Weighted generalized quasi-likelihood estimation in a survey population setup for longitudinal count data

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Motivation

Longitudinal binary survey data

Example: Statistics Canada SLID (Survey of labor income dynamics) data

- (i) Initially the sample s^* had 35,000 individuals to survey for six years from 1993 to 1998.
- (ii) Binary response $y_{it} = 1$ denote that the ith individual is unemployed at time t $(i \epsilon s^*)$, $t = 1, \ldots, T_i$.
- (iii) 5 covariates: gender, age, geographic location, education level, and marital status of the individual. Here gender, age and geographic location are held as observed in 1993, education level and marital status are considered to be time dependent covariates.

Motivation-continued

Some of these five covariates are categorical with more than two levels. Express these 5 covariates through 12 renamed covariates. Let x_{it} denote the vector for these 12 covariates corresponding to y_{it}. To be specific, the 12 covariates are constructed as follows.

$$x_{it1} = \begin{cases} 0 & \text{for female} \\ 1 & \text{for male} \end{cases} \qquad (x_{it2}, x_{it3}) \equiv \begin{cases} (0,0) & \text{age group 16 and 24} \\ (1,0) & \text{age group 25 and 54} \\ (0,1) & \text{age group 55 and 65} \end{cases}$$

Motivation-continued

$$(x_{it4}, x_{it5}, x_{it6}, x_{it7}) \equiv \begin{cases} (0,0,0,0) & \text{Atlantic region} \\ (1,0,0,0) & \text{Quebec region} \\ (0,1,0,0) & \text{Ontario region} \\ (0,0,1,0) & \text{Praries} \\ (0,0,0,1) & \text{British Columbia-Alberta region} \end{cases}$$

$$(x_{it8}, x_{it9}) \equiv \begin{cases} (0,0) & \text{low education} \\ (1,0) & \text{medium education} \\ (0,1) & \text{high education} \end{cases}$$

and

$$(x_{it,10}, x_{it,11}, x_{it,12}) \equiv \begin{cases} (0,0,0) & \text{married and common law spouse grou} \\ (1,0,0) & \text{separated and divorce group} \\ (0,1,0) & \text{widow group} \\ (0,0,1) & \text{single group} \end{cases}$$

Motivation-continued

Sampling weights:

Sampling weights are usually denoted by $w_i(i \epsilon s^*)$ or equivalently w_{is^*}, for simplicity. However, the notation for these design weights may further be changed showing the strata and cluster (two stage cluster sampling based) that the individual belongs to. Let

1. L denote the total number of stratum;
2. n_h be the number of clusters in the hth $(h = 1, \ldots, L)$ stratum;
3. n_{hc} denote the size of the cth $(c = 1, \ldots, n_h)$ cluster under the hth stratum;
4. w_{hcis^*} represent the design weight for the ith individual to be included in the sample s^* based on his/her origin in the cth cluster of the hth stratum.

Motivation-continued

Notice that

- (i) Conditional on the random effect $\gamma_i \overset{i.i.d.}{\sim} N(0, \sigma_\gamma^2)$, the repeated binary responses, i.e., $(y_{i1}, \ldots, y_{it}, \ldots, y_{iT_i})|\gamma_i$ are correlated through σ_γ^2 and a correlation index parameter, say ρ.
- (ii) ith individual is chosen in the sample s^* , say with sampling weight $w_i(i \epsilon s^*)$ or equivalently w_{is^*}, or equivalently w_{hcis^*}.
- (iii) It is of interest to compute the effects (β) of x_{it} on y_{it} after taking the overdispersion (σ_γ^2) and correlation structure (based on ρ) from (i), and sampling weights (w_{is^*}) from (ii) into account.

Longitudinal count survey data

- Example 1: Statistics Canada NLSCY (National Longitudinal Survey of Children and Youth) data: [1994-1995; 1996-1997; 1998-1999; 2000-2001; 2002-2003; 2004-2005; 2006-2007] The NLSCY was developed to gather information on the factors that influence the social and emotional development of children and youth, as well as their overall behaviour. Here y_{it} can be the yearly number of extra school sports/programs activities.
- Example 2: Statistics Canada NPHS (National Population Health Survey) data: The NPHS gathers information on the health of the Canadian population, as well as relevant socio-demographic information. Here y_{it} can be number of yearly physician visits, for example.

Some references for survey data analysis in panel data setup:

- Sutradhar and Kovacevic (Biometrika, 2000); with applications to SLID (survey of labour and income dynamics) data
- Sutradhar, B. C. (2008). Inferences in familial Poisson mixed models for survey data. *Sankhya B*, 70, Part 2, 18-33.
- Sutradhar, B. C., R. Prabhakar Rao & Pandit, V. N. (2010). Inferences in longitudinal mixed models for survey data. *Journal of the Indian Society of Agricultural Statistics, a special issue in Memory of Dr. G. R. Seth*, 64, 177-189.

Finite/Survey population of counts in a longitudinal setup

- Consider a survey population of N individuals which may also be referred to as a finite population of size N.
- Suppose that the ith $(i = 1, \ldots, N)$ individual of this survey population provides T repeated count responses, and y_{it} denotes the tth $(t = 1, \ldots, T)$ response of this ith individual.
- Thus, the survey population under consideration contains NT elements or responses.
- Also suppose that $x_{it} = (x_{it1}, \ldots, x_{itp})'$ denote the $p-$dimensional time dependent covariate vector corresponding to y_{it}.

Super-population model (SM)

- We assume that the repeated count responses $y_{i1}, \ldots, y_{it}, \ldots, y_{iT}$ in the survey population follow a super population model **(SM)** to explain the effect of x_{it} on y_{it}.
- A reference, for example, for survey population and super-population : Godambe and Thompson (1986, International Statistical Review).
- **(SM)**:

$$\begin{aligned} y_{i1}|\gamma_i &\sim Poi(\mu^*_{i1} = \exp(x'_{i1}\beta)) \\ y_{it}|\gamma_i &= \rho * y_{i,t-1}|\gamma_i + d_{it}|\gamma_i, \; t = 2, \ldots, T, \end{aligned} \quad (1)$$

(Sutradhar and Bari (2007, Sankhya B)) where
(a). γ_i is the individual common random effect shared by all T repeated responses of the ith individual, and it is assumed that $\gamma_i \overset{i.i.d.}{\sim} N(0, \sigma^2_\gamma)$.

Super-population model (SM) (Continued)

(b). For $t = 2, \ldots, T$,

$$y_{i,t-1}|\gamma_i \sim Poi(\mu^*_{i,t-1}) \text{ and } d_{it}|\gamma_i \sim Poi(\mu^*_{it} - \rho\mu^*_{i,t-1}),$$

with $\mu^*_{it} = \exp(x'_{it}\beta + \gamma_i)$.
(c). Conditional on γ_i, d_{it} and $y_{i,t-1}$ are independent.
(d). For given $y_{i,t-1}$,

$$\rho * y_{i,t-1} = \sum_{j=1}^{y_{i,t-1}} b_j(\rho)$$

is a binomial thinning operation, where $b_j(\rho)$ stands for a binary variable with $Pr[b_j(\rho) = 1] = \rho$ and $Pr[b_j(\rho) = 0] = 1 - \rho$.

Basic properties of the (SM):

- Means and variances:

$$\begin{aligned} E_{\mathbf{SM}}[Y_{it}] &= E_{\gamma_i} E_{\mathbf{SM}}[Y_{it}|\gamma_i] = E_{\gamma_i} E_{\mathbf{SM}}[\exp(x'_{it}\beta + \gamma_i)] \\ &= \exp(x'_{it}\beta + \sigma^2_\gamma/2) = \mu_{it}(\beta, \sigma^2_\gamma) \end{aligned} \quad (2)$$

$$\text{var}_{\mathbf{SM}}[Y_{it}] = \mu_{it}(\beta, \sigma^2_\gamma) + [\exp(\sigma^2_\gamma) - 1]\mu^2_{it}(\beta, \sigma^2_\gamma) = \sigma_{itt}(\beta, \sigma^2_\gamma) \quad (3)$$

- Covariances:

$$\begin{aligned} \text{cov}_{\mathbf{SM}}(Y_{iu}, Y_{it}) &= E_{\gamma_i}[\text{cov}_{\mathbf{SM}}\{(Y_{iu}, Y_{it})|\gamma_i\}] \\ &+ \text{cov}_{\gamma_i}[E_{\mathbf{SM}}(Y_{iu}|\gamma_i),\ E_{\mathbf{SM}}(Y_{it}|\gamma_i)] \\ &= \rho^{t-u}\mu_{iu}(\beta, \sigma^2_\gamma) + [\exp(\sigma^2_\gamma) - 1] \\ &\times \mu_{iu}(\beta, \sigma^2_\gamma)\mu_{it}(\beta, \sigma^2_\gamma) \\ &= \sigma_{iut}(\beta, \sigma^2_\gamma, \rho),\ \text{for } u < t. \end{aligned} \quad (4)$$

Understanding survey population parameters β_N, $\sigma^2_{\gamma,N}$, and ρ_N

What is this β_N parameter?
For $i = 1, \ldots, N$, in survey population setup, we write

$$\begin{aligned}\mu_i(\beta, \sigma^2_\gamma) &= E_{\mathbf{SM}}[Y_i] == [\mu_{i1}(\beta, \sigma^2_\gamma), \ldots, \mu_{it}(\beta, \sigma^2_\gamma), \ldots, \mu_{iT}(\beta, \sigma^2_\gamma)]\\ \Sigma_i(\beta, \sigma^2_\gamma, \rho) &= \mathrm{cov}_{\mathbf{SM}}[Y_i] = (\sigma_{iut}(\beta, \sigma^2_\gamma, \rho)) : T \times T.\end{aligned}$$

It then follows that β_N is the solution of the survey (finite) population (of size N) based GQL estimating equation

$$\begin{aligned}&\sum_{i=1}^{N} \frac{\partial \mu_i'(\beta, \sigma^2)}{\partial \beta} \Sigma_i^{-1}(\beta, \sigma^2_\gamma, \rho)[y_i - \mu_i(\beta, \sigma^2)]\\ =& \sum_{i=1}^{N} g^*_{i1}(\beta, \sigma^2_\gamma, \rho) = G^*_1(\beta, \sigma^2_\gamma, \rho) = 0, \qquad (6)\end{aligned}$$

[Sutradhar and Bari (2007, Sankhya B), Sutradhar (2003, Statistical Science)] for β.

Understanding survey population parameters β_N, $\sigma^2_{\gamma,N}$, and ρ_N (Continued)

What is this $\sigma^2_{\gamma,N}$ parameter?
Let

$$u_i = (y^2_{i1}, \ldots, y^2_{iT}, y_{i1}y_{i2}, \ldots, y_{it}y_{i,t+1}, \ldots, y_{i,T-1}y_{iT})'.$$

Under the **SM** one obtains

$$\begin{aligned} E_{\mathbf{SM}}(U_i) &= \lambda_i(\beta, \sigma^2_\gamma, \rho) \quad (7 \\ &= (\lambda_{i11}(\cdot), \ldots, \lambda_{itt}(\cdot), \ldots, \lambda_{iTT}(\cdot), \lambda_{i12}(\cdot), \ldots, \lambda_{iut}(\cdot), \ldots, \lambda_{i,T-1,T}(\cdot)) \end{aligned}$$

with

$$\begin{aligned} \lambda_{itt}(\cdot) &\equiv \lambda_{itt}(\beta, \sigma^2_\gamma) = E_{\mathbf{SM}}(Y^2_{it}) = \mu_{it}(\beta, \sigma^2_\gamma) + \mu^2_{it}(\beta, \sigma^2_\gamma)\exp(\sigma^2_\gamma) \\ \lambda_{iut}(\cdot) &\equiv \lambda_{iut}(\beta, \sigma^2_\gamma, \rho) = E_{\mathbf{SM}}(Y_{iu}Y_{it}) \\ &= \rho^{t-u}\mu_{iu}(\beta, \sigma^2_\gamma) + \mu_{iu}(\beta, \sigma^2_\gamma)\mu_{it}(\beta, \sigma^2_\gamma)\exp(\sigma^2_\gamma). \end{aligned}$$

Understanding $\sigma^2_{\gamma,N}$ continued

Also compute

$$\text{cov}_{\mathbf{SM}}(U_i|\rho = 0) = \Omega_{iw}(\beta, \sigma^2_\gamma, \rho = 0), \tag{8}$$

using, for example,

$$\begin{aligned}
& \text{cov}_{\mathbf{SM}}[\{Y^2_{iu}, Y_{it}Y_{iv}\}|\rho = 0] \\
= \;& E_{\mathbf{SM}}[\{Y^2_{iu}Y_{it}Y_{iv}\}|\rho = 0] - E_{\mathbf{SM}}[Y^2_{iu}]E_{\mathbf{SM}}[\{Y_{it}Y_{iv}\}|\rho = 0] \\
= \;& E_{\gamma_i}\left[E_{\mathbf{SM}}(Y^2_{iu}|\gamma_i)E_{\mathbf{SM}}(Y_{it}|\gamma_i)E_{\mathbf{SM}}(Y_{iv}|\gamma_i)\right] - E_{\mathbf{SM}}[Y^2_{iu}]E_{\mathbf{SM}}[\{Y_{it}Y_{iv}\} \\
= \;& E_{\gamma_i}\left[\{\mu^*_{iu} + \mu^{*2}_{iu}\}\mu^*_{it}\mu^*_{iv}\right] \\
- \;& \lambda_{itt}(\beta, \sigma^2_\gamma)E_{\mathbf{SM}}[\{Y_{it}Y_{iv}\}|\rho = 0] \\
= \;& \mu_{iu}(\beta, \sigma^2_\gamma)\mu_{it}(\beta, \sigma^2_\gamma)\mu_{iv}(\beta, \sigma^2_\gamma)\exp(3\sigma^2_\gamma)[1 + \mu_{iu}(\beta, \sigma^2_\gamma)\exp(3\sigma^2_\gamma)] \\
- \;& \lambda_{itt}(\beta, \sigma^2_\gamma)\mu_{it}(\beta, \sigma^2_\gamma)\mu_{iv}(\beta, \sigma^2_\gamma)\exp(\sigma^2_\gamma).
\end{aligned}$$

Understanding $\sigma^2_{\gamma,N}$ continued

Similar to (6), it then follows that for known β and ρ, $\sigma^2_{\gamma,N}$ is the solution of the survey (finite) population (of size N) based GQL estimating equation

$$\sum_{i=1}^{N} \frac{\partial \lambda_i'(\beta, \sigma^2_\gamma, \rho)}{\partial \sigma^2_\gamma} \Omega^{-1}_{iw}(\beta, \sigma^2_\gamma, \rho = 0)[u_i - \lambda_i(\beta, \sigma^2_\gamma, \rho)]$$
$$= \sum_{i=1}^{N} g^*_{i2}(\beta, \sigma^2_\gamma, \rho) = G^*_2(\beta, \sigma^2_\gamma, \rho) = 0, \quad (9)$$

[Sutradhar and Bari (2007, Sankhya B)] for the estimation of σ^2_γ.

Understanding ρ_N

Suppose that $y^*_{it} = [y_{it} - \mu_{it}(\beta, \sigma^2_\gamma)]/[\sigma_{itt}(\beta, \sigma^2_\gamma)]^{1/2}$. One may then exploit (7) and obtain ρ_N, the moment estimate of ρ, as

$$\rho_N = \frac{a_{1,N} - b_{1,N}}{g_{1,N}}, \text{ where} \tag{10}$$

$$\begin{aligned}
a_{1,N} &= \sum_{i=1}^{N}\sum_{t=1}^{T-1} y^*_{it} y^*_{i(t+1)}/N(T-1)/\sum_{i=1}^{N}\sum_{t=1}^{T} {y^*_{it}}^2/NT \\
b_{1,N} &= (\exp(\sigma^2_\gamma) - 1)\sum_{i=1}^{N}\sum_{t=1}^{T-1} m_{it}(\cdot) m_{i,t+1}(\cdot)/N(T-1) \\
g_{1,N} &= \sum_{i=1}^{N}\sum_{t=1}^{T-1} \mu_{it}(\beta, \sigma^2_\gamma)\left[\sigma_{itt}(\beta, \sigma^2_\gamma)\sigma_{i,t+1,t+1}(\beta, \sigma^2_\gamma)\right]^{-\frac{1}{2}}/N(T-1),
\end{aligned}$$

with $m_{it}(\cdot) \equiv m_{it}(\beta, \sigma^2_\gamma) = \mu_{it}(\beta, \sigma^2_\gamma)/[\sigma_{itt}(\beta, \sigma^2_\gamma)]^{\frac{1}{2}}$.

Sampling weights based unbiased estimation

- Notice from (6) and (9) that the survey population parameters β_N and $\sigma^2_{\gamma,N}$ are defined as the solutions of the GQL estimating equations

$$\sum_{i=1}^{N} g^*_{i1}(\beta, \sigma^2_\gamma, \rho) = 0, \text{ and}$$
$$\sum_{i=1}^{N} g^*_{i2}(\beta, \sigma^2_\gamma, \rho) = 0,$$

respectively.

- We now have a finite sample s^* of size n collected from the survey population of size N using the sampling design (D) based weights $w_{is^*} \equiv w_{hcis^*}$ explained in the last section. This sample s^* is used as follows to obtain consistent estimates for the survey population parameters β_N and $\sigma^2_{\gamma,N}$.

Construction of unbiased survey estimating functions

By weighting the estimating functions $g_{i1}^*(\beta, \sigma_\gamma^2, \rho)$ (6) and $g_{i2}^*(\beta, \sigma_\gamma^2, \rho)$ (9), we first write two survey estimating functions $\sum_{i\epsilon s^*} w_{is^*} g_{i1}^*(\beta, \sigma_\gamma^2, \rho)$ and $\sum_{i\epsilon s^*} w_{is^*} g_{i2}^*(\beta, \sigma_\gamma^2, \rho)$ such that

$$E_D[\sum_{i\epsilon s^*} w_{is^*} g_{i1}^*(\beta, \sigma_\gamma^2, \rho)] = \sum_{i=1}^{N} g_{i1}^*(\beta, \sigma_\gamma^2, \rho),$$

and

$$E_D[\sum_{i\epsilon s^*} w_{is^*} g_{i2}^*(\beta, \sigma_\gamma^2, \rho)] = \sum_{i=1}^{N} g_{i2}^*(\beta, \sigma_\gamma^2, \rho),$$

where $E_D(\cdot)$ denote the sampling design based expectation.

Weighted GQL estimating equations for β and σ^2_γ

- Consequently, one can obtain consistent estimates for β_N and $\sigma^2_{\gamma,N}$ by solving the estimating equations

$$\hat{G}^*_1(\beta, \sigma^2_\gamma, \rho) = \sum_{i \epsilon s^*} w_{is^*} g^*_{i1}(\beta, \sigma^2_\gamma, \rho) = 0, \text{ and} \quad (11)$$

$$\hat{G}^*_2(\beta, \sigma^2_\gamma, \rho) = \sum_{i \epsilon s^*} w_{is^*} g^*_{i2}(\beta, \sigma^2_\gamma, \rho) = 0, \quad (12)$$

  respectively.
- Notice that the quantities $g^*_{i1}(\beta, \sigma^2_\gamma, \rho)$ in (11) and $g^*_{i2}(\beta, \sigma^2_\gamma, \rho)$ in (12) are the same GQL functions for the ith individual as shown in (6) and (9), respectively.
- Consequently, the equations in (11)-(12) are referred to as the weighted (GQL) (WGQL) estimating equations.

WGQL estimating equations in terms of survey design weights

Note that by writing

$$g^*_{i1}(\beta,\sigma^2_\gamma,\rho) = \frac{\partial \mu'_i}{\partial \beta}\Sigma_i^{-1}(\beta,\sigma^2_\tau,\rho)(y_i-\mu_i) = z^*_{is^*},$$

we can re-express the WGQL estimating equation (11) in terms of survey design weights, as

$$\sum_{i\in s^*} w_{is^*} z^*_{is^*} \equiv \sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*} z^*_{hcis^*} = \sum_{h=1}^{L}\sum_{c=1}^{n_h} z^*_{hc} = 0, \qquad (13)$$

where the stratum and cluster specific individual function now has the form

$$z^*_{hcis^*} \equiv \frac{\partial \mu'_{hcis^*}}{\partial \beta}\Sigma^{-1}_{hcis^*}(\beta,\sigma^2_\tau,\rho)(y_{hcis^*}-\mu_{hcis^*}).$$

Final iterative equations for β

The solution for (11) (same as (13)) for β, that is, $\hat{\beta}_N \equiv \hat{\beta}_{WGQL}$ may now be obtained by using the iterative equation

$$\hat{\beta}_{WGQL}(m+1) = \hat{\beta}_{WGQL}(m)$$

$$+ \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial \mu'_{hcis^*}}{\partial \beta}\Sigma^{-1}_{hcis^*}(\beta,\sigma^2_\gamma,\rho)\frac{\partial \mu_{hcis^*}}{\partial \beta'}\right]^{-1}_{m}$$

$$\times \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial \mu'_{hcis^*}}{\partial \beta}\Sigma^{-1}_{hcis^*}(\beta,\sigma^2_\gamma,\rho)(y_{hcis^*}-\mu_{hcis^*})\right]_{m} \quad (14)$$

Final iterative equation for σ^2_γ

In the manner similar to the estimation of β_N, the solution for (12) for σ^2_γ, that is, $\hat{\sigma}^2_{\gamma,N} \equiv \hat{\sigma}^2_{\gamma,WGQL}$ may be obtained by using the iterative equation

$$\hat{\sigma}^2_{\gamma,WGQL}(m+1) = \hat{\sigma}^2_{\gamma,WGQL}(m)$$

$$+ \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*} \frac{\partial \lambda'_{hcis^*}}{\partial \sigma^2_\gamma} \Omega^{-1}_{hcis^*}(\beta, \sigma^2_\gamma, \rho = 0) \frac{\partial \lambda_{hcis^*}}{\partial \sigma^2_\gamma}\right]^{-1}_m$$

$$\times \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*} \frac{\partial \lambda'_{hcis^*}}{\partial \sigma^2_\gamma} \Omega^{-1}_{hcis^*}(\beta, \sigma^2_\gamma, \rho = 0)(u_{hcis^*} - \lambda_{hcis^*})\right]_m \quad (15)$$

Asymptotic variance of the WGQL estimator

Asymptotic variance of $\hat{\beta}_{WGQL}$
Following Lindeberg-Feller central limit theorem (see Amemiya (1985, Theorem 3.3.6, p. 92; Sutradhar, Jowaheer, and Rao (2014, BJPS)), for example, under some mild regularity conditions on covariates, it may be shown that asymptotically (as $n = \sum_{h=1}^{L} \sum_{c=1}^{n_h} n_{hc} \rightarrow N \rightarrow \infty$)

$$\sqrt{n}(\hat{\beta}_{WGQL} - \beta) \sim N_p\left(0, nV\right), \quad (16)$$

where $\hat{\beta}_{WGQL}$ is obtained from (14). The covariance matrix V has the formula

Asymptotic variance of $\hat{\beta}_{WGQL}$

$$
\begin{aligned}
V &= \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial\mu'_{hcis^*}}{\partial\beta}\Sigma^{-1}_{hcis^*}(\beta,\sigma^2_\gamma,\rho)\frac{\partial\mu_{hcis^*}}{\partial\beta'}\right]^{-1} \\
&\times \mathrm{cov}\left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial\mu'_{hcis^*}}{\partial\beta}\Sigma^{-1}_{hcis^*}(\beta,\sigma^2_\gamma,\rho)(y_{hcis^*}-\mu_{hcis^*})\right] \\
&\times \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial\mu'_{hcis^*}}{\partial\beta}\Sigma^{-1}_{hcis^*}(\beta,\sigma^2_\gamma,\rho)\frac{\partial\mu_{hcis^*}}{\partial\beta'}\right]^{-1}. \qquad (17)
\end{aligned}
$$

A consistent estimator of the covariance matrix V

This consistent estimator is obtained as

$$\hat{V}_{WGQL} = \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial \mu'_{hcis^*}}{\partial \beta}\Sigma_{hcis^*}^{-1}(\beta,\sigma_\gamma^2,\rho)\frac{\partial \mu_{hcis^*}}{\partial \beta'}\right]^{-1}$$
$$\times \widehat{\text{cov}}\left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial \mu'_{hcis^*}}{\partial \beta}\Sigma_{hcis^*}^{-1}(\beta,\sigma_\gamma^2,\rho)(y_{hcis^*}-\mu_{hcis^*})\right]$$
$$\times \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial \mu'_{hcis^*}}{\partial \beta}\Sigma_{hcis^*}^{-1}(\beta,\sigma_\gamma^2,\rho)\frac{\partial \mu_{hcis^*}}{\partial \beta'}\right]^{-1}, \quad (18)$$

where, the middle term in (18) may be computed as

Consistent estimation of V -Continued

$$
\begin{aligned}
& \widehat{\text{cov}}\left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial \mu'_{hcis^*}}{\partial \beta}\Sigma^{-1}_{hcis^*}(\beta,\sigma^2_{\gamma},\rho)(y_{hcis^*}-\mu_{hcis^*})\right] \\
= & \widehat{\text{cov}}\left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*} z^*_{hcis^*}\right] \\
= & \left[\sum_{h=1}^{L}\left\{n_h(n_h-1)^{-1}\sum_{c=1}^{n_h}(z^*_{hc}-\bar{z}^*_h)(z^*_{hc}-\bar{z}^*_h)'\right\}\right], \qquad (19)
\end{aligned}
$$

where $\bar{z}^*_h = \sum_{c=1}^{n_h} z^*_{hc}/n_h$ is the $p \times 1$ mean vector.

Asymptotic variance of $\hat{\sigma}^2_{\gamma,WGQL}$

By using (15), it can be shown that asymptotically (as $n = \sum_{h=1}^{L}\sum_{c=1}^{n_h} n_{hc}$
$N \to \infty$)

$$\sqrt{n}(\hat{\sigma}^2_{\gamma,WGQL} - \sigma^2_\gamma) \sim N_1\left(0, nS\right), \tag{20}$$

where S has the form

$$S = \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial\lambda'_{hcis^*}}{\partial\sigma^2_\gamma}\Omega^{-1}_{hcis^*}(\beta,\sigma^2_\gamma,\rho=0)\frac{\partial\lambda_{hcis^*}}{\partial\sigma^2_\gamma}\right]^{-2} \tag{21}$$
$$\times\text{var}\left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial\lambda'_{hcis^*}}{\partial\sigma^2_\gamma}\Omega^{-1}_{hcis^*}(\beta,\sigma^2_\gamma,\rho=0)(u_{hcis^*}-\lambda_{hcis^*})\right].$$

Consistent estimator of S

$$\hat{S}_{WGQL} = \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial\lambda'_{hcis^*}}{\partial\sigma^2_\gamma}\Omega^{-1}_{hcis^*}(\beta,\sigma^2_\gamma,\rho=0)\frac{\partial\lambda_{hcis^*}}{\partial\sigma^2_\gamma}\right]^{-2}$$
$$\times \hat{\text{var}}\left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial\lambda'_{hcis^*}}{\partial\sigma^2_\gamma}\Omega^{-1}_{hcis^*}(\beta,\sigma^2_\gamma,\rho=0)(u_{hcis^*}-\lambda_{hcis^*})\right]$$
$$= \left[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\frac{\partial\lambda'_{hcis^*}}{\partial\sigma^2_\gamma}\Omega^{-1}_{hcis^*}(\beta,\sigma^2_\gamma,\rho=0)\frac{\partial\lambda_{hcis^*}}{\partial\sigma^2_\gamma}\right]^{-2}$$
$$\times\left[\sum_{h=1}^{L}\left\{n_h(n_h-1)^{-1}\sum_{c=1}^{n_h}(v^*_{hc}-\bar{v}^*_h)(v^*_{hc}-\bar{v}^*_h)'\right\}\right], \quad (22$$

where, for $v^*_{hc} = \sum_{i=1}^{n_{hc}} w_{hcis^*} v*_{hcis^*}$, $\bar{v}^*_h = \sum_{c=1}^{n_h} v^*_{hc}/n_h$.

Weighted method of moments (WMM) estimate for ρ_N

Recall that ρ_N is the survey population parameter representing dynamic dependence between two consecutive count responses of an individual in the survey population of size N. Suppose that

$$y^*_{it} = [y_{it} - \mu_{it}(\beta, \sigma^2_\gamma)]/[\sigma_{itt}(\beta, \sigma^2_\gamma)]^{1/2},$$

where $\mu_{it}(\cdot)$ and $\sigma_{itt}(\cdot)$ are the SM parameters defined in (2) and (3), respectively. One may then exploit (4) and obtain ρ_N, the moment estimate of ρ, as

$$\rho_N = \frac{a_{1,N} - b_{1,N}}{g_{1,N}}, \tag{23}$$

where

Weighted method of moments (WMM) estimate for ρ_N- Continued

$$
\begin{aligned}
a_{1,N} &= \sum_{i=1}^{N}\sum_{t=1}^{T-1} y^*_{it} y^*_{i(t+1)}/N(T-1)/\sum_{i=1}^{N}\sum_{t=1}^{T} {y^*_{it}}^2/NT \\
b_{1,N} &= (\exp(\sigma^2_\gamma)-1)\sum_{i=1}^{N}\sum_{t=1}^{T-1} m_{it}(\cdot)m_{i,t+1}(\cdot)/N(T-1) \\
g_{1,N} &= \sum_{i=1}^{N}\sum_{t=1}^{T-1} \mu_{it}(\beta,\sigma^2_\gamma)\left[\sigma_{itt}(\beta,\sigma^2_\gamma)\sigma_{i,t+1,t+1}(\beta,\sigma^2_\gamma)\right]^{-\frac{1}{2}}/N(T-1),
\end{aligned}
$$

with $m_{it}(\cdot) \equiv m_{it}(\beta,\sigma^2_\gamma) = \mu_{it}(\beta,\sigma^2_\gamma)/[\sigma_{itt}(\beta,\sigma^2_\gamma)]^{\frac{1}{2}}$.

Weighted method of moments (WMM) estimate for ρ_N- Continued

We now modify this formula (23) to reflect the estimation based on a two stage cluster sample (TSCS) s^* of size $n = \sum_{h=1}^{L} \sum_{c=1}^{n_h} n_{hc}$ taken from the survey population of N individuals. This modification is done by inserting the sampling weight w_{hcis^*} in (23) and summing appropriately over the individual $i \in s^*$. Thus the WMM (weighted method of moment) estimator of ρ_N has the formula given by

$$\hat{\rho}_N = \frac{a^*_{1,n} - b^*_{1,n}}{g^*_{1,n}}, \tag{24}$$

where

$$a^*_{1,n} = \frac{\frac{\sum_{t=1}^{T-1} \sum_{h=1}^{L} \sum_{c=1}^{n_h} \sum_{i=1}^{n_{hc}} w_{hcis^*} y^*_{hcis^*,t} y^*_{hcis^*,(t+1)}}{(T-1)[\sum_{h=1}^{L} \sum_{c=1}^{n_h} \sum_{i=1}^{n_{hc}} w_{hcis^*}]}}{\frac{\sum_{t=1}^{T} \sum_{h=1}^{L} \sum_{c=1}^{n_h} \sum_{i=1}^{n_{hc}} w_{hcis^*} {y^*_{hcis^*,t}}^2}{T[\sum_{h=1}^{L} \sum_{c=1}^{n_h} \sum_{i=1}^{n_{hc}} w_{hcis^*}]}},$$

and

Weighted method of moments (WMM) estimate for ρ_N- Continued

$$b^*_{1,n} = (\exp(\sigma^2_\gamma) - 1) \sum_{t=1}^{T-1} \sum_{h=1}^{L} \sum_{c=1}^{n_h} \sum_{i=1}^{n_{hc}} w_{hcis^*} m_{hcis^*,t}(\cdot) m_{hcis^*,t+1}(\cdot)$$

$$/ \quad (T-1)[\sum_{h=1}^{L} \sum_{c=1}^{n_h} \sum_{i=1}^{n_{hc}} w_{hcis^*}],$$

$$g^*_{1,n} = \sum_{t=1}^{T-1} \sum_{h=1}^{L} \sum_{c=1}^{n_h} \sum_{i=1}^{n_{hc}} w_{hcis^*} \mu_{hcis^*,t}(\beta, \sigma^2_\gamma) \left[\sigma_{hcis^*,tt}(\beta, \sigma^2_\gamma) \sigma_{hcis^*,t+1,}\right.$$

$$/ \quad (T-1)[\sum_{h=1}^{L} \sum_{c=1}^{n_h} \sum_{i=1}^{n_{hc}} w_{hcis^*}],$$

where $\mu_{hcis^*,t}$, and $\sigma_{hcis^*,tt}$ a are defined in (14), and it follows from (23) that $m_{hcis^*,t} = \mu_{hcis^*,t} / [\sigma_{hcis^*,tt}]^{1/2}$.

Weighted GQL method for longitudinal binary survey data

- We go back to the motivational SLID data problem with y_{it} as the unemployment status of the ith individual at time t.
- **SM: LDCP (linear dynamic conditional probability) super population model** For all $t = 1, \ldots, T$, let

$$\begin{aligned} Pr[Y_{it} = 1|x_{it}] &= \pi_{it}(\beta, x_{it}) \\ &= \left[1 + \exp(x'_{it}\beta)\right]^{-1} \exp(x'_{it}\beta), \end{aligned} \qquad (26)$$

and for $t = 2, \ldots, T$, we use the LDCP as

$$\begin{aligned} & Pr[Y_{it} = 1|y_{i,t-1}, x_{it}, x_{i,t-1}] \\ = & \pi_{it}(\beta, x_{it}) + \rho[y_{i,t-1} - \pi_{i,t-1}(\beta, x_{i,t-1})] \text{ for } t = 2, \ldots, T \\ = & \psi_{i,t|t-1}(\beta, \rho, x_{it}, x_{i,t-1}) \text{ (say)} \end{aligned} \qquad (27)$$

Basic properties of the LDCP SM model

$$
\begin{aligned}
E[Y_{it}|x_{it}] &= \pi_{it}(\beta, x_{it}) \text{ for all } t = 1, \ldots, T \\
\text{var}[Y_{it}|x_{it}] &= \sigma_{i,tt}(\beta, x_{it}) \\
&= \pi_{it}(\beta, x_{it})[1 - \pi_{it}(\beta, x_{it})], \text{ for all } t = 1, \ldots, T, \qquad (28)
\end{aligned}
$$

and

$$
\begin{aligned}
& corr(Y_{iu}, Y_{ik}|x_{iu}, x_{ik}) \\
&= \begin{cases} \rho^{k-u}\sqrt{\frac{\pi_{iu}(\beta,x_{iu})[1-\pi_{iu}(\beta,x_{iu})]}{\pi_{ik}(\beta,x_{ik})[1-\pi_{ik}(\beta,x_{ik})]}} & u < k \\ \rho^{u-k}\sqrt{\frac{\pi_{ik}(\beta,x_{ik})[1-\pi_{ik}(\beta,x_{ik})]}{\pi_{iu}(\beta,x_{iu})[1-\pi_{iu}(\beta,x_{iu})]}} & u > k, \end{cases} \qquad (29)
\end{aligned}
$$

respectively.

Basic properties of the LDCP SM model- Continued

Using (28) and (29), construct the mean vector

$$\begin{aligned} \pi_i(\cdot) &= (\pi_{i1}, \ldots, \pi_{it}, \ldots, \pi_{iT})' \\ \Sigma_{i,b}(\cdot) &= (\text{cov}(Y_{iu}, Y_{ik})), \; u, k = 1, \ldots, T. \end{aligned} \tag{30}$$

WGQL estimating equation for β

By writing

$$\tilde{g}_{i1}(\beta,\rho) = \frac{\partial \pi_i'}{\partial \beta} \Sigma_{i,b}^{-1}(\beta,\rho)(y_i - \pi_i) = \tilde{z}_{is^*},$$

following (13), the sampling design weights based GQL estimating equation is written as

$$\sum_{i \in s^*} w_{is^*} \tilde{z}_{is^*} \equiv \sum_{h=1}^{L} \sum_{c=1}^{n_h} \sum_{i=1}^{n_{hc}} w_{hcis^*} \tilde{z}_{hcis^*} = \sum_{h=1}^{L} \sum_{c=1}^{n_h} \tilde{z}_{hc} = 0, \qquad (31)$$

where the stratum and cluster specific individual function has the form

$$\tilde{z}_{hcis^*} \equiv \frac{\partial \pi_{hcis^*}'}{\partial \beta} \Sigma_{hcis^*,b}^{-1}(\beta,\rho)(y_{hcis^*} - \pi_{hcis^*}).$$

Weighted method of moments (WMM) estimate for ρ

Let

$$y^*_{it} = [y_{it} - \pi_{it}]/\sqrt{\sigma_{itt,b}}$$

with $\sigma_{itt,b} = \pi_{it}(1 - \pi_{it})$. Then equating the lag 1 correlation (29) under the **SM** with its counter part from the survey population, one obtains

$$\rho_N = \frac{\sum_{i=1}^{N}\sum_{t=2}^{T} y^*_{it} y^*_{i,t-1}}{\sum_{i=1}^{N}\sum_{t=1}^{T} y^{*2}_{it}} \frac{NT}{\sum_{i=1}^{N}\sum_{t=2}^{T}\left[\frac{\sigma_{i,t-1,t-1,b}}{\sigma_{i,tt,b}}\right]^{\frac{1}{2}}}. \quad (32)$$

Weighted method of moments (WMM) estimate for ρ

Consequently, sampling weights based estimator of ρ has the formula

$$\hat{\rho}_N = \frac{\sum_{t=2}^{T}\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*} y^*_{hcis^*,t} y^*_{hcis^*,(t-1)}}{\sum_{t=1}^{T}\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*} y^{*2}_{hcis^*,t}} \times \frac{T[\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}]}{\sum_{h=1}^{L}\sum_{c=1}^{n_h}\sum_{i=1}^{n_{hc}} w_{hcis^*}\left[\frac{\sigma_{hcis^*,t-1,t-1,b}}{\sigma_{hcis^*,tt,b}}\right]^{\frac{1}{2}}}. \quad (33)$$

SLID data analysis: An illustration

	Set up			
	Infinite population		Finite population	
Parameters	Estimate	SE	Estimate	SE
Male vs Female (x_1)	-0.638	0.066	-0.502	0.074
Age group 2 vs 1 (x_2)	-1.688	0.057	-1.285	0.087
Age group 3 vs 1 (x_3)	-2.489	0.115	-1.864	0.154
Quebec vs Atlantic (x_4)	-0.762	0.078	-1.249	0.157
Ontario vs Atlantic (x_5)	-1.052	0.088	-1.528	0.110
Praries vs Atlantic (x_6)	-1.702	0.114	-2.061	0.136
BC & Alberta vs Atlantic (x_7)	-1.482	0.169	-1.955	0.191
Education medium vs low (x_8)	-1.681	0.058	-1.589	0.092
Education high vs low (x_9)	-2.446	0.153	-2.609	0.236
Marital status 2 vs 1 (x_{10})	0.193	0.096	0.243	0.142
Marital status 3 vs 1 (x_{11})	-0.688	0.248	-0.480	0.379
Marital status 4 vs 1 (x_{12})	-0.566	0.077	-0.343	0.146
ρ	0.393	–	0.360	–

BDL SM that generates recursive means over time

$$
\begin{aligned}
& Pr[Y_{i1} = 1|x_{i1}] \\
= \;& \pi_{i1}(\beta|x_{i1}) = \frac{\exp(x'_{i1}\beta)}{1+\exp(x'_{i1}\beta)}, \qquad (34) \\
& Pr[Y_{it} = 1|y_{i,t-1}, x_{it}] \\
= \;& \frac{\exp(x'_{it}\beta + \theta y_{i,t-1})}{1+\exp(x'_{it}\beta + \theta y_{i,t-1})}\; t = 2, \ldots, T, \\
= \;& p_{i,t|t-1}(\beta, \theta|x_{it}, y_{i,t-1}), \qquad (35)
\end{aligned}
$$

where θ is a dynamic dependence parameter which is quite different than ρ in (27).

BDL SM that generates recursive means over time- Continued

In order to derive the recursive unconditional mean, it is convenient to use the notation

$$\begin{aligned}
\pi_{it}(\beta|x_{ij}) &= p_{i,t|t-1}(\beta,\theta|x_{it},y_{i,t-1}=0) \\
&= \frac{\exp(x'_{it}\beta)}{1+\exp(x'_{it}\beta)},\ t=2,\ldots,T \text{ and} \\
\tilde{p}_{it}(\beta,\theta) &= p_{i,t|t-1}(\beta,\theta|x_{it},y_{i,t-1}=1) \\
&= \frac{\exp(x'_{it}\beta+\theta)}{1+\exp(x'_{it}\beta+\theta)},\ t=2,\ldots,T.
\end{aligned}$$

Notice that because of (34), the above formula for $\pi_{it}(\beta|x_{it})$ is true for all $t=1,\ldots,T$.

Basic properties of the BDL SM- Continued

Using these notations, by (34) and (35) one may then write the unconditional expectation as

$$\mu_{it}(\beta,\theta) = E[Y_{it}] = Pr[Y_{it}=1]$$
$$= \begin{cases} \pi_{i1}(\beta) & t=1 \\ \pi_{it}(\beta) + \mu_{i,t-1}(\beta,\theta)[\tilde{p}_{it}(\beta,\theta) - \pi_{it}(\beta)] & t=2,\ldots,T, \end{cases} \quad (36)$$

which yields the unconditional variance as

$$\sigma_{itt}(\beta,\theta) = \mu_{it}(\beta,\theta)[1-\mu_{it}(\beta,\theta)], \text{ for } t=1,\ldots,T. \quad (37)$$

Basic properties of the BDL SM- Continued

Also for $j < k$, following Sutradhar and Farrell (2007, Sankhya B) for example, we may compute the unconditional pair-wise covariances as

$$\begin{aligned} \sigma_{ijk}(\beta,\theta) &= \text{cov}(Y_{ij}, Y_{ik}) \\ &= \mu_{ij}(\beta,\theta)[1-\mu_{ij}(\beta,\theta)] \\ &\times \Pi_{u=j+1}^{k}[\tilde{p}_{iu}(\beta,\theta) - \pi_{iu}(\beta)], \end{aligned} \quad (38)$$

and may further obtain the pair-wise lag $(k-j)$ correlations given by

$$\begin{aligned} \text{corr}(Y_{ij}, Y_{ik}) &= \sqrt{\frac{\sigma_{ijj}(\beta,\theta)}{\sigma_{ikk}(\beta,\theta)}} \\ &\times \Pi_{u=j+1}^{k}[\tilde{p}_{iu}(\beta,\theta) - \pi_{iu}(\beta)] \end{aligned} \quad (39)$$

which satisfies the full range from -1 to 1, as

$$0 < \tilde{p}_{iu}(\beta,\theta), \pi_{iu}(\beta) < 1.$$

Survey population based likelihood estimation

Using (34) and (35), write the likelihood for β and θ as

$$\begin{aligned} L(\beta,\theta) &= \Pi_{i=1}^{N}\left[\{\pi_{i1}(\beta)\}^{y_{i1}}\right. \\ \times & \left.\Pi_{t=2}^{T}\{p_{i,t|t-1}(\beta,\theta|x_{it},y_{i,t-1})\}^{y_{it}}\right], \end{aligned} \quad (40)$$

leading to the log likelihood estimating equation for $\phi = (\beta', \theta)'$ given by

$$\frac{\partial \log L}{\partial \phi} = \sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it} - p_{it|t-1})\begin{pmatrix} x_{it} \\ y_{i,t-1} \end{pmatrix} = 0, \quad (41)$$

where we have used $y_{i0} = 0$ as a conventional notation.

WGQL estimating equations in terms of survey design weights

Note that by writing

$$\tilde{g}^*_{i1}(\beta,\theta) = \sum_{t=1}^{T} (y_{it} - p_{it|t-1}) \begin{pmatrix} x_{it} \\ y_{i,t-1} \end{pmatrix} = \tilde{z}^*_{is^*},$$

we may follow (41) and write the WML (weighted maximum likelihood) estimating equation in terms of survey design weights, as

$$\sum_{i \in s^*} w_{is^*} \tilde{z}^*_{is^*} \equiv \sum_{h=1}^{L} \sum_{c=1}^{n_h} \sum_{i=1}^{n_{hc}} w_{hcis^*} \tilde{z}^*_{hcis^*} = \sum_{h=1}^{L} \sum_{c=1}^{n_h} \tilde{z}^*_{hc} = 0, \qquad (42)$$

where the stratum and cluster specific individual function now has the form

$$\tilde{z}^*_{hcis^*} \equiv \sum_{t=1}^{T} (y_{hcis^*,t} - p_{hcis^*,t|t-1}) \begin{pmatrix} x_{hcis^*,t} \\ y_{hcis^*,t-1} \end{pmatrix}.$$

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