Stellar magnetic activity

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Model solutions for exercise IV

1. The dependence of the kinematic alpha effect α scales with ϑ as

$$\alpha \propto -\cos\vartheta\,, \tag{1}$$

where ϑ relates to the latitude θ through $\vartheta = \theta + 90°$. The kinematic alpha effect is positive in the northern and negative in the southern hemisphere.

The magnetic field will migrate in the direction

$$\boldsymbol{s} = \alpha \nabla \boldsymbol{\Omega} \times \hat{\boldsymbol{y}}\,, \tag{2}$$

where $\boldsymbol{\Omega}$ is the angular velocity vector, and $\hat{\boldsymbol{y}}$ is an azimuthal unit vector. The cross product in Eq. 2 is the radial gradient of differential rotation:

$$\nabla \boldsymbol{\Omega} \times \hat{\boldsymbol{y}} = \frac{\partial \Omega}{\partial r}\,. \tag{3}$$

Let us take the rotation profile of the Sun within the convective zone (CZ, Figure 1). The radial gradient of the angular velocity is positive between the tachocline and bottom of the leptocline (at $r \approx 0.7\ R_\odot$ and $r \approx 0.95\ R_\odot$, respectively) for latitudes below 45°. Above 45° the radial gradient of the angular velocity is almost continuously around or below zero.

Table 1 shows the migration direction of the latitudinal dynamo wave for positive and negative α and $\partial\Omega/\partial r$ values.

Table 1: Direction of shear $\boldsymbol{s}$ for positive and negative α and $\partial\Omega/\partial r$ values.

	$\partial\Omega/\partial r > 0$	$\partial\Omega/\partial r < 0$
$\alpha > 0$	$\boldsymbol{s} > 0$	$\boldsymbol{s} < 0$
$\alpha < 0$	$\boldsymbol{s} < 0$	$\boldsymbol{s} > 0$

Comparing Table 1 and Fig. 1 we see that:

- Between the tachocline and bottom of the leptocline, at low latitudes the dynamo wave migrates towards the poles, and at high latitudes it migrates towards the equator.
- From the bottom of the leptocline upwards the dynamo wave migrates towards the equator. This is the same that is recovered in observations.

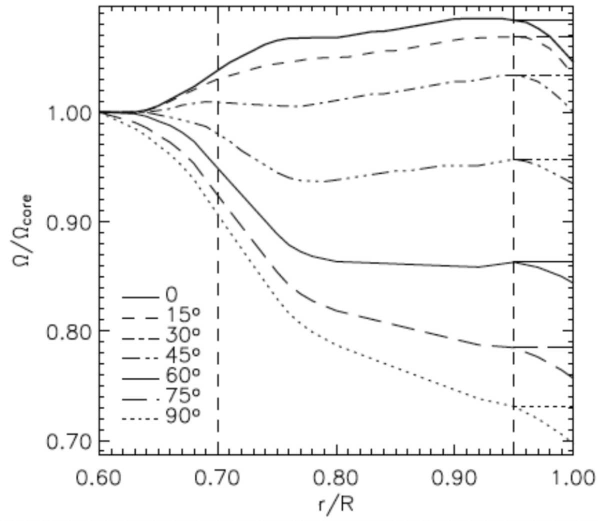

Figure 1: Rotation profile of the Sun within the convective zone for different latitudes (Figure from Lecture Notes).

2. An axisymmetric magnetic field $\mathbf{B}$ can be decomposed to poloidal-toroidal components as

$$\mathbf{B} = \mathbf{B}_P + \mathbf{B}_T = \nabla \times (A\hat{\boldsymbol{y}}) + B\hat{\boldsymbol{y}}\,, \tag{4}$$

where $\mathbf{B}_P$ and $\mathbf{B}_T$ are the poloidal and toroidal components of the magnetic field, A and B are the vector potentials for the poloidal and toroidal fields, and $\hat{\boldsymbol{y}}$ is a unit vector, in the same plane as the stellar equator.

The mean field induction equation without the α effect is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B})\,, \tag{5}$$

where $\mathbf{u} = \boldsymbol{\Omega} \times \boldsymbol{r}$, where $\boldsymbol{\Omega}$ is the angular velocity vector of the star, and $\boldsymbol{r}$ is a position vector, and η is the magnetic diffusivity. Looking at Eq. 4, we see that the toroidal (B) and poloidal (A) components of the mean field induction equation can be given as the y-component, and the uncurled y-component of Eq. 5:

$$\begin{aligned} \frac{\partial B}{\partial t} &= [\nabla \times (\mathbf{u} \times \mathbf{B}) - \eta \nabla \times \mathbf{B}]_y \\ \frac{\partial A}{\partial t} &= [\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}]_y\,. \end{aligned} \tag{6}$$

Cowling's antidynamo argument states that a steady, axisymmetric poloidal field cannot be maintained by motional induction (Hide and Palmer, 1982). Looking at Eq. 6, we see that motional induction (a gradient of $\mathbf{u}$) only takes place in the generation of the toroidal field. This means that the poloidal field decays over time, in accordance with Cowling's antidynamo argument.

If we re-instate the α effect term, $\partial A/\partial t$ will have a non-axisymmetric term, which allows the regeneration of the poloidal field.

3. The diffusion time is

$$\tau_D = \frac{R_\odot^2}{\eta_t}\,, \tag{7}$$

where the solar radius is $R_\odot \approx 7 \times 10^8$ m, and η_t is the magnetic diffusivity.

The turnover time of the meridional circulation, i.e., the time that it takes for the flow to make a full revolution can be estimated by dividing the travelled distance of the flow by the velocity of the flow along its trajectory

$$\begin{aligned}\tau_T &= t_t + t_b + 2t_s \\ &= \frac{s_t}{v_t} + \frac{s_b}{v_b} + 2\frac{s_s}{v_s} \\ &= \frac{\frac{\pi}{2}R_\odot}{v_t} + \frac{\frac{\pi}{2}r_b R_\odot}{v_b} + 2\frac{(1-r_b)R_\odot}{v_s} \\ &= \frac{\pi}{2}R_\odot\left(\frac{1}{v_t} + \frac{r_b}{v_b}\right) + 2\frac{(1-r_b)R_\odot}{v_s}\,,\end{aligned} \tag{8}$$

where t_t, t_b, and t_s are the times spent on the top of the CZ, the bottom of the CZ, and moving vertically between the two layers, s_t, s_b and s_s are the distances travelled at the top and the bottom of the CZ and between the two layers, v_t, v_b and v_s are the speed of the flow at the top and the bottom of the CZ and moving between the two layers (here we assume that $v_s = (v_t + v_b)/2$), and $r_b = 0.7$ is a scale factor that determines how deep the meridional circulation reaches within the Sun.

Using 10 m/s and 1 m/s for flow speed at the top and bottom of the CZ, we get $\tau_T = 30.29$ yr.

The diffusion times for different η_t values are given in Table 2. From this we see, that these are

Table 2

Dynamo type	η_t (m^2/s)	τ_D (yr)
Turbulent dynamo	5×10^8	31.05
Flux-transport dynamo	5×10^6	3105.50

Thus we can conclude:

- When diffusion dominates over advection, the diffusion determines the cycle.
- When advection by meridional circulation dominates, the meridional speed determines the cycle.

4. There are several possible studies to refer to. There are some contradictions between these in details, but the general notion is that the relative differential rotation $\alpha = \frac{\Delta\Omega}{\Omega}$ decreases with the angular velocity Ω, while the absolute differential rotation $\Delta\Omega$ is constant or slightly increases with Ω. As for the temperature dependence, some results indicate and increase of α with temperature among main-sequence stars of F and G-types. In the following we refer to the study by Reinhold et al. (2013).

   Using Kepler observations, Reinhold et al. (2013) estimated the rotation periods P and the absolute shear, which serves as a proxy for differential rotation, for over 20 000 field stars. Absolute shear is defined as $\Delta\Omega = \Omega_{\mathrm{eq}} - \Omega_{\mathrm{pol}}$, where Ω_{eq} and Ω_{pol} represent the angular velocity of a star at its equator and poles, respectively.

   Reinhold et al. (2013) found that:

- The differential rotation shows a temperature dependency. Between effective temperatures $T_{\text{eff}} = 3500$ K and $T_{\text{eff}} = 6000$ K, $\Delta\Omega$ slightly increases from $\Delta\Omega = 0.079$ rad/d to $\Delta\Omega = 0.096$ rad/d. This is consistent with the calculations of Küker and Rüdiger (2011), who found that $\Delta\Omega$ only slightly increases with temperature for stars below $T_{\text{eff}} \approx 5800$ K.
- Hotter, F-type stars with thinner convective envelopes show stronger differential rotation than cooler G and K-type stars with deeper convective zones.
- The dependency of absolute differential rotation (= shear) on the rotation period of the star is weak over a large period range.

References

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