Astrofysiikan peruskurssi – Kaavakokoelma

- Säteilyn perusmääritelmiä:

  $I_\nu(\theta,\phi) = \frac{dE_\nu}{dt\cos\theta dA d\nu d\omega}$

  $F_\nu = \int_\Omega I_\nu(\theta,\phi)\cos\theta d\omega$

  $J_\nu = \frac{1}{4\pi}\int_\Omega I_\nu(\theta,\phi)d\omega$

  $H_\nu = \frac{1}{4\pi}\int_\Omega I_\nu(\theta,\phi)\cos\theta d\omega = \frac{F_\nu}{4\pi}$

  $K_\nu = \frac{1}{4\pi}\int_\Omega I_\nu(\theta,\phi)\cos^2\theta d\omega = \frac{c}{4\pi}P_\mathrm{R}$

  $u = \frac{1}{c}\int_\Omega I_\nu(\theta,\phi)d\omega$

- Mustan kappaleen säteily:

  $I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$

  $\lambda_\mathrm{max}T = 0.289782$ cmK

  $F_\mathrm{tot} = \pi\int_0^\infty B_\nu d\nu = \sigma T_\mathrm{eff}^4, \quad \sigma = 5.669\cdot 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$

- Säteilyn emissio ja absorptio:

  $j_\nu = \frac{1}{4\pi}\frac{dE_\nu}{dt d\nu dm}$

  $k_\nu = \frac{\kappa_\nu}{\rho}$

  $\tau_\nu(x) = \int_0^x \rho(x')k_\nu(x')dx'$

  Kirchoffin laki: $\frac{j_\nu}{k_\nu} = B_\nu(T)$

- Säteilynkuljetus:

  $\cos\theta\frac{dI_\nu(\tau_\nu,\theta)}{d\tau_\nu} = I_\nu(\tau_\nu,\theta) - S_\nu(\tau_\nu), \qquad S_\nu = \frac{j_\nu}{k_\nu}$

  $\cos\theta\frac{dI_\nu}{d\tau_\nu} = I_\nu - \left(\frac{k_\nu}{k_\nu+\sigma_\nu}B_\nu + \frac{\sigma_\nu}{k_\nu+\sigma_\nu}J_\nu\right)$

  $I_\nu(\tau_\nu,\theta) = \int_{\tau_\nu}^\infty S_\nu(\tau'_\nu)e^{-(\tau'_\nu-\tau_\nu)\sec\theta}\sec\theta d\tau'_\nu, \qquad \sec\theta = 1/\cos\theta$

  $J_\nu(\tau_\nu) = \frac{1}{2}\int_0^\infty S_\nu(\tau'_\nu)E_1(|\tau'_\nu-\tau_\nu|)d\tau'_\nu$

  $F_\nu(\tau_\nu) = 2\pi\int_{\tau_\nu}^\infty S_\nu(\tau'_\nu)E_2(\tau'_\nu-\tau_\nu)d\tau'_\nu - 2\pi\int_0^{\tau_\nu} S_\nu(\tau'_\nu)E_2(\tau_\nu-\tau'_\nu)d\tau'_\nu$

  Integraalieksponenttifunktio: $E_n(x) = \int_1^\infty \frac{e^{-xy}}{y^n}dy$

- Kaasumaisen tilan fysiikkaa:

  $PV = NRT, \qquad P = NkT, \qquad P = \frac{\rho kT}{m}$

  $P = \frac{1}{3}m\overline{v^2}, \qquad \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$

  Adiabaattinen muutos: $PV^\gamma =$ vakio, $\quad TV^{\gamma-1} =$ vakio, $\quad P^{1-\gamma}T^\gamma =$ vakio

- Maxwellin nopeusjakautuma, Boltzmannin virityskaava sekä Ionisaatioyhtälö:

$$dN(v_x, v_y, v_z) = N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}(v_x^2+v_y^2+v_z^2)} dv_x dv_y dv_z$$

$$dN(v) = 4\pi v^2 N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{m}{2kT}v^2} dv$$

$$\frac{N_n}{N_m} = \frac{g_n}{g_m} e^{-(E_n - E_m)/kT}$$

$$\frac{N_n}{N} = \frac{g_n}{u(T)} e^{-\chi_n/kT}$$

$$\frac{N_{i+1}N_e}{N_i} = \frac{(2\pi mkT)^{3/2}}{h^3}\frac{2u_{i+1}(T)}{u_i(T)} e^{-\chi_i/kT}$$

$$\frac{N_{i+1}P_e}{N_i} = 0.331T^{5/2}\cdot\frac{2u_{i+1}(T)}{u_i(T)}10^{-\frac{5040}{T}\chi_i},$$

$[P_e] = \mathrm{dyn/cm^2}$, $[T] = \mathrm{K}$, $[\chi_i] = \mathrm{eV}$

- Klassisen dipolin absorptio cgs-yksiköissä:

$$F = \frac{\ddot{p}^2}{4\pi c^3}\frac{\sin^2\vartheta}{r^2}, \quad \langle F\rangle = \frac{c}{8\pi}E_0^2$$

$$\frac{dW}{dt} = -\frac{2}{3}\frac{e^2\ddot{z}^2}{c^3}, \quad \left\langle\frac{dW}{dt}\right\rangle = -\frac{16\pi^4\nu^4}{3c^3}p_0^2 = -\frac{8\pi^2\nu^2e^2}{3mc^3}\cdot W$$

Atomin efektiivinen pinta-ala: $\sigma = \int_0^\infty \alpha_\nu d\nu = \frac{\pi e^2}{m_e c}$

$$k_\nu\rho = \frac{N_{0\nu}e^2}{mc}\frac{\gamma/4\pi}{(\nu-\nu_0)^2+(\gamma/4\pi)^2} = N_{0\nu}\alpha_\nu$$

- Säteilyn sirottuminen klassisesta oskillaattorista cgs-yksiköissä:

$$\sigma_S(\theta,\phi) = \left(\frac{e^2}{mc^2}\right)^2\cos^2\theta, \quad \sigma_T = \frac{8}{3}\pi\left(\frac{e^2}{mc^2}\right)^2 = 6.65\cdot 10^{-25}\ \mathrm{cm^2}$$

$$\rho k_\nu = N\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2\left(\frac{\nu}{\nu_0}\right)^4 = N\sigma_T\left(\frac{\lambda_0}{\lambda}\right)^4, \quad \sigma_{\mathrm{at}} = \sigma_T\left(\frac{\lambda_0}{\lambda}\right)^4$$

- Kontinuumiabsorptio:

$$\alpha_\nu^{\mathrm{bf}} = \overline{f}\frac{\pi e^2}{mc}\frac{K^3}{2R_\nu Z^2}$$

$$f_{\mathrm{nk}} = \frac{32}{3\pi\sqrt{3}}\frac{R_\nu^3 Z^6}{n^5\nu^3}\left|\frac{1}{K^3}\right| g^{\mathrm{bf}}$$

$$\alpha_\nu^{\mathrm{bf}} = \frac{32}{3\sqrt{3}}\frac{\pi^2e^6}{ch^3}\frac{R_\nu Z^4}{n^5\nu^3}g^{\mathrm{bf}}$$

$$\kappa_\nu(\mathrm{b-f}) = 2.81\cdot 10^{29}\frac{\mathrm{Z}^4}{\mathrm{n}^5\nu^3}\mathrm{g_{bf}}\mathrm{N_{0,n}}, \quad [\kappa_\nu] = \mathrm{cm^{-1}}$$

$$\alpha_K^{\mathrm{ff}}(\nu)dv = \frac{2}{3\sqrt{3}}\frac{R_\nu Z^2h^2e^2}{m^3\pi cv}\frac{g^{\mathrm{ff}}}{\nu^3}dv$$

$$k_\nu^{\mathrm{ff}} = \frac{8}{3\sqrt{3}}\frac{e^2R_\nu k}{chm\cdot m_H}\frac{T_eg^{\mathrm{ff}}}{\nu^3}e^{-\chi_1}$$

- Einsteinin kertoimet:

$$A_{n,n'} = \frac{2h\nu^3}{c^2}B_{n,n'},$$ säteilykentässä I_ν

$$A_{n,n'} = \frac{8\pi h\nu^3}{c^3}B_{n,n'},$$ isotrooppisessa säteilykentässä: $(u = \frac{4\pi}{c}I_\nu)$

$$g_nB_{n,n'} = g_{n'}B_{n',n}$$

$$4\pi I_\nu\overline{k'}\rho = (N_{n'}B_{n',n} - N_nB_{n,n'})I_\nu h\nu_0$$

- Spektriviivaprofiilit:

  $\frac{1}{k_R} = \frac{\pi}{4\sigma T^3} \int_0^\infty \frac{1}{k_\nu} \frac{\partial B_\nu(T)}{\partial T} d\nu$

  $\gamma = \frac{8\pi^2 e^2 \nu^2}{3mc^3}$

  $f_{n'n} = \frac{1}{3\gamma} \frac{g_n}{g_{n'}} A_{nn'}$

  $\text{FWHP} = 2|\Delta\nu_{1/2}| = \frac{\gamma}{2\pi}$, luonnollisen viivaprofiilin puoliarvoleveys.

  $\text{FWHP} = 2|\Delta\nu_{1/2}| = \frac{2\nu_0}{c}\sqrt{\frac{2kT}{m}\log 2}$, terminen viivan leveneminen.

  $k_\nu \rho = N f \frac{\pi e^2}{mc} \frac{\Gamma_{\text{eff}}}{4\pi^2} \int_{-\infty}^{\infty} \frac{\sqrt{\frac{m}{2\pi kT}} e^{-mv^2/(2kT)}}{(\nu - \nu_0 - \frac{v}{c}\nu_0)^2 + (\Gamma_{\text{eff}}/4\pi)^2} dv$, Lorentz-profiili.

  $W_\nu = \int \frac{I_0 - I_\nu}{I_0} d\nu$, ekvivalenttileveys.

- Vedyn fotoionisaatio ja HII-alueet:

  $N_0 R_{0,1} = 4\pi \int_{\nu_0}^{\infty} \frac{\kappa_\nu(\text{b}-\text{f})\text{I}_\nu}{h\nu} d\nu$

  $\frac{dL_c(r)}{dr} = -4\pi r^2 x^2 N_H^2 (\alpha_0 - \alpha_{0,1})$

  $r_S = 1.23 \cdot 10^{-7} \left(\frac{R_*}{R_\odot}\right)^{2/3} N_{\text{Ly}}^{1/3} N_{\text{H}}^{-2/3} \quad [\text{pc}]$

- Säteily tähtienvälisessä kaasussa

  $\frac{dN_n}{dt} = -N_n \sum_m (R_{n,m} + C_{n,m}) + \sum_m N_m (R_{m,n} + C_{m,n}) = 0$

  $n' \rightarrow n$: $N_{n'} \frac{4\pi}{c} I_\nu B_{n',n}$

  $n \rightarrow n'$: $N_n (A_{n,n'} + \frac{4\pi}{c} I_\nu B_{n,n'})$

  $\frac{Q_{m,n}}{Q_{n,m}} = \frac{g_n}{g_m} e^{-\Delta E/kT}, \quad T = T_{\text{kin}}$

  $T_{\text{ex}} = \frac{T + xT_b}{1+x}, \quad x = \frac{A_{21}kT/h\nu}{N'Q_{21}}$

  $\tau = 1/A_0, \quad \Delta\nu_N = \frac{1}{2\pi\tau}$