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Small area estimation with calibration methods

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Outline

Preliminaries

Calibration weighting systems

Monte Carlo experiments

Discussion

Literature

This is joint work with my colleague Dr Ari Veijanen of Statistics Finland

Design-based calibration methods for domain estimation to be discussed

Traditional model-free calibration MFC

Deville J.-C. and Särndal C.-E. (1992), Särndal C.-E. (2007)

Deville J.-C., Särndal C.-E. and Sautory (1993) (CALMAR I,II,...)

Estevao & Särndal (1999, 2004), Lehtonen & Veijanen (2009)

Model-assisted calibration MC

Lehtonen & Veijanen (2012, 2016)

Wu & Sitter (2001), Montanari & Ranalli (2005) (Model calibration)

Hybrid calibration HC

Lehtonen & Veijanen (2015)

Montanari & Ranalli (2009) (Multiple model calibration)

Two-level hybrid calibration HC2

Lehtonen and Veijanen (2017)

Some key properties

(under complete response)

	Study variable y	Estimator type	Models	Expected gain
MFC	continuous	Direct	No explicit model statement (Implicit linear fixed-effects model)	Multi-purpose weighting Coherence with published statistics
MC	continuous binary polytomous count	Indirect (essentially)	Explicit model statement GLMM family e.g. logistic mixed model	Accuracy improvement
HC				Accuracy improvement (MC part) Partial coherence (MFC part)
HC2				Same as HC1 and more: Reduction of instability of MFC part in small domains

Questions of interest

Relative design-based properties of MFC, MC, HC and HC2

Accuracy properties

Distributional properties of calibrated weights

Comparison with model-based SAE

Design bias and accuracy of model-assisted calibration vs. model-based EB method

Main interest: What happens in minor domains (with small domain sample size)?

Empirical framework

Design-based simulation experiments

Real population data

Mixed models

Target parameters

Domain proportions

$$p_d = \frac{t_d}{N_d} = \frac{\sum_{k \in U_d} y_k}{N_d}, \quad d = 1, \ldots, D$$

where our study variable y is binary (poverty indicator)

1: in poverty, 0: otherwise

Populations and sub-populations

$U_d \subset U,\ d = 1, \ldots, D$ domains of interest

N_d known domain size, $\sum_{d=1}^{D} N_d = N$

$R_d \supset U_d,\ d = 1, \ldots, D$ higher-level regions

$U = \{1, \ldots, k, \ldots, N\}$ unit-level population

Sample data

$s \subset U$ sample from U with sampling design $p(s)$

π_k inclusion probability, $k \in U$

$a_k = 1/\pi_k$ design weight

$s_d = s \cap U_d$ part of sample s falling in domain U_d

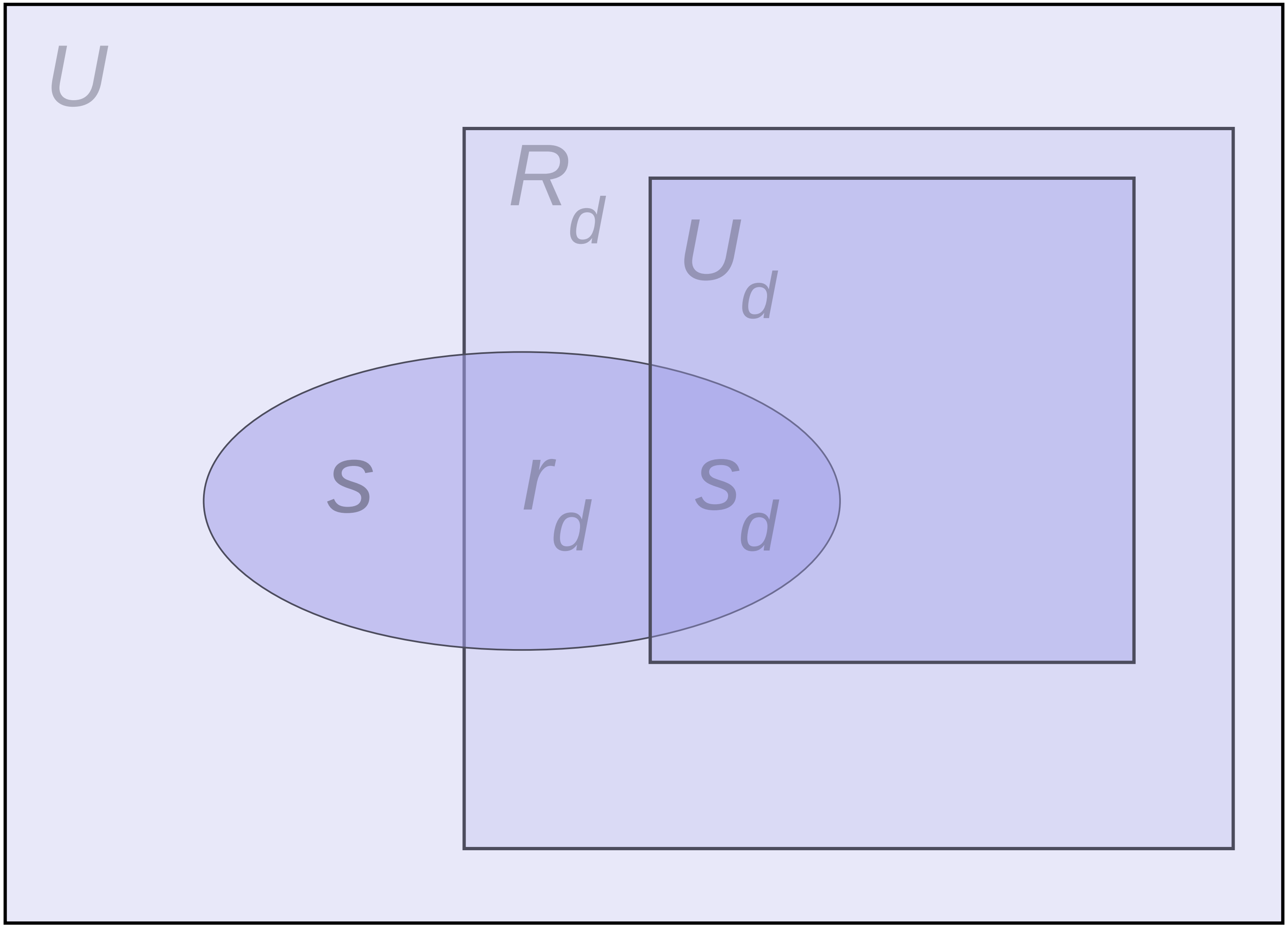
(unplanned domains)

$r_d = s \cap R_d$ part of sample s falling in higher-level area R_d

$d = 1,...,D$

Values y_k of study variable y are measured for $k \in s$

We assume complete response


U
R_d
U_d
s
r_d
s_d

Auxiliary data

We assume access to unit - level auxiliary data for variables $x_j, j = 1,...,J$ with known vector value for every $k \in U$:

$$\mathbf{x}_k = (x_{1k},...,x_{Jk})'$$

We usually add a value $x_{0k} = 1$ in the vector

For estimation purposes the sample data and auxiliary data are merged at the unit level by using unique identifiers that are assumed available for both data sources

This option is available in increasing number of statical infrastructures

Assisting mixed models

Logistic mixed model for binary study variable y

$$E_m(y_k|u_d) = \frac{\exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d)}{1 + \exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d)}, \quad k \in U_d,\ d = 1,\ldots,D \qquad (1)$$

where $\mathbf{x}_k = (x_{0k}, x_{1k}, \ldots, x_{Jk})'$, known for all $k \in U$

$\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_J)'$ vector of fixed effects

u_d are domain-level random intercepts, $u_d \sim N(0, \sigma_u^2)$

Estimate $\boldsymbol{\beta}$ and σ_u^2 from sample data set (`lme4`, GLIMMIX)

Calculate estimates $\hat{u}_d,\ d = 1,\ldots,D$ and predicted values

$$\hat{y}_k = \frac{\exp(\mathbf{x}_k'\hat{\boldsymbol{\beta}} + \hat{u}_d)}{1 + \exp(\mathbf{x}_k'\hat{\boldsymbol{\beta}} + \hat{u}_d)} \quad \text{for all } k \in U_d,\ d = 1,\ldots,D$$

Calibration weighting system - 1

Calibration equations for single-level calibration methods (MFC, MC and HC) for domains

$$\sum_{i \in s_d} w_{di} \mathbf{z}_i = \sum_{i \in U_d} \mathbf{z}_i, \quad d = 1, \ldots, D \qquad (2)$$

w_{di} calibration weight for element i in domain d

$\mathbf{z}_i$ generic calibration vector

Distance measure approach with a chi-square distance

We minimize

$$\sum_{k \in s_d} \frac{(w_{dk} - a_k)^2}{a_k} - \boldsymbol{\lambda}'_d \left(\sum_{i \in s_d} w_{di} \mathbf{z}_i - \sum_{i \in U_d} \mathbf{z}_i \right) \qquad (3)$$

subject to the calibration equations (2)

NOTE: Distance measure in (3) corresponds to GREG weighting

Calibration weighting system - 2

The equation (3) is minimized by weights

$$w_{dk} = a_k \left(1 + \boldsymbol{\lambda}_d' \mathbf{z}_k\right) \qquad (4)$$

where Lagrange coefficient vectors are

$$\boldsymbol{\lambda}_d' = \left(\sum_{i \in U_d} \mathbf{z}_i - \sum_{i \in s_d} a_i \mathbf{z}_i\right)' \left(\sum_{i \in s_d} a_i \mathbf{z}_i \mathbf{z}_i'\right)^{-1}, \quad d = 1, \ldots, D \qquad (5)$$

The resulting calibration estimator of domain total t_d is of the form:

$$\hat{t}_{dCAL} = \sum_{k \in s_d} w_{dk} y_k, \quad d = 1, \ldots, D \qquad (6)$$

where w_{dk} is method-specific calibration weight

Calibration vectors for single-level methods

MFC	$\mathbf{z}_i = \mathbf{x}_i = (1, x_{1i}, \ldots, x_{Ji})', \; i \in U_d$
MC	$\mathbf{z}_i = (1, \hat{y}_i)', \; i \in U_d$ $\hat{y}_i = f(\mathbf{x}_i'(\hat{\boldsymbol{\beta}} + \hat{\mathbf{u}}_d))$ with $\mathbf{x}_i = (1, x_{1i}, \ldots, x_{Ji})'$
HC	$\mathbf{z}_i = (1, \hat{y}_i, x_{1i}, \ldots, x_{ji})', \; i \in U_d$ $\hat{y}_i = f(\mathbf{x}_i'(\hat{\boldsymbol{\beta}} + \hat{\mathbf{u}}_d))$ with $\mathbf{x}_i = (1, x_{j+1,i}, \ldots, x_{Ji})'$

Calibration equations $\sum_{i \in s_d} w_{di} \mathbf{z}_i = \sum_{i \in U_d} \mathbf{z}_i, \; d = 1, \ldots, D$

Two-level hybrid calibration - 1

Calibration equations for HC2

$$\sum_{i \in r_d} w_{ri} \mathbf{z}_i^{(1)} = \sum_{i \in U_d} \mathbf{z}_i^{(1)} = \left(\sum_{i \in U_d} x_{0i}, \sum_{i \in U_d} \hat{y}_i \right)' \quad \text{MC part (lower level)} \qquad (7)$$

$$\mathbf{z}_i^{(1)} = (x_{0i}^{(1)}, \hat{y}_i^{(1)})', \; r_d = s \cap R_d, \; R_d \supset U_d, \; d = 1, \ldots, D$$

$x_{0i}^{(1)} = 1, \; i \in U_d$, 0 otherwise (extended x_0 variable)

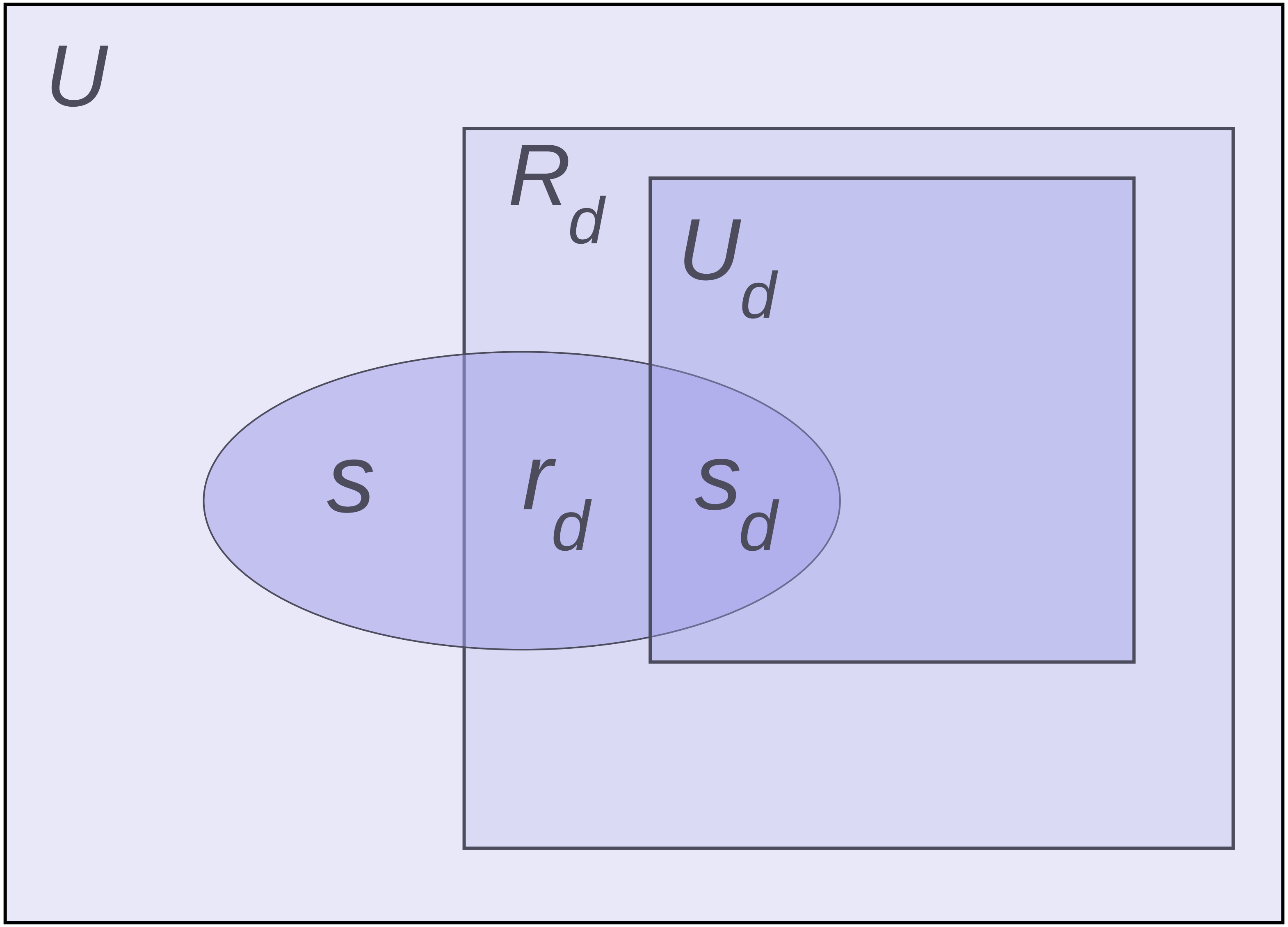
$\hat{y}_i^{(1)} = \hat{y}_i \,, \; i \in U_d$, 0 otherwise (extended predictions)

$\hat{y}_i = f(\mathbf{x}_i'(\hat{\boldsymbol{\beta}} + \hat{\mathbf{u}}_d))$ predictions calculated for chosen GLMM

$$\mathbf{x}_i = (x_{0i}, x_{j+1,i}, \ldots, x_{Ji})', \; i \in U_d$$

$$\sum_{i \in r_d} w_{ri} \mathbf{z}_i^{(2)} = \sum_{i \in R_d} \mathbf{z}_i^{(2)} = \left(\sum_{i \in R_d} x_{1i}, \ldots, \sum_{i \in R_d} x_{ji} \right)' \quad \text{MFC part (higher level)} \qquad (8)$$

$$\mathbf{z}_i^{(2)} = (x_{1i}, \ldots, x_{ji})', \; i \in R_d$$


U
R_d
U_d
s
r_d
s_d

Two-level hybrid calibration - 2

Minimize function

$$\sum_{k\in r_d}\frac{\left(w_{rk}-a_k\right)^2}{a_k}-\boldsymbol{\lambda}_r'\left(\sum_{i\in r_d}w_{ri}\begin{pmatrix}\mathbf{z}_i^{(1)}\\ \mathbf{z}_i^{(2)}\end{pmatrix}-\begin{pmatrix}\sum_{i\in R_d}\mathbf{z}_i^{(1)}\\ \sum_{i\in R_d}\mathbf{z}_i^{(2)}\end{pmatrix}\right) \tag{9}$$

subject to calibration constraints (7) and (8)

Equation (9) is minimized by weights $w_{rk}=a_k\left(1+\boldsymbol{\lambda}_r'\mathbf{z}_k\right)$,

$$\boldsymbol{\lambda}_r'=\left(\sum_{i\in R_d}\begin{pmatrix}\mathbf{z}_i^{(1)}\\ \mathbf{z}_i^{(2)}\end{pmatrix}-\sum_{i\in r_d}a_i\begin{pmatrix}\mathbf{z}_i^{(1)}\\ \mathbf{z}_i^{(2)}\end{pmatrix}\right)'\left(\sum_{i\in r_d}a_i\begin{pmatrix}\mathbf{z}_i^{(1)}\\ \mathbf{z}_i^{(2)}\end{pmatrix}\begin{pmatrix}\mathbf{z}_i^{(1)}\\ \mathbf{z}_i^{(2)}\end{pmatrix}'\right)^{-1}$$

The resulting two-level HC estimator of domain total is given by

$$\hat{t}_{dHC2}=\sum_{k\in r_d}w_{rk}y_k,\quad d=1,\ldots,D \tag{10}$$

NOTE: Weights for $k\in r_d$ outside s_d tend to be small

Estimators of domain proportions

Calibration estimators of domain proportions

$$p_d = \sum_{k \in U_d} y_k / N_d,\ d = 1,\ldots,D$$

a) Horvitz-Thompson type estimators (using known N_d)

$$\hat{p}_{dCAL\_HT} = \frac{\hat{t}_{dCAL}}{N_d} = \frac{\sum_{k \in s_d} w_{dk} y_k}{N_d},\ d = 1,\ldots,D \qquad (11)$$

b) Hájek type estimators (using estimate $\hat{N}_d$)

$$\hat{p}_{dCAL\_HA} = \frac{\hat{t}_{dCAL}}{\hat{N}_d} = \frac{\sum_{k \in s_d} w_{dk} y_k}{\sum_{k \in s_d} w_{dk}},\ d = 1,\ldots,D \qquad (12)$$

where w_{dk} are method-specific calibration weights as in (6) or as in (10)

Model-based EB predictor

EB estimator of domain totals:

$$\hat{t}_{dEB} = \sum_{k \in U_d} \hat{y}_k, \quad d = 1,\ldots,D \qquad (13)$$

where fitted values are $\hat{y}_k = f(\mathbf{x}_k' \hat{\boldsymbol{\beta}} + \hat{u}_d))$

with $\mathbf{x}_k = (1, x_{1k},\ldots,x_{Jk})'$, $k \in U_d$

and f refers to the chosen member of the GLMM family

EB estimator of domain proportions is:

$$\hat{p}_{dEB\_HT} = \frac{\hat{t}_{dEB}}{N_d}, \quad , \quad d = 1,\ldots,D \qquad (14)$$

Model-based SAE: see Rao & Molina (2015) Small Area Estimation. 2nd Ed. Wiley.

Some known differences

	Design-based calibration estimators	**Model-based EB predictor**
Design bias	(Nearly) design unbiased even if model is wrong	Design biased under model mis-specification
Precision (Variance)	Variance may be large for small domains	Variance can be small even for small domains
Accuracy (MSE)	MSE = Variance (or nearly so)	MSE = Variance + squared bias Can be large if bias dominates

EXAMPLE: Poverty rate for regions

Design-based simulation experiment with real data

Fixed finite population of about 600,000 persons

- Western Finland
- Register databases of Statistics Finland

Regional hierarchy: NUTS4 (LAU1) regions within NUTS3 regions

- Domains of interest: 36 NUTS4 regions
- Higher level regions: 7 NUTS3 regions

SRSWOR sampling of n = 2000 persons

Limited simulation experiments

- Calibration methods: K=1000 simulated samples
- Weight distributions: K=100 simulated samples

Variables

Study variable *y*:

Binary indicator with values 1=in poverty, 0=otherwise

European Union definition, one of the AROPE indicators: The poverty indicator shows when a person's equivalized income is smaller than or equal to the poverty threshold, 60% of the median equivalized income in the population

Equivalized income variable was taken from taxation registers

Overall poverty rate in population: 14.3%
lowest(NUTS4): 9.9%, highest(NUTS4): 22.4%

Auxiliary x-variables

Labour force status (3 classes)

Gender (2 classes)

Age group (3 classes)

We generated five indicator variables for the x-vector

$$\mathbf{x}_k = (1, x_{1k}, x_{2k}, x_{3k}, x_{4k}, x_{5k})'$$

Estimators

Target parameters: At-risk-of poverty rate in domains

$p_d = t_d / N_d$

where $t_d = \sum_{k \in U_d} y_k$ and y_k is binary poverty indicator

Estimators: $\hat{p}_{dCAL\_HT} = \hat{t}_{dCAL} / N_d$

where $\hat{t}_{dCAL} = \sum_{k \in s_d} w_{dk} y_k$, $d = 1,...,36$

Weights w_{dk} are method specific as in (6) or in (10)

Estimators $\hat{p}_d$ are of HT type (11)

Model-assisted estimators use logistic mixed model

$$E_m(y_k | u_d) = \frac{\exp(\mathbf{x}_k' \boldsymbol{\beta} + u_d)}{1 + \exp(\mathbf{x}_k' \boldsymbol{\beta} + u_d)}, \quad k \in U_d$$

Quality measures of estimators

Absolute relative bias (ARB) of poverty rate estimate: Table 1

$$\mathrm{ARB}(\hat{p}_d) = \left| \frac{1}{1000} \sum_{i=1}^{1000} \hat{p}_d(s_i) - p_d \right| / \, p_d$$

Relative root mean squared error (RRMSE): Table 2

$$\mathrm{RRMSE}(\hat{p}_d) = \sqrt{\frac{1}{1000} \sum_{i=1}^{1000} (\hat{p}_d(s_i) - p_d)^2} \, / \, p_d$$

where

$\hat{p}_d(s_i)$ is an estimate from sample s_i for domain d

p_d is known parameter value in domain d, $d = 1,\ldots,36$

Table 1 Average absolute relative bias (ARB) (%) of HT type calibration estimators of poverty rate in domains in three domain sample size classes						
Estimator	Assisting model & calibration scheme		Expected domain sample size			All
			Minor <25	Medium 25-50	Major >50	
Direct estimators						
Model-free calibration	NUTS4	$\mathbf{z}_k = (1, x_{1k}, \ldots, x_{5k})'$	1.2	1.2	0.6	1.1
Indirect (Semi-direct) estimators						
Model: $E_m(y_k \mid u_d) = \exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d) / (1 + \exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d))$, $\mathbf{x}_k = (1, x_{1k}, x_{2k}, x_{3k}, x_{4k}, x_{5k})'$						
Model MC calibration	NUTS4	$\mathbf{z}_k = (1, \hat{y}_k)'$	1.3	1.2	0.6	1.2
Model: $E_m(y_k \mid u_d) = \exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d) / (1 + \exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d))$, $\mathbf{x}_k = (1, x_{3k}, x_{4k}, x_{5k})'$						
Hybrid HC calibration	NUTS4	$\mathbf{z}_k = (1, \hat{y}_k, x_{1k}, x_{2k})'$	1.3	1.3	0.5	1.2
Hybrid HC2 calibration	NUTS4	$\mathbf{z}_k^{(1)} = (1, \hat{y}_k)'$	1.5	1.5	0.6	1.4
	NUTS3	$\mathbf{z}_k^{(2)} = (x_{1k}, x_{2k})'$				

Table 2 Average relative root mean squared error (RRMSE) (%) of HT type calibration estimators of poverty rate in domains in three domain sample size classes

Estimator	Assisting model & calibration scheme		Expected domain sample size			All
			Minor <25	Medium 25-50	Major >50	
Direct estimators						
Model-free calibration	NUTS4	$\mathbf{z}_k = (1, x_{1k}, \ldots, x_{5k})'$	61.1	40.4	20.1	47.3
Indirect (Semi-direct) estimators						
Model: $E_m(y_k \mid u_d) = \exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d)/(1+\exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d))$, $\mathbf{x}_k = (1, x_{1k}, x_{2k}, x_{3k}, x_{4k}, x_{5k})'$						
Model MC calibration	NUTS4	$\mathbf{z}_k = (1, \hat{y}_k)'$	54.1	37.6	19.8	43.0
Model: $E_m(y_k \mid u_d) = \exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d)/(1+\exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d))$, $\mathbf{x}_k = (1, x_{3k}, x_{4k}, x_{5k})'$						
Hybrid HC calibration	NUTS4	$\mathbf{z}_k = (1, \hat{y}_k, x_{1k}, x_{2k})'$	58.0	39.1	20.1	45.4
Hybrid HC2 calibration	NUTS4	$\mathbf{z}_k^{(1)} = (1, \hat{y}_k)'$	54.2	38.1	20.2	43.3
	NUTS3	$\mathbf{z}_k^{(2)} = (x_{1k}, x_{2k})'$				

Summary of results on calibration

Design bias: All estimators are nearly design unbiased

Design accuracy

Major domains: All estimators show pretty similar accuracy

Minor and medium-sized domains:
Model-assisted methods outperform direct model-free calibration

Model-assisted calibration shows best accuracy and is preferred

Hybrid calibration offers coherence property for selected x-variables but can suffer from instability in small areas

Two-level hybrid calibration accounts for the instability and can provide a good compromise if coherence is required for some x-variables

NOTE: Preliminary results on Hájek type estimators (12) indicate that accuracy differences to HT type methods are small and exist in small domains and are systematically in favour of Hájek methods

Distributional properties of calibrated weights

Problems of practical concern in model-free calibration:

- Possible large variation of weights
- Weights smaller than one
- Positive but extremely small weights
- Negative weights

To what extent model-assisted calibration methods can help?

Small simulation experiment:

- 100 SRSWOR samples of size 2,000 elements from *U*

Reporting:

- Distribution of weights by domain size (log scale) - Figure 1
- Medians of maximum interdecile range of calibrated weights in domain sample size classes - Table 3

Fig. 1. Distribution of weights by domain size

100 simulated SRSWOR samples, n=2000

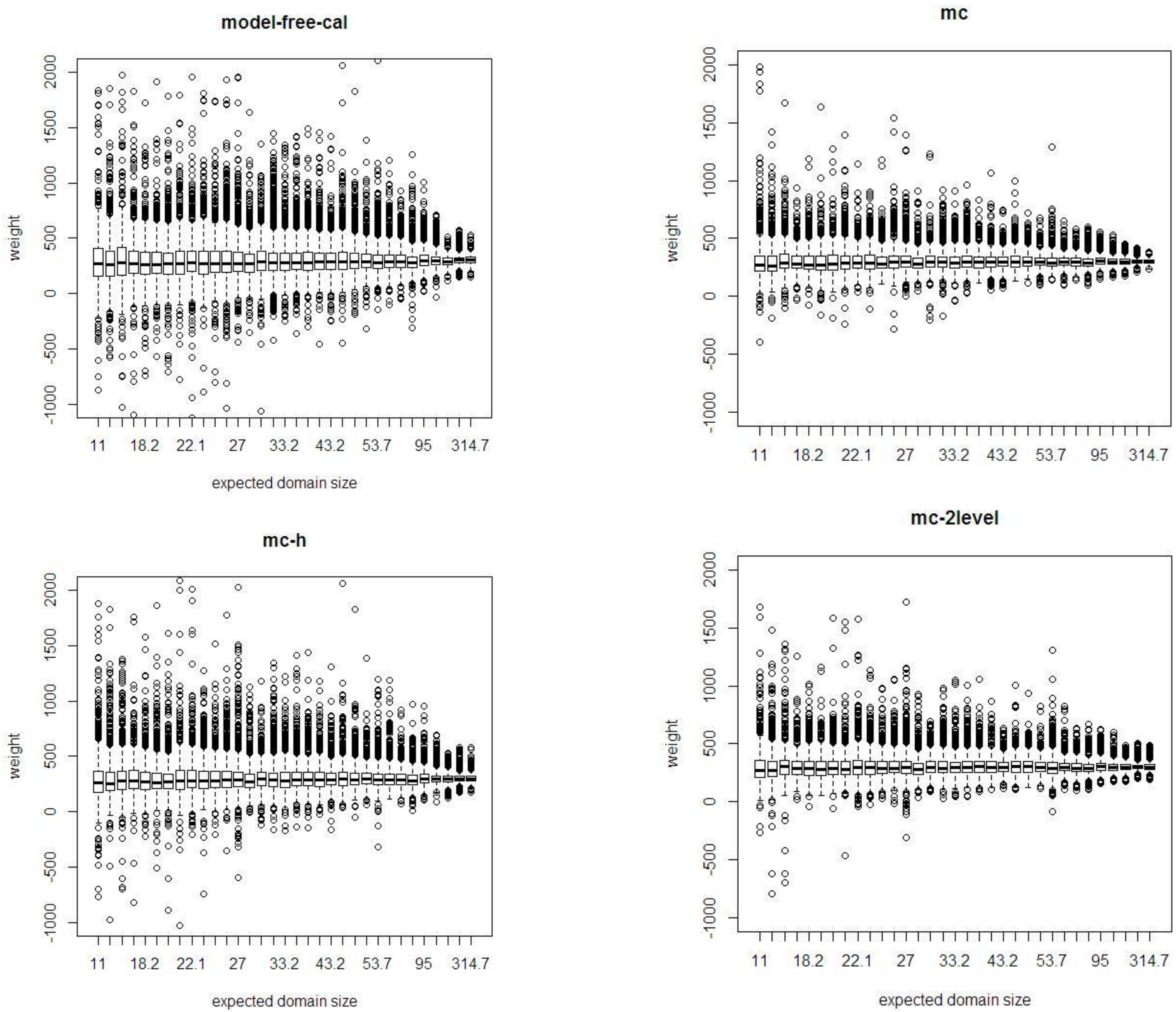


Table 3. Median of maximum interdecile range of calibrated weights over 100 simulations in three domain sample size classes.

Method	Expected domain sample size		
	Minor <25	Medium 25-50	Major >50
Model-free calibration MFC	1620	673	210
Model-assisted calibration MC	984	418	172
Single-level hybrid calibration HC	1415	665	245
Two-level hybrid calibration HC2	780	430	214

NOTE: Maximum of interdecile range over the 100 simulations is first computed for each domain and the median of these is then computed in each domain sample size class.

Summary of distributional properties

Model-free calibration MFC shows worst performance

Model-assisted calibration MC stabilizes substantially the distribution of weights, in small domains in particular

Model-assisted calibration MC and two-level hybrid calibration HC2 indicate best weight performance

But: negative weights still remain
Can we live with that?
Rather use other solutions?

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Wu C. and Lu W.W. (2016) Calibration weighting methods for complex surveys. International Statistical Review 84, 79-98.

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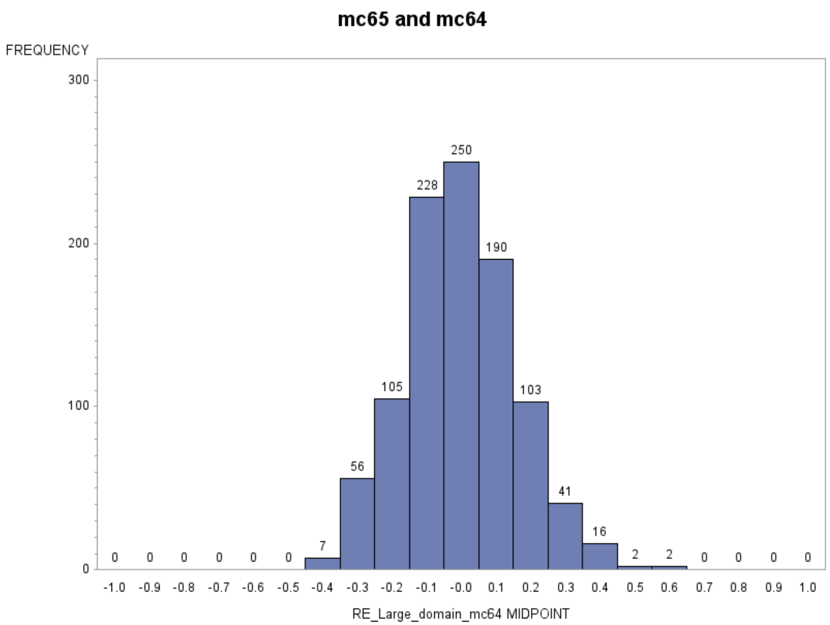
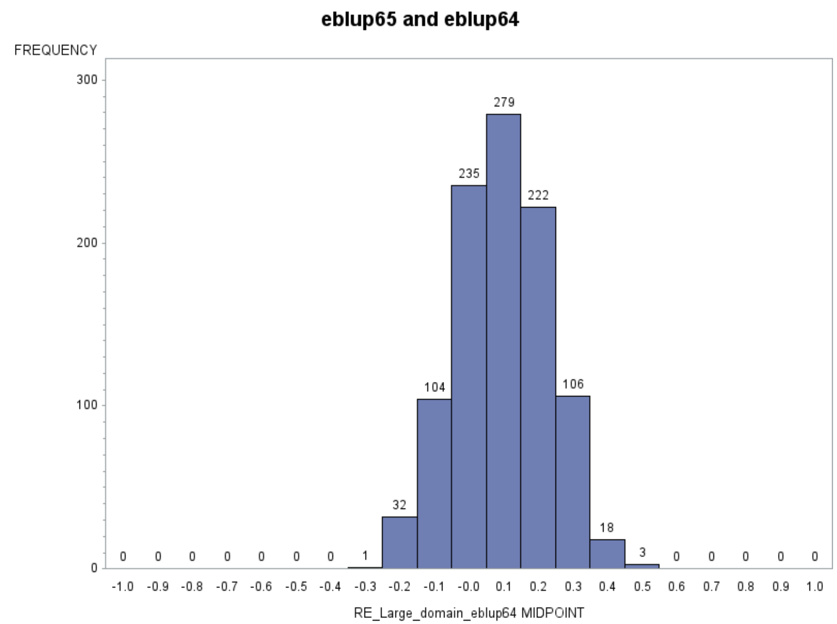
Gelman: “Survey weighting is a mess.”

Table 4 Average absolute relative bias (ARB) (%) and average relative root mean squared error (RRMSE) (%) of model-assisted calibration estimator MC and model-based EB predictor of poverty rate in domains in three domain sample size classes

	Bias: ARB (%)			**Accuracy: RRMSE (%)**		
Estimator	Expected domain sample size			Expected domain sample size		
	Minor <25	Medium 25-50	Major >50	Minor <25	Medium 25-50	Major >50
Model: $E_m(y_k \mid u_d) = \frac{\exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d)}{1 + \exp(\mathbf{x}_k'\boldsymbol{\beta} + u_d)}$, $\mathbf{x}_k = (1, x_{1k}, x_{2k}, x_{3k}, x_{4k}, x_{5k})'$						
Design-based calibration estimator						
Model MC calibration	1.3	1.2	0.56	54.1	37.6	19.8
Model-based small area estimator						
EB estimator	16.2	11.1	9.0	21.7	18.4	16.3

Relative error of MC and EB estimators in a certain large domain





Distribution of relative error
$RE(\hat{p}_{dMC}) = (\hat{p}_{dMC}(s_i) - p_d) / p_d$
of design-based model assisted calibration MC estimator of poverty rate in large domain 64

NOTE
Nearly design unbiased
Outperforms EB in design bias

Distribution of relative error
$RE(\hat{p}_{dEB}) = (\hat{p}_{dEB}(s_i) - p_d) / p_d$
of model-based EB estimator of poverty rate in large domain 64

NOTE
Design biased
Outperforms MC in design accuracy

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merci beaucoup!