Stellar magnetic activity, Exercise V

April 22, 2024

Model solutions

1. (a) The momentum equation is

$$\rho u \frac{\mathrm{d}u}{\mathrm{d}r} = -\frac{\mathrm{d}P}{\mathrm{d}r} - \frac{GM_S}{r^2} \,. \tag{1}$$

The MHD equation for pressure in a steady state with radial flow is

$$u\frac{\mathrm{d}P}{\mathrm{d}r} = -\gamma P \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2 u) \,. \tag{2}$$

Substituting $\mathrm{d}P/\mathrm{d}r$ fom Eq. 2 into Eq. 1, we get

$$u^2\frac{\mathrm{d}u}{\mathrm{d}r} = \frac{\gamma P}{\rho}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2 u) - \frac{GM_S}{r^2}u \,. \tag{3}$$

P can be obtained from integrating

$$\frac{1}{P}\frac{\mathrm{d}u}{\mathrm{d}r} = \frac{\gamma}{\rho u^2 r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2 u) - \frac{GM_S}{r^2}u \,. \tag{4}$$

Introducing the notation for the sound speed $c_s = \sqrt{\gamma P/\rho}$, Eq. 3 becomes

$$u^2\frac{\mathrm{d}u}{\mathrm{d}r} = c_s^2\left(\frac{\mathrm{d}u}{\mathrm{d}r} + \frac{2u}{r}\right) - \frac{GM_S}{r^2}u_r \,. \tag{5}$$

Rearranging this to $\mathrm{d}u/\mathrm{d}r$, we get

$$\frac{\mathrm{d}u}{\mathrm{d}r} = \frac{u}{r}\left(\frac{2c_s^2 - GM_S/r}{u^2 - c_s^2}\right) \,. \tag{6}$$

(b) The hydrodynamic solution is zero when

$$\frac{u}{r}\left(\frac{2c_s^2 - GM_S/r}{u^2 - c_S^2}\right) = 0 \,. \tag{7}$$

This is true if the radius r is the critical radius

$$r = \frac{GM_S}{2c_s^2} \tag{8}$$

as the sound speed is

$$c_s = \sqrt{\frac{\gamma P}{\rho}} \tag{9}$$

Eq. 8 becomes

$$r = \frac{GM_S \rho}{2\gamma P} \, . \tag{10}$$

An isothermal solution corresponds to choosing $\gamma = 1$, while an adiabatic solution with a monatomic gas corresponds to $\gamma = 5/3$. In the isothermal case, the critical radius is

$$r_{c,\mathrm{iso}} = \frac{GM_S \rho}{2P} \, , \tag{11}$$

while in the adiabatic case with a monatomic gas, the critical radius is

$$r_{c,\mathrm{adi}} = \frac{3}{5} \frac{GM_S \rho}{2P} \, , \tag{12}$$

so $r_{c,\mathrm{iso}} > r_{c,\mathrm{adi}}$.

2. The critical radius r_c obtained from the isothermal model is

$$r_c = \frac{GM_S m}{4k_B T} \, , \tag{13}$$

where G is the gravitational constant, $M_S = 1.989 \times 10^{30}$ kg is the mass of the Sun, m is the mass of a proton, k_B is the Boltzmann constant, and T is the solar coronal temperature.

Substituting values from $T = 10^5$ K to $T = 10^7$ K into Eq. 13, we get critical radii as shown on Fig. 1.

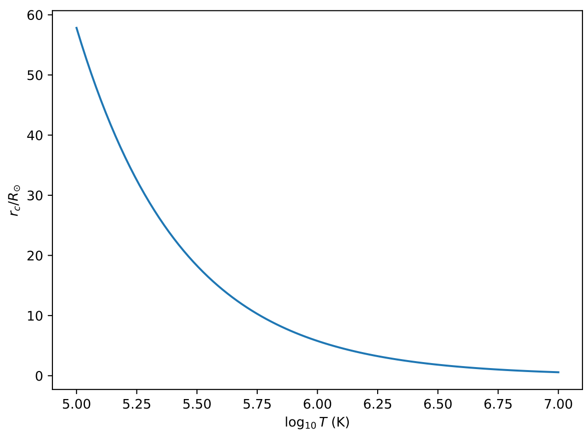

Figure 1: Critical radius r_c in terms of solar radius $R_\odot$, as a function of the solar coronal temperature T.

3. Gradient and curvature drift are the two key processes that contribute to the formation of the ring current around Earth within the magnetosphere:

Gradient Drift: Gradient drift occurs when charged particles move in a magnetic field where there is a gradient or variation in the strength of the magnetic field. In the case of Earth's magnetosphere, the magnetic field strength varies with distance from the Earth's surface. The magnetic field is stronger closer to the Earth and weaker further away. In the context of the ring current, gradient drift contributes to the inward radial transport of charged particles towards the Earth. This occurs because charged particles tend to move from regions of weaker magnetic field strength to regions of stronger magnetic field strength. As a result, charged particles from the magnetotail and other regions of the magnetosphere are transported towards the Earth along the magnetic field lines, accumulating in the region around the equator to form the ring current.

Curvature Drift: Curvature drift arises from the curvature of the magnetic field lines in the magnetosphere. The Earth's magnetic field lines are not straight but are curved, particularly in the near-Earth region. Charged particles moving along curved magnetic field lines experience a force perpendicular to both their velocity and the curvature of the magnetic field lines. In the context of the ring current, curvature drift contributes to the azimuthal motion of charged particles around the Earth. As charged particles move along the curved magnetic field lines, they tend to spiral around the Earth in an eastward direction due to the Coriolis force. This eastward motion leads to the accumulation of charged particles in a ring-like structure around the equator, forming the ring current.

In summary, gradient drift facilitates the radial transport of charged particles towards the Earth, while curvature drift contributes to their azimuthal motion around the planet. Together, these processes play crucial roles in the formation and maintenance of the ring current around Earth within the magnetosphere.