Galactic dynamics – Problem set 1. Spring 2023

The answers should be returned by ***Thursday (2.2) 4pm (16.00) in Moodle****, link through the official course homepage. A problem set help session will be held on* ***Thursday (26.1) at 14.15-16.00 in Room D115, Physicum****. The correct solutions will appear in Moodle after the due date.*

1. A general two-power density model can be expressed as:

$$\rho(r) = \frac{\rho_0}{(r/a)^{\alpha}(1 + r/a)^{\beta-\alpha}}$$

Let us then study the following three popular models, the Hernquist model for which $\alpha = 1, \beta = 4$, the Jaffe model for which $\alpha = 2, \beta = 4$ and the NFW model for which $\alpha = 1, \beta = 3$. Calculate now for each model:

   (a) The mass distribution as a function of radius, $M(r)$.

   (b) The circular rotation speed $v_c(r)$, also plot the rotation speed and compare it to Fig 2.5 in the lecture notes.

2. Calculate now the gravitational potential, $\Phi(r)$ for the Hernquist, Jaffe and NFW density profiles, as defined in the previous problem.

3. Let $\Phi(R, z)$ be the Galactic potential. At the solar location, $(R, z) = (R_0, 0)$, using Poisson's equation for a flattened system show that:

$$\frac{\partial^2 \Phi}{\partial z^2} = 4\pi G\rho_0 + 2(A^2 - B^2),$$

where ρ_0 is the density in the solar neighbourhood and A and B are the usual Oort constants. *Hint: use equation (2.73) from the Lecture notes.*

4. Show that $\Phi = \ln[r(1 + |\cos\theta|)]$ solves Laplace's equation everywhere except when $r = 0$ or $\theta = \pi/2$. By applying Gauss's theorem near $\phi = \pi/2$, find the potential of the Mestel disk (Eq. 2.158 in the lecture notes) in the limit $R_{\max} \to \infty$.

5. The r^{-1} dependence of the gravitational potential on distance arises because the graviton, which carries the gravitational field is, massless. If the graviton had a mass m_g, the gravitational potential due to a body of mass M would be

$$\Phi(r) = -\frac{GMe^{-\alpha r}}{r},$$

where $\alpha = m_g c/\hbar$ (the Yukawa potential), which reduces to the Newtonian potential in the limit $\alpha \to 0$. What is the analogue of Poisson's equation for the Yukawa potential?