

Dent & Min (1978), J. of Econometrics, 7, 23-55.

Table 1
The first-order autoregressive process: Monte Carlo results for various estimators based on sample size 100 using 100 simulation trials.^a

Parameter	True value	Max. likelihood			Least squares			Approx. max. likelihood			Yule-Walker			Regression			Kendall			Quenouille		
		Bias	Var.	MSE	Bias	Var.	MSE	Bias	Var.	MSE	Bias	Var.	MSE	Bias	Var.	MSE	Bias	Var.	MSE	Bias	Var.	MSE
ϕ^1	-0.9	0.0079	0.0017	0.0017	-0.0014	0.0018	0.0018	0.0077	0.0017	0.0017	0.0180	0.0018	0.0074	0.0017	0.0017	0.0112	0.0021	0.0021	-0.0116	0.0017	0.0034	
σ^2	1.0	-0.0053	0.0217	0.0217	-0.0058	0.0217	0.0217	-0.0054	0.0217	0.0217	-0.0018	0.0220	0.0221	-0.0048	0.0217	0.0218	-0.0028	0.0219	0.0219	0.0073	0.0252	0.0253
ϕ^1	-0.7	0.0180	0.0067	0.0070	0.0110	0.0068	0.0070	0.0179	0.0067	0.0070	0.0255	0.0068	0.0075	0.0179	0.0067	0.0070	0.0237	0.0075	0.0081	-0.0011	0.0095	0.0095
σ^2	1.0	-0.0361	0.0196	0.0209	-0.0362	0.0196	0.0209	-0.0361	0.0196	0.0209	-0.0354	0.0196	0.0209	-0.0360	0.0196	0.0209	-0.0345	0.0197	0.0209	-0.0329	0.0199	0.0210
ϕ^1	-0.6	0.0017	0.0054	0.0054	0.0078	0.0055	0.0057	0.0017	0.0054	0.0054	0.0032	0.0056	0.0056	0.0023	0.0056	0.0056	0.0012	0.0073	0.0073	-0.0146	0.0059	0.0061
σ^2	1.0	-0.0311	0.0224	0.0234	-0.0312	0.0224	0.0234	-0.0311	0.0224	0.0234	-0.0308	0.0225	0.0234	-0.0311	0.0224	0.0234	-0.0259	0.0224	0.0234	-0.0305	0.0224	0.0233
ϕ^1	-0.5	0.0004	0.0068	0.0068	0.0054	0.0069	0.0070	0.0003	0.0068	0.0068	-0.0035	0.0067	0.0067	-0.0010	0.0068	0.0068	0.0003	0.0079	0.0079	-0.0118	0.0068	0.0070
σ^2	1.0	-0.0038	0.0152	0.0152	-0.0038	0.0152	0.0152	-0.0038	0.0152	0.0152	-0.0036	0.0152	0.0152	-0.0037	0.0152	0.0152	-0.0025	0.0153	0.0153	-0.0034	0.0152	0.0152
ϕ^1	-0.3	0.0092	0.0086	0.0087	0.0063	0.0088	0.0089	0.0092	0.0086	0.0087	0.0123	0.0086	0.0087	0.0093	0.0088	0.0088	0.0162	0.0091	0.0093	0.0041	0.0093	0.0093
σ^2	1.0	-0.0214	0.0207	0.0211	-0.0214	0.0207	0.0211	-0.0214	0.0207	0.0211	-0.0213	0.0207	0.0211	-0.0214	0.0207	0.0211	-0.0207	0.0207	0.0211	-0.0213	0.0207	0.0211
ϕ^1	-0.1	0.0106	0.0111	0.0112	0.0117	0.0114	0.0115	0.0106	0.0111	0.0112	0.0096	0.0110	0.0111	0.0102	0.0111	0.0112	0.0102	0.0111	0.0112	-0.0132	0.0117	0.0119
σ^2	1.0	-0.0269	0.0193	0.0200	-0.0269	0.0193	0.0200	-0.0269	0.0193	0.0200	-0.0269	0.0193	0.0200	-0.0269	0.0193	0.0200	-0.0267	0.0193	0.0200	-0.0266	0.0193	0.0200
ϕ^1	0.1	0.0061	0.0092	0.0093	0.0051	0.0094	0.0095	0.0061	0.0092	0.0093	0.0070	0.0091	0.0092	0.0059	0.0093	0.0093	0.0090	0.0089	0.0090	-0.0046	0.0096	0.0096
σ^2	1.0	-0.0293	0.0287	0.0295	-0.0293	0.0287	0.0295	-0.0293	0.0287	0.0295	-0.0293	0.0287	0.0295	-0.0293	0.0287	0.0295	-0.0292	0.0287	0.0295	-0.0293	0.0287	0.0295
ϕ^1	0.3	0.0147	0.0076	0.0078	0.0118	0.0078	0.0079	0.0147	0.0076	0.0078	0.0180	0.0074	0.0078	0.0145	0.0075	0.0077	0.0205	0.0077	0.0081	-0.0094	0.0080	0.0080
σ^2	1.0	-0.0143	0.0191	0.0193	-0.0143	0.0191	0.0193	-0.0143	0.0191	0.0193	-0.0142	0.0191	0.0193	-0.0143	0.0191	0.0193	-0.0137	0.0191	0.0193	-0.0142	0.0191	0.0193
ϕ^1	0.5	0.0360	0.0089	0.0102	0.0313	0.0091	0.0101	0.0360	0.0089	0.0102	0.0415	0.0085	0.0102	0.0367	0.0087	0.0101	0.0388	0.0094	0.0109	-0.0289	0.0090	0.0098
σ^2	1.0	-0.0038	0.0230	0.0230	-0.0038	0.0230	0.0230	-0.0038	0.0230	0.0230	-0.0036	0.0230	0.0230	-0.0037	0.0230	0.0230	-0.0027	0.0230	0.0231	-0.0035	0.0230	0.0230
ϕ^1	0.6	0.0338	0.0080	0.0092	0.0280	0.0082	0.0090	0.0338	0.0080	0.0092	0.0404	0.0078	0.0095	0.0351	0.0079	0.0092	0.0383	0.0090	0.0105	-0.0245	0.0084	0.0090
σ^2	1.0	-0.0242	0.0193	0.0199	-0.0242	0.0193	0.0200	-0.0242	0.0193	0.0199	-0.0237	0.0193	0.0199	-0.0240	0.0194	0.0199	-0.0229	0.0194	0.0199	-0.0238	0.0193	0.0199
ϕ^1	0.7	0.0405	0.0053	0.0069	0.0337	0.0054	0.0065	0.0404	0.0053	0.0069	0.0471	0.0055	0.0077	0.0405	0.0055	0.0071	0.0472	0.0063	0.0085	-0.0260	0.0069	0.0076
σ^2	1.0	-0.0318	0.0235	0.0245	-0.0319	0.0235	0.0245	-0.0318	0.0235	0.0245	-0.0313	0.0235	0.0245	-0.0317	0.0235	0.0245	-0.0304	0.0235	0.0244	-0.0305	0.0236	0.0245
ϕ^1	0.9	0.0402	0.0032	0.0048	0.0311	0.0033	0.0043	0.0398	0.0032	0.0048	0.0557	0.0036	0.0068	0.0425	0.0035	0.0053	0.0503	0.0041	0.0067	-0.0294	0.0042	0.0051
σ^2	1.0	-0.0335	0.0180	0.0191	-0.0338	0.0180	0.0191	-0.0335	0.0180	0.0191	-0.0329	0.0185	0.0193	-0.0323	0.0181	0.0191	-0.0300	0.0183	0.0192	-0.0282	0.0181	0.0189

^a Asymptotic variances of maximum likelihood estimators are, respectively, 0.0019, 0.0051, 0.0064, 0.0091, 0.0099, 0.0099, 0.0075, 0.0064, 0.0075, 0.0091, 0.0099, 0.0099, 0.0091, 0.0075, 0.0064, 0.0051, 0.0019.

Table 4
The first-order moving average process: Monte Carlo results for various estimators based on sample size 100 using 100 simulation trials.^a

Parameter value	Max. likelihood			Uncond. least sq.			Cond. least sq.			Walker			Method of moments		
	Bias	Var.	MSE	Bias	Var.	MSE	Bias	Var.	MSE	Bias	Var.	MSE	Bias	Var.	MSE
θ_1	-0.95	0.0131	0.0010	0.0016	0.0010	0.0010	0.0290	0.0026	0.0035	0.1754	0.0081	0.0389	0.3267	0.0169	0.1237
σ^2	1.0	-0.0083	0.0176	-0.0079	0.0165	0.0166	0.0318	0.0206	0.0217	0.1703	0.0553	0.0843	0.2781	0.0514	0.1287
θ_1	-0.7	0.0008	0.0081	-0.0073	0.0085	0.0085	0.0117	0.0079	0.0081	0.0229	0.0114	0.0119	0.1315	0.0196	0.0369
σ^2	1.0	-0.0297	0.0185	-0.0300	0.0195	0.0204	-0.0162	0.0197	0.0199	-0.0313	0.0217	0.0227	0.0532	0.0285	0.0313
θ_1	-0.6	-0.0075	0.0082	-0.0139	0.0089	0.0091	-0.0029	0.0078	0.0078	-0.0024	0.0103	0.0103	0.0962	0.0176	0.0269
σ^2	1.0	-0.0199	0.0181	-0.0223	0.0179	0.0184	-0.0130	0.0179	0.0180	-0.0263	0.0207	0.0214	0.0176	0.0275	0.0278
θ_1	-0.5	-0.0005	0.0110	-0.0061	0.0114	0.0114	0.0041	0.0105	0.0105	0.0094	0.0117	0.0118	0.0367	0.0221	0.0235
σ^2	1.0	-0.0185	0.0209	-0.0185	0.0203	0.0206	-0.0140	0.0207	0.0209	-0.0203	0.0206	0.0210	-0.0143	0.0241	0.0243
θ_1	-0.3	0.0113	0.0082	0.0075	0.0079	0.0080	0.0107	0.0079	0.0080	0.0139	0.0080	0.0081	0.0184	0.0127	0.0131
σ^2	1.0	-0.0083	0.0260	-0.0085	0.0257	0.0258	-0.0083	0.0262	0.0262	-0.0107	0.0253	0.0254	-0.0111	0.0265	0.0266
θ_1	-0.1	0.0039	0.0127	0.0026	0.0129	0.0129	0.0034	0.0128	0.0128	0.0054	0.0125	0.0125	0.0118	0.0112	0.0113
σ^2	1.0	-0.0203	0.0171	-0.0189	0.0170	0.0173	-0.0191	0.0171	0.0175	-0.0212	0.0169	0.0173	-0.0191	0.0167	0.0170
θ_1	0.1	0.0186	0.0084	0.0190	0.0085	0.0089	0.0173	0.0083	0.0086	0.0180	0.0084	0.0087	0.0187	0.0094	0.0098
σ^2	1.0	-0.0093	0.0180	-0.0102	0.0179	0.0180	-0.0107	0.0179	0.0180	-0.0114	0.0176	0.0178	-0.0125	0.0177	0.0178
θ_1	0.3	0.0190	0.0091	0.0203	0.0092	0.0096	0.0159	0.0091	0.0094	0.0117	0.0089	0.0090	-0.0059	0.0142	0.0142
σ^2	1.0	-0.0122	0.0184	-0.0134	0.0186	0.0188	-0.0119	0.0182	0.0184	-0.0169	0.0187	0.0190	-0.0105	0.0191	0.0192
θ_1	0.5	0.0110	0.0072	0.0154	0.0074	0.0076	0.0098	0.0070	0.0071	0.0026	0.0079	0.0079	-0.0318	0.0202	0.0212
σ^2	1.0	-0.0161	0.0162	-0.0152	0.0168	0.0170	-0.0105	0.0170	0.0171	-0.0098	0.0185	0.0186	-0.0020	0.0208	0.0208
θ_1	0.6	0.0271	0.0079	0.0346	0.0082	0.0094	0.0202	0.0074	0.0078	0.0219	0.0118	0.0123	-0.0343	0.0201	0.0212
σ^2	1.0	-0.0097	0.0265	-0.0111	0.0273	0.0274	-0.0021	0.0273	0.0273	-0.0105	0.0330	0.0331	-0.0003	0.0305	0.0305
θ_1	0.7	0.0254	0.0060	0.0337	0.0065	0.0076	0.0228	0.0067	0.0072	0.0245	0.0115	0.0121	-0.0737	0.0194	0.0248
σ^2	1.0	-0.0106	0.0185	-0.0142	0.0180	0.0182	-0.0058	0.0182	0.0182	-0.0137	0.0213	0.0215	0.0337	0.0388	0.0399
θ_1	0.9	0.0112	0.0017	0.0226	0.0020	0.0025	-0.0065	0.0032	0.0032	-0.1222	0.0074	0.0223	-0.2994	0.0204	0.1100
σ^2	1.0	-0.0081	0.0203	-0.0105	0.0199	0.0200	0.0227	0.0223	0.0228	0.1303	0.0455	0.0625	0.2583	0.0446	0.1113
θ_1	0.95	-0.0234	0.0017	-0.0117	0.0017	0.0018	-0.0388	0.0031	0.0046	-0.1769	0.0089	0.0402	-0.2928	0.0187	0.1045
σ^2	1.0	0.0066	0.0183	0.0019	0.0180	0.0180	0.0396	0.0207	0.0223	0.2139	0.0622	0.1080	0.2929	0.0519	0.1377

^aAsymptotic variances of the maximum likelihood estimators of θ_1 are, respectively, 0.0010, 0.0051, 0.0064, 0.0075, 0.0091, 0.0099, 0.0099, 0.0091, 0.0075, 0.0064, 0.0051, 0.0064, 0.0075, 0.0091, 0.0099, 0.0099, 0.0091, 0.0075, 0.0064, 0.0051, 0.0064, 0.0075, 0.0091, 0.0099, 0.0099, 0.0091.

$$(1 - \phi B)y_t = (1 - \theta B)\epsilon_t$$

Table 5
 Estimated coefficient biases and mean squared errors and estimated forecast mean squared errors for ARMA(1, 1) processes (n = 50; 600 replications).

ϕ	θ	Bias $\hat{\phi}$			MSE $\hat{\phi}$ ($\times 10^3$)			Bias $\hat{\theta}$			MSE $\hat{\theta}$ ($\times 10^3$)			V(1)		
		ML	ELS	CLS	ML	ELS	CLS	ML	ELS	CLS	ML	ELS	CLS	ML	ELS	CLS
-0.95	-1.00	0.172	0.096	0.505	90.5	28.1	502	0.112	0.042	0.468	63.8	23.2	510	1.054	1.050	1.065
-0.95	-0.85	0.090	0.067	0.298	52.0	50.7	298	0.074	0.050	0.286	70.8	86.2	320	1.036	1.058	1.057
-0.95	-0.40	0.039	0.017	0.046	9.41	7.09	13.2	0.026	0.006*	0.029	33.4	34.3	36.8	1.037	1.053	1.048
-0.95	0.40	0.028	0.012	0.033	4.78	4.85	5.54	0.015	0.033	0.017	22.7	30.2	23.6	1.043	1.051	1.049
-0.95	0.85	0.009	0.009	0.024	4.40	3.86	4.36	0.025	0.053	-0.012	9.04	12.5	10.1	1.039	1.063	1.047
-0.95	1.00	0.026	0.007	0.023	3.91	3.20	5.19	-0.028	-0.008	-0.098	3.11	1.12	17.8	1.055	1.050	1.101
-0.50	-1.00	0.064	0.054	0.184	29.0	20.9	91.7	0.041	0.011	0.180	9.60	2.09	104	1.059	1.053	1.114
-0.50	-0.85	0.061	0.053	0.191	57.6	59.2	122	0.001*	-0.017	0.143	38.6	40.1	121	1.041	1.059	1.104
-0.50	-0.40	0.119	0.064	0.191	234	214	283	0.114	0.047	0.178	287	255	323	1.044	1.047	1.045
-0.50	0.40	0.038	0.031	0.016	34.1	35.4	29.5	0.032	0.040	0.018	40.8	43.0	40.3	1.049	1.049	1.050
-0.50	0.85	0.007*	0.004*	-0.001*	17.1	17.1	18.1	0.025	0.056	-0.016	11.1	12.9	12.6	1.047	1.061	1.054
-0.50	1.00	0.005*	-0.008	-0.026	14.4	14.7	18.4	-0.032	-0.008	-0.105	4.18	0.95	20.0	1.066	1.047	1.100
0.50	-1.00	-0.013	0.006*	0.010	13.8	13.8	20.1	0.027	0.007	0.105	3.46	0.82	20.8	1.055	1.046	1.095
0.50	-0.85	-0.020	-0.010	0.007*	19.5	17.6	17.8	-0.021	-0.057	0.018	10.5	13.7	12.5	1.045	1.068	1.058
0.50	-0.40	-0.033	-0.020	0.016	31.9	31.3	30.0	-0.036	-0.034	-0.013*	40.8	45.0	42.9	1.046	1.053	1.051
0.50	0.40	-0.120	-0.064	-0.181	241	199	282	-0.114	-0.056	-0.164	294	253	329	1.043	1.044	1.046
0.50	0.85	-0.072	0.046	-0.183	60.9	54.8	118	-0.009*	0.025	-0.128	40.4	38.2	104	1.046	1.062	1.059
0.50	1.00	-0.064	0.055	-0.182	30.1	24.8	90.3	-0.040	-0.011	-0.181	9.36	4.76	96.9	1.059	1.054	1.104
0.95	-1.00	-0.028	-0.004	-0.024	3.71	3.08	4.08	0.030	0.008	0.105	3.72	1.10	20.3	1.056	1.049	1.101
0.95	-0.85	-0.026	-0.010	-0.025	3.64	4.06	4.76	-0.020	-0.056	0.019	10.1	12.6	11.6	1.039	1.065	1.046
0.95	-0.40	-0.030	-0.007	-0.035	5.16	4.12	6.02	-0.019	-0.028	-0.018	23.8	23.4	23.9	1.039	1.051	1.044
0.95	0.40	-0.046	-0.017	-0.047	14.0	9.91	13.2	-0.034	-0.012*	-0.015*	38.4	38.9	38.6	1.042	1.060	1.054
0.95	0.85	-0.096	-0.050	-0.281	67.4	39.9	258	-0.071	-0.028	-0.264	87.6	71.7	276	1.041	1.070	1.051
0.95	1.00	-0.168	-0.103	-0.526	93.9	42.4	520	-0.110	-0.051	-0.500	68.6	36.6	547	1.048	1.050	1.060

*Bias is not statistically significant at 5% level.

Table 7

Percentage times 95% confidence intervals contain the true parameter values for first-order autoregressive models with ML estimation (1,000 replications).

ϕ	$n=50$	$n=100$
0.40	95.6	95.9
0.75	94.5	94.3
0.95	60.6	63.2

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Table 8

Percentage times 95% confidence intervals contain the true parameter values for first-order moving average models with ML estimation (1,000 replications).

θ	$n=50$	$n=100$
-0.9	99.0	97.8
-0.8	96.3	88.2
-0.4	91.5	92.0

Table 9
 Percentage times 95% confidence intervals contain the true parameter values for ARMA(1,1) models with ML estimation (1,000 replications, $n=50$; 600 replications, $n=100$).

ϕ	θ	ϕ Interval		θ Interval	
		$n=50$	$n=100$	$n=50$	$n=100$
0.50	-0.85	91.3	93.7	98.5	89.0
0.50	-0.40	83.1	85.7	78.2	84.3
0.50	0.40	35.3	43.2	33.7	43.2
0.50	0.85	77.7	74.2	91.1	76.3
0.95	-0.85	96.1	94.8	98.8	90.5
0.95	-0.40	93.4	94.8	90.6	92.8
0.95	0.40	87.6	89.8	84.6	90.3
0.95	0.85	66.2	68.7	73.0	68.5

Table 10
 Estimated coefficient biases and mean squared errors and percentage times estimates fall in the interval $-1.00 < \hat{\theta} \leq -0.99$ for first-order moving average processes (1,000 replications).

θ	$n=50$			$n=100$		
	Bias	MSE	Percentage $-1.00 < \hat{\theta} \leq -0.99$	Bias	MSE	Percentage $-1.00 < \hat{\theta} \leq -0.99$
-1.00	0.031	0.0042	65.0%	0.015	0.0009	67.1%
-0.99	0.021	0.0036	64.1	0.006	0.0008	65.4
-0.95	-0.005	0.0042	52.6	-0.010	0.0017	36.1
-0.90	-0.017	0.0067	32.8*	-0.011	0.0031	11.9
-0.85	-0.015	0.0107	19.5	-0.012	0.0044	4.9
-0.80	-0.026	0.0117	10.3	-0.011	0.0051	1.8
-0.70	-0.016	0.0163	4.5	-0.006	0.0064	0.2
-0.60	-0.017	0.0175	1.3	-0.007	0.0081	0.1
-0.40	-0.008	0.0222	0.1	-0.005	0.0100	0.0
0.00	0.001	0.0258	0.0	-0.002	0.0121	0.0

*Based on 10,000 replications.

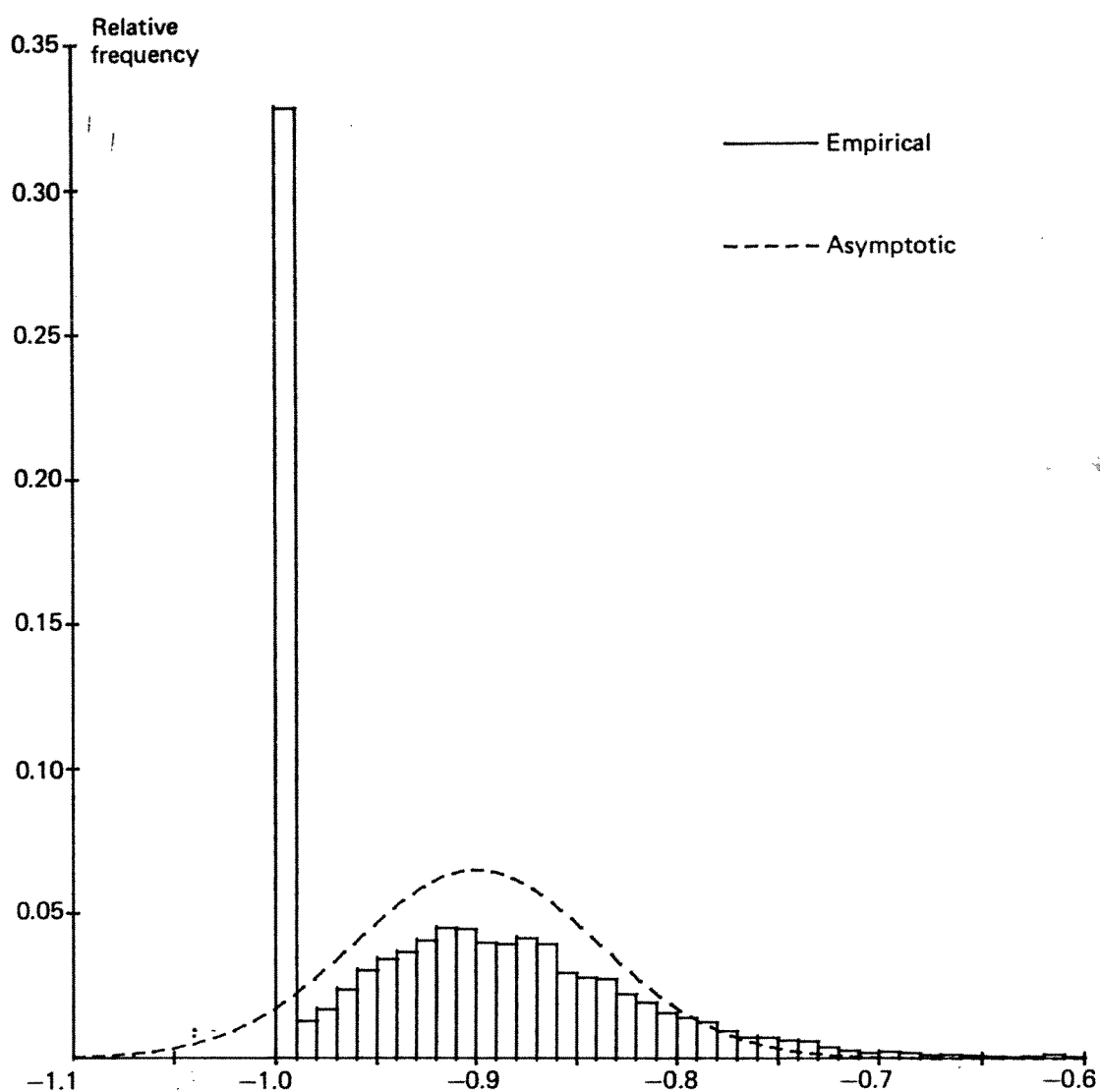


Fig. 1. Comparison of empirical and asymptotic distributions of ML estimator $\hat{\theta}$ for the MA(1) model $X_t = a_t + 0.9a_{t-1}$ ($n = 50$; 10,000 replications).