Ex. 5 Part I Fall 2013 / SOLUTIONS

1.1. a) a.1) $P = 1/6 \times 1/6 = 1/36$
a.2) $P = 1/6$
a.3) $P = 0$ (impossible event)
b) 0.5565
c) $z = 0.405$

1.2. 100 voters => $n = 100$ and $p^* = 0.55$ (= 55 %)
a) 95 % confidence limits for p can be obtained by:
$0.55 \pm 1.96 \sqrt{0.55 \times 0.44 / 100} \iff 0.55 \pm 0.10$
Thus, the 95 % confidence interval is: $0.45 < p < 0.65$.
b) We want to be sure that according to the opinion poll the candidate will get more than 50 % of the votes. Determinene the needed sample size (by using the assumption that $p^* = 0.55$).
Thus, the estimation error Δ must be less than 55 % - 50 % = 5 % (= 0.05)
95 % confident: Carry out the calculations by using similar argumentation as in a) yielding

$$1.96 \times \sqrt{0.55 \times 0.45 / n} = 0.05 \iff \sqrt{n} = 1.96 \times 0.5 / 0.05 \Rightarrow \sqrt{n} = 19.6$$

Thus: $n = (19.6)^2$, leading to $n \geq 385$ (too large due to some rounding: $n \geq 381$ is the correct answer).

1.3. Two groups: A (treatment group) and B (control group) 100 people in each.
Proportion of recovery

A	75 %	(= p(A))
B	65 %	(= p(B))

Hypothesis: The treatment helps to cure the disease.

H(0): $p(A) = p(B)$
H(1): $P(A) > P(B)$ (one-tailed test)

Use normal approximation (z –transformation) to test H(0): $p(A) – p(B) = 0$.

Under H(0): $\mu = 0$

$$\sigma = \sqrt{p\, q\, (1 / N(A) + 1 / N(B))} = \sqrt{0.3 \times 0.7\, (1 / 100 + 1 / 100)} = 0.0684$$

How to estimate p ? It is the weighted average of the observed proportions in the two groups:

$$p = (N(A) \times p(A) + N(B) \times p(B)) / (N(A) + N(B)) = (0.75 + 0.65) / 2 = 0.7.$$

$$z = (P(A) – P(B)) / \sigma = (0.75 – 0.65) / 0.0648 = 1.54.$$

a) $\alpha = 0.05$ & one-tailed test. The critical z-value = 1.645.
Cannot reject H(0).

b) $\alpha = 0.10$ & one-tailed test. The critical z-value = 1.28.
Can reject H(0)

Statistical conclusion: The serum is effective at a 0.10 level of significance.

1.4. Two sample polls of voters (different districts):

A	300 voters	56 % in favor of the candidate
B	200 voters	48 % in favor of the candidate

a) Is there a difference between the districts ?

H(0): p(A) = p(B)
H(1): p(A) ≠ p(B) (two-tailed test)

Use the same formulas as earlier:
$\sigma = \sqrt{p\,q\,(1 / N(A) + 1 / N(B))} = \sqrt{0.528 \times 0.472\,(1 / 300 + 1 / 200)} = 0.0456$

Here: $p = (N(A) \times p(A) + N(B) \times p(B)) / (N(A) + N(B))$
$= (300 \times 0.56 + 200 \times 0.48) / 500$
$= 0.528$
$q = 1 - p = 0.472$

$z = (p(A) - p(B)) / \sigma = (0.560 - 0.480) / 0.0456 = 1.75$

Conclusion: Cannot reject H(0).

b) Test whether the candidate is preferred in district A.

H(0): p(A) = p(B)
H(1): p(A) > p(B) (one-tailed test)

Critical z-value (5 %) = 1.645

Conclusion: H(0) is rejected.

1.5. 6 tosses of a coin gives the result "6 Heads". Can we conclude that the coin is not fair ?

p = probability of "Heads" in a single toss.

Use Binomial distribution with 6 repetitions, with p = 0.5 (= H(0))

P(6 Heads) = 1/64 = 0.01562
(P(5 or 6 Heads) = 7/64 = 0.1095)

a) One-tailed test.
We can reject H(0) at a 0.05 level but not at a 0.01 level of significance.

b) Two tailed test. Now the critical region is symmetric (= {0 or 6 Heads})

P(0 or 6 Heads) = 1/64 + 1/64 = 0.03125.
We can reject H(0) at a 0.05 level but not at a 0.01 level of significance.