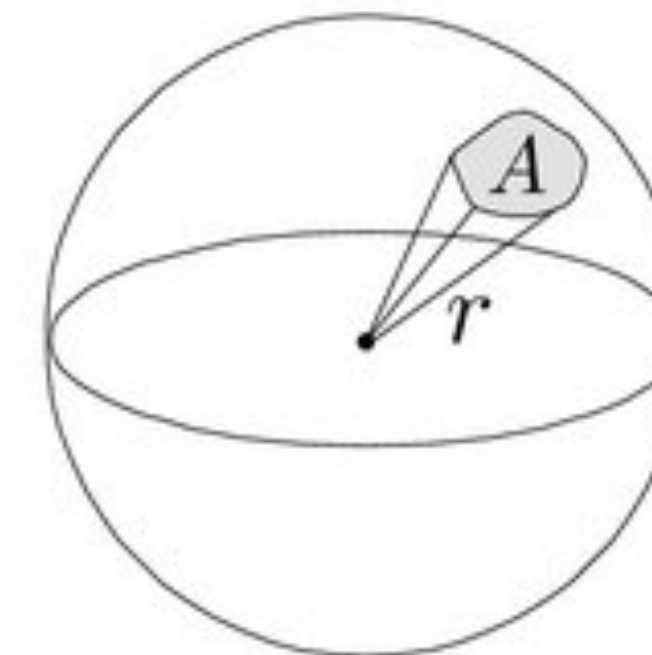
WHAT HAVE WE LEARNT?

- **Disc galaxies have flat rotation curves and exponential surface brightness profiles, which a successful model of galaxy formation should explain.**
- **The key physical reason for the formation of disc galaxies is that radiative cooling is an isotropic process, thus the gas contracts while approximately conserving angular momentum resulting in extended discs.**
- **Adiabatic contraction steepens significantly the inner density profiles resulting in peak rotation velocities, well above the virial velocity.**
- **In order to produce exponential discs, the feedback processes must leave the highest angular momentum disc material in the halo and eject preferentially low angular momentum material.**



EXTRA SLIDES

- An astronomical image depicts the surface brightness distribution (I) of an object, defined as the energy of the received photons from a unit solid angle per unit area per unit time [$\text{erg sr}^{-1} \text{cm}^{-2} \text{s}^{-1}$]
 - Solid angle $\Omega = A/r^2$ sr, where A is the surface and r the distance
- Integrating I over the entire image gives the flux (F) in [$\text{erg cm}^{-2} \text{s}^{-1}$]
- Integrating F over a sphere gives the luminosity (L) in [erg s^{-1}]





EXTRA SLIDES

- The apparent magnitude of an object in a band X can be calculated based on its flux and a reference object's flux (historically the star Vega)

$$m_X = -2.5 \log(f_X / f_{X,0})$$

- The absolute magnitude can be written as

$$\mathcal{M}_X = -2.5 \log \left(\frac{L_X}{L_{\odot X}} \right) + \mathcal{M}_{\odot X}$$

- Usually, the surface brightness of an object in a band X can be written in units of magnitude per arcsecond squared (1 arcsec = 1/3600 of a degree)

$$\mu_X = -2.5 \log \left(\frac{I_X}{L_{\odot} \text{pc}^{-2}} \right) + 21.572 + \mathcal{M}_{\odot X}$$

- If there are available observations in more than one bands, the difference between magnitudes in two bands defines the colour index, e.g.

$$(B - V) \equiv m_B - m_V = \mathcal{M}_B - \mathcal{M}_V$$



EXTRA SLIDES

- **Discs are often modelled by infinitesimally thin, exponential discs for which we get the following surface mass density and mass**

$$\Sigma(R) = \Sigma_0 e^{-R/R_d} \qquad M_d = 2\pi \int_0^\infty \Sigma(R) R dR = 2\pi \Sigma_0 R_d^2$$

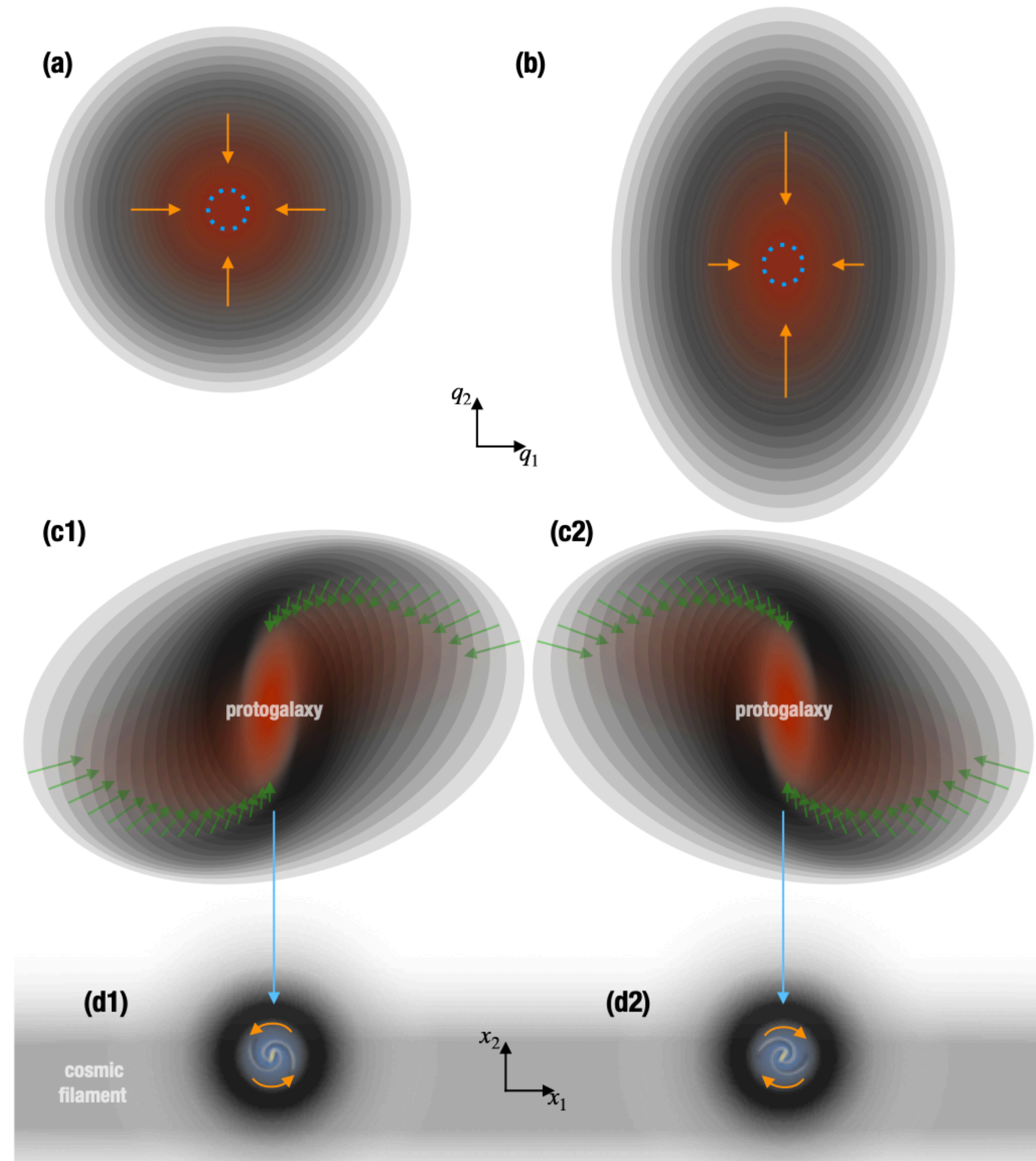
- **The gravitational potential and circular velocity of an infinitesimally thin disc can be found with the help of Bessel functions (J_0 the cylindrical Bessel function of zero order. I_0 and K_0 are modified Bessel functions):**

$$\Phi(R, z) = -2\pi G \int_0^\infty J_0(kR) \bar{\Sigma}(k) e^{-k|z|} dk$$

$$V_{c,d}^2(R) = -4\pi G \Sigma_0 R_d y^2 [I_0(y) K_0(y) - I_1(y) K_1(y)], \quad y = R/(2R_d)$$

EXTRA SLIDES

Figure 1 — Predicting the galaxy spin [arxiv:2003.04800](https://arxiv.org/abs/2003.04800)



Grey ellipses represent tidal tensor $\mathbf{T} = (T_{ij}) = \partial_{q_i} \partial_{q_j} \phi$. Their major axes are oriented along the major principal axes of \mathbf{T} and show directions of (relatively) stronger gravitational clustering. Concentric ellipses represent \mathbf{T} averaged over increasingly larger smoothing scales, with the largest ellipse corresponding to the largest smoothing scale.

- (a) **Isotropic tidal field.** Isotropic Lagrangian region (red cloud) is gravitationally collapsed (orange arrows) into a virtualized object (blue dotted circle).
- (b) **Anisotropic, scale-independent tidal field.** The gravitational force is stronger along the Lagrangian coordinate q_2 , as represented by long orange arrows. Consequently, the Lagrangian region is elongated along this direction. Here, the direction of the major axis of \mathbf{T} does not depend on the smoothing scale.
- (c) **Anisotropic, scale-dependent tidal field.** Similar to (b), but as we move to larger scales, major axis of \mathbf{T} rotates clockwise (c1) / anticlockwise (c2). This is illustrated by the pairs of green arrows. As a result, the Lagrangian regions will be slightly tilted as we go further from the center.
- (d) **Evolution of the two systems from (c)** into two halo-galaxy systems inside a filament (grey bar). The tidal torques have opposite signs, spinning the halo-galaxy systems in opposite directions. As a result, (d1) ends up with spin pointing out of the screen and we observe an “S”-shaped spiral galaxy, while (d2) has spin pointing into the screen and we observe a “Z”-shaped spiral galaxy.

Notes:

- (1) In our Universe, we are able to reconstruct the initial gravitational potential (c1,c2) from the positions of galaxies, and thus predict the oriented spins of galaxies (d1,d2).
- (2) We are able to predict the full 3D direction of the spin, so we can explicitly distinguish between j and $-j$. Measurements of galaxy shapes (intrinsic alignments) cannot provide such a distinction.