Galactic dynamics – Problem set 6. Spring 2023

The answers should be returned by ***Thursday (4.5) 4pm (16.00) in Moodle****, link through the official course homepage. A problem set help session will be held on* ***Thursday (27.4) at 14.15-16.00 in Room D115, Physicum****.The correct solutions will appear in Moodle after the due date.*

1. At typical sea-level conditions ($p = 1.01 \times 10^5$ N/m^2 and $T = 15$°C), the density of air is $\rho = 1.22$ kg/m^3 and the speed of sound is $v_S = 340$ m/s. Find (i) the fractional change in frequency due to the self-gravity of the air, for a sound wave with wavelength $\lambda = 1$ m and (ii) the Jeans length.

2. Prove that a system of N self-gravitating point masses with positive energy must disrupt, in the sense that at least one star must escape. Hint: use the virial theorem, and prove that the moment of inertia must increase without limit.

3. Consider a system in which the interparticle potential energy has the form $\Phi_{\alpha\beta} = -C|\mathbf{x}_\alpha - \mathbf{x}_\beta|^{-p}$, where p and C are positive constants.

   (a) Show that the scalar virial theorem has the form

   $$2K + pW = 0,$$

   where K is the kinetic energy and W is the potential energy.

   (b) For what values of p does the system have negative heat capacity, in the sense of Equation 7.51 in the lecture notes.

4. Two identical galaxies are initially at rest, at a large distance from one another. They are spherical, composed solely of identical stars, and their light distributions obey the Seŕsic law (eq 1.17 in the lecture notes) with Seŕsic index m and effective radius R_e. The galaxies fall together and merge. If the merger product also satisfies the Seŕsic law with the same index m, what is its effective radius?

5. Show that the mass arm (the maximum of the surface density) in a tightly wound spiral leads the potential arm (the minimum of the gravitational potential) at a given radius by an angle

   $$\Delta\phi = \frac{1}{km}\frac{d}{dR}\ln R^{1/2}|\Phi(R)|$$

   where $|\Phi(R)|$ is the amplitude of the spiral potential and the result has fractional error $\mathcal{O}(|kR|^{-1})$. Thus the mass arm can either lead or lag the potential arm, depending on the radial dependence of the strength of the spiral. Hint: use equation (6.33) in the lecture notes.