Galactic dynamics – Problem set 2. Spring 2023

The answers should be returned by ***Thursday (16.2) 4pm (16.00) in Moodle****, link through the official course homepage. A problem set help session will be held on* ***Thursday (9.2) at 14.15-16.00 in Room D115, Physicum****.The correct solutions will appear in Moodle after the due date.*

1. Prove that if a homogeneous sphere of a pressureless fluid with density ρ is released from rest, it will collapse to a point in time $t_{\rm ff} = \frac{1}{4}\sqrt{3\pi/(2G\rho)}$. The time $t_{\rm ff}$ is called the free-fall time of a system with density ρ.

2. Show that for a Kepler orbit the eccentric anomaly η and the true anomaly $\psi - \psi_0$ are related by:

$$\cos(\psi - \psi_0) = \frac{\cos\eta - e}{1 - e\cos\eta}; \quad \sin(\psi - \psi_0) = \sqrt{1 - e^2}\frac{\sin\eta}{1 - e\cos\eta} \tag{1}$$

3. Show that the energy of a circular orbit in the isochrone potential (Eq. 2.47 in the lecture notes) is $E = -GM/(2a)$, where $a = \sqrt{b^2 + r^2}$. Let the angular momentum of this orbit be $L_c(E)$. Show that

$$L_c = \sqrt{GMb}(x^{-1/2} - x^{1/2}), \quad \text{where} \quad x = -\frac{2Eb}{GM}$$

4. A star orbiting in a spherical potential suffers an arbitrary instantaneous velocity change while it is as pericenter. Show that the pericenter distance of the ensuing orbit cannot be larger than the initial pericenter distance.

5. Astronauts orbiting an unexplored planet find that (i) the surface of the planet is precisely spherical and centered on $r = 0$; and (ii) the potential exterior to the planetary surface is $\Phi = -GM/r$ exactly, that is, there are no non-zero multipole moments other than the monopole. Can they conclude from their observations that the mass distribution in the interior of the planet is spherically symmetric? If not, give a simple example of a non-spherical mass distribution that would reproduce the observations.