Stellar magnetic activity, Spring 2024

March 7, 2024

Models for exercises III

1. (a) The Coriolis number is expressed as

$$\mathrm{Co} = \frac{2\Omega l}{u}\,, \tag{1}$$

where l and u are the length and velocity scales, and Ω is the angular velocity of the star, which is related to the rotational period P_{rot} as $\Omega = 2\pi/P_{\mathrm{rot}}$.

Let us assume that the young Sun when reaching the main sequence (MS) was rotating with $P_{\mathrm{rot}} = 1$ d, and that and towards the end of its MS lifetime it will rotate with $P_{\mathrm{rot}} = 50$ d. We can approximate the current rotational period of the Sun as $P_{\mathrm{rot}} = 27$ d.

Let us further choose the convective length and velocity to be $l_{\mathrm{bottom}} = 50\,000$ km and $v_{\mathrm{bottom}} = 10$ m/s at the bottom of the convective zone, and $l_{\mathrm{top}} = 100$ km and $v_{\mathrm{top}} = 1$ km/s at the top of the convective zone. Substituting these numbers into Eq. 1, we obtain the Coriolis numbers at the bottom and the top at the convection zone $\mathrm{Co}_{\mathrm{bottom}}$, $\mathrm{Co}_{\mathrm{top}}$, for different stages of the solar life cycle. The Coriolis numbers along with the parameters describing the stellar rotation are listed in Table 1.

Table 1: Coriolis numbers Co at the bottom and the top at the convection zone for different stages of the solar life cycle.

Stage	P_{orb} (d)	Ω (1/s)	$\mathrm{Co}_{\mathrm{bottom}}$	$\mathrm{Co}_{\mathrm{top}}$
Young Sun	1	7.27×10^{-5}	727.22	1.5×10^{-2}
Current Sun	27	2.69×10^{-6}	26.93	5.4×10^{-4}
Old Sun	50	1.45×10^{-6}	14.54	2.9×10^{-4}

(b) The magnetic Prandtl number is (Brandenburg and Subramanian, 2005)

$$P_{\mathrm{m}} = 1.1 \times 10^{-4} \left(\frac{T}{10^6\ \mathrm{K}}\right)^4 \left(\frac{\rho}{0.1\ \mathrm{g\,cm^{-3}}}\right)^{-1} \left(\frac{\ln\Lambda}{20}\right)^{-2}, \tag{2}$$

where T denotes the temperature, ρ is the density, and $\ln\Lambda$ is the Coulomb logarithm of the plasma. To calculate the magnetic Prandtl number of different spectral type stars, we adapt the effective temperature T_{eff}, radius R, and mass

M from Pecaut and Mamajek (2013). For the calculation, we use mean stellar density $\rho = M/V$ where V is the volume of a spherical star. Following Brandenburg and Subramanian (2005), we assume $\ln \Lambda = 20$. The resulting magnetic Prandtl numbers are listed in Table 2.

Table 2: Magnetic Prandtl numbers Pm for different spectral type stars and relevant parameters for its calculation.

Spectral type	T_{eff} (K)	$R/R_\odot$	$M/M_\odot$	ρ (g/cm^3)	Pm
O5V	41 400	11.45	43.00	0.04	8×10^{-10}
B5V	15 700	3.36	4.70	0.17	4×10^{-12}
A5V	8 100	1.78	1.88	0.47	1×10^{-13}
F5V	6 550	1.47	1.33	0.59	3×10^{-14}
G5V	5 660	0.98	0.98	1.48	8×10^{-15}
K5V	4 440	0.70	0.70	2.87	1×10^{-15}
M5V	3 060	0.20	0.16	30.34	3×10^{-17}

(c) The Reynolds numbers are Rm (magnetic) and Re (fluid):

$$\mathrm{Rm} = \frac{ul}{\eta}, \tag{3}$$

and

$$\mathrm{Re} = \frac{\mathrm{Rm}}{\mathrm{Pm}}, \tag{4}$$

where u and l are the velocity and length scales, and η is the resistivity in the convection zone.

We take $\eta = 0.1$ m^2/s for all stars. Adopting the l and u values for a Sun-like star from problem 1a, we get $\mathrm{Rm}_{\text{bottom}} = 5 \times 10^9$ and $\mathrm{Rm}_{\text{top}} = 10^9$ for the magnetic Reynolds number at the bottom and top of the convection zone.

We assume that in other stars, the length and velocity scales at the top and bottom of the convection zone may vary by a factor of 10, and that the resistivity is constant. Using these assumptions, the magnetic Reynolds numbers at the bottom and top of the convection zone might show variations on the order of 10^2 from the values calculated for a solar type star.

Taking $\mathrm{Rm} \sim 10^9$ and $\mathrm{Pm} \sim 10^{-14}$ we get

$$\mathrm{Re} \sim \frac{10^9}{10^{-14}} = 10^{23}$$

2. The kinetic and magnetic energy spectra (marked as E_k and E_m, respectively) are sketched out on Fig. 1.

   At high Pm values and high k values, the magnetic energy E_m is higher than the kinetic energy E_k. Thus, small-scale dynamo action is easiest to occur if $\mathrm{Pm} > 1$. In these conditions, small scale magnetic structures can develop more easily than with lower Pm values.

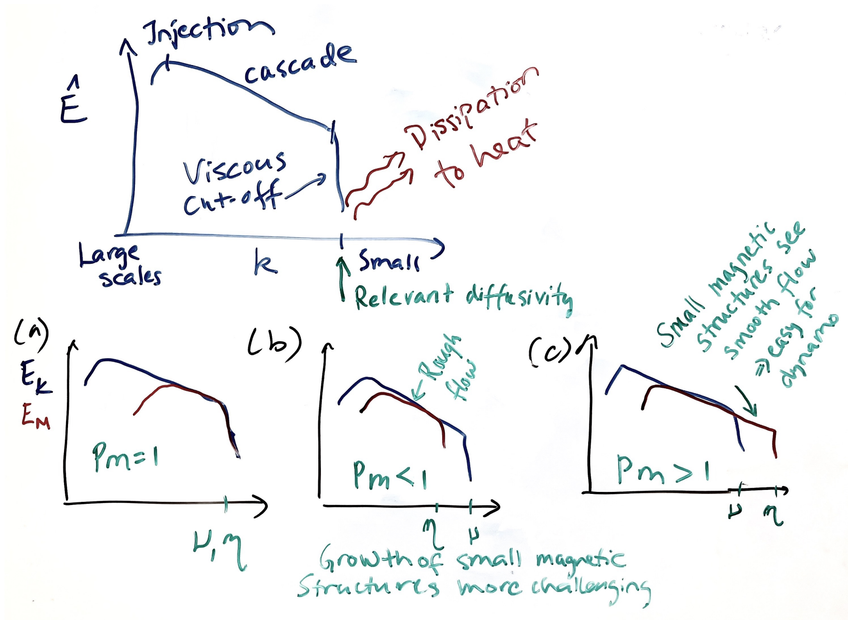

Figure 1: Upper panel: General scheme of energy injection, cascade and dissipation as function of wavenumber k. Lower panels: Kinetic E_{k} and magnetic E_{m} energy as functions of wavenumber k, with Prandtl numbers (a) $\mathrm{Pm} = 1$, (b) $\mathrm{Pm} < 1$, (c) $\mathrm{Pm} > 1$.

3. The kinetic helicity H_k is calculated by

$$H_k = \int_V \boldsymbol{u} \cdot (\nabla \times \boldsymbol{u})\, dV \,. \tag{5}$$

where $\boldsymbol{u}$ is the velocity of the cell. We denote $\boldsymbol{w} = \nabla \times \boldsymbol{u}$ and $h = \boldsymbol{w} \cdot \boldsymbol{u}$.

Depending on the relative pressure of the cells, and the strength of the stratification, we can draw a table of conditions, shown in Table 3. Assuming that the conditions are isothermal, and that a convective cell contains a constant number of particles, the product of the pressure P and volume V of some cell is constant in time $PV = \mathrm{const}$.

Table 3

	Stratification	
	Weak	Strong
Cells have higher P than surroundings	(a)	(c)
Cells have lower P than surroundings	(b)	(d)

The time evolution of convective cells with the different conditions listed in Table 3 are sketched out in Fig. 2. The change in cell sizes depend on the stratification (hence the density gradient). If the stratification is weak ($\Delta\rho \approx 0$), the cells will (more or less) keep their form. This leads to

$$\boldsymbol{w} \approx \boldsymbol{0}$$

If the stratification is large ($|\Delta\rho| \gg 0$), the form of the cells will change significantly. Thus

$$\boldsymbol{w} \neq \mathbf{0}.$$

This leads to the different situations showed in Fig. 2. Note that the sign of h for each case will be opposite at the North and South poles.

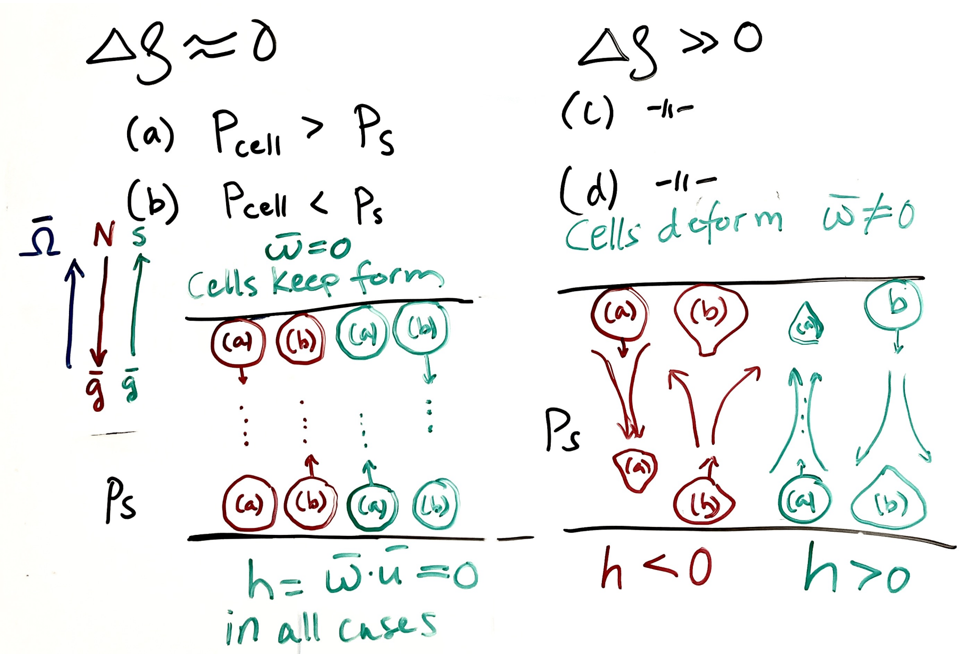

Figure 2: Kinetic helicity of convective cells around the stellar poles.

4. The kinematic alpha effect is

$$\alpha_K = -\frac{1}{3}\tau \langle \boldsymbol{w} \cdot \boldsymbol{u} \rangle \tag{6}$$

where τ is the relaxation time, $\boldsymbol{w} = \nabla \times \boldsymbol{u}$ and $\boldsymbol{u}$ is the velocity vector. The angle between $\boldsymbol{w}$ and $\boldsymbol{u}$ is $\vartheta = \theta + 90°$, where θ is the latitude. Thus the dot product in Eq. 6 is

$$\langle \boldsymbol{w} \cdot \boldsymbol{u} \rangle = |\boldsymbol{w}||\boldsymbol{u}| \cos\vartheta\,. \tag{7}$$

Substituting Eq. 7 into Eq. 6, we get

$$\alpha_K = -\frac{1}{3}\tau |\boldsymbol{w}||\boldsymbol{u}| \cos\vartheta\,. \tag{8}$$

From here, we can see that the kinetic alpha effect will scale as $\alpha_K \propto -\cos\vartheta$. α_K is thus positive at the North pole, 0 at the equator and negative at the South pole.

References

A. Brandenburg and K. Subramanian. Astrophysical magnetic fields and nonlinear dynamo theory. *Physics Reports*, 417(1-4):1–209, Oct. 2005. doi: 10.1016/j.physrep.2005.06.005.

M. J. Pecaut and E. E. Mamajek. Intrinsic Colors, Temperatures, and Bolometric Corrections of Pre-main-sequence Stars. *The Astrophysical Journal Supplement*, 208(1):9, Sept. 2013. doi: 10.1088/0067-0049/208/1/9.