Local and global DNS-like simulations of solar and stellar magnetism

Learning outcomes

- Continuing to learn to master but also to apply the concepts discussed during the last two weeks
- Understand how and why full MHD solvers can be useful to understand stellar magnetism
- Basic principles of the numerical methods
- Restrictions of the methods
- Most prominent results
- Recap of the whole contents of the MHD module

Magnetohydrodynamics: basic equations

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U} \times \boldsymbol{B} - \eta\mu_0 \boldsymbol{J},$$

$$\frac{\mathrm{D}\ln\rho}{\mathrm{D}t} = -\,\boldsymbol{\nabla}\cdot\boldsymbol{U},$$

$$\frac{\mathrm{D}\boldsymbol{U}}{\mathrm{D}t} = \boldsymbol{g} - 2\boldsymbol{\Omega}_0 \times \boldsymbol{U} - \frac{1}{\rho}(\boldsymbol{\nabla} p + \boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{\nabla}\cdot 2\nu\rho\mathbf{S}),$$

$$T\frac{\mathrm{D}s}{\mathrm{D}t} = \frac{1}{\rho}\left[\eta\mu_0\boldsymbol{J}^2 - \boldsymbol{\nabla}\cdot(\boldsymbol{F}^{\mathrm{rad}} + \boldsymbol{F}^{\mathrm{SGS}}) - \Gamma_{\mathrm{cool}}\right] + 2\nu\mathbf{S}^2$$

$$\nabla \cdot \boldsymbol{B} = 0$$

Magnetohydrodynamics: non-dimensional parameters

$$\left|\frac{\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}}{\partial\boldsymbol{u}/\partial t}\right| \approx \frac{u^2\tau}{ul} = \frac{u\tau}{l} \equiv \mathrm{St},$$

$$\left|\frac{\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}}{2\,\boldsymbol{\Omega}\times\boldsymbol{u}}\right| \approx \frac{u^2}{2\,\Omega l u} = \frac{u}{2\,\Omega l} \equiv \mathrm{Ro} = \mathrm{Co}^{-1},$$

$$\left|\frac{\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}}{\boldsymbol{g}}\right| \approx \frac{u^2}{lg} \equiv \mathrm{Fr} = \mathrm{Ri}^{-1},$$

$$\left|\frac{\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}}{\nu\boldsymbol{\nabla}^2\boldsymbol{u}}\right| \approx \frac{u^2l^2}{\nu l u} = \frac{ul}{\nu} \equiv \mathrm{Re},$$

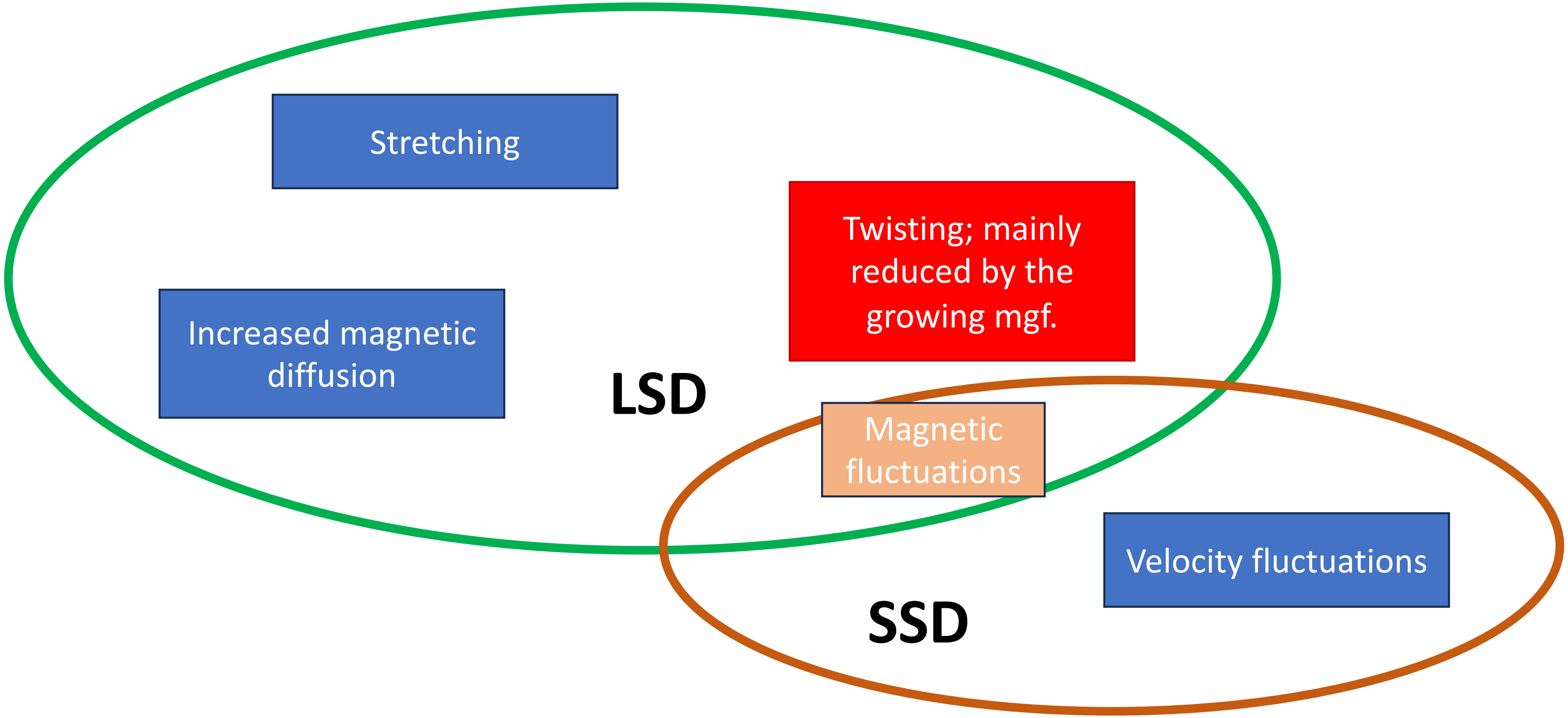
$$\mathrm{Rm} \equiv \frac{ul}{\eta},$$

$$\mathrm{Ma} = \frac{u}{c_s},$$

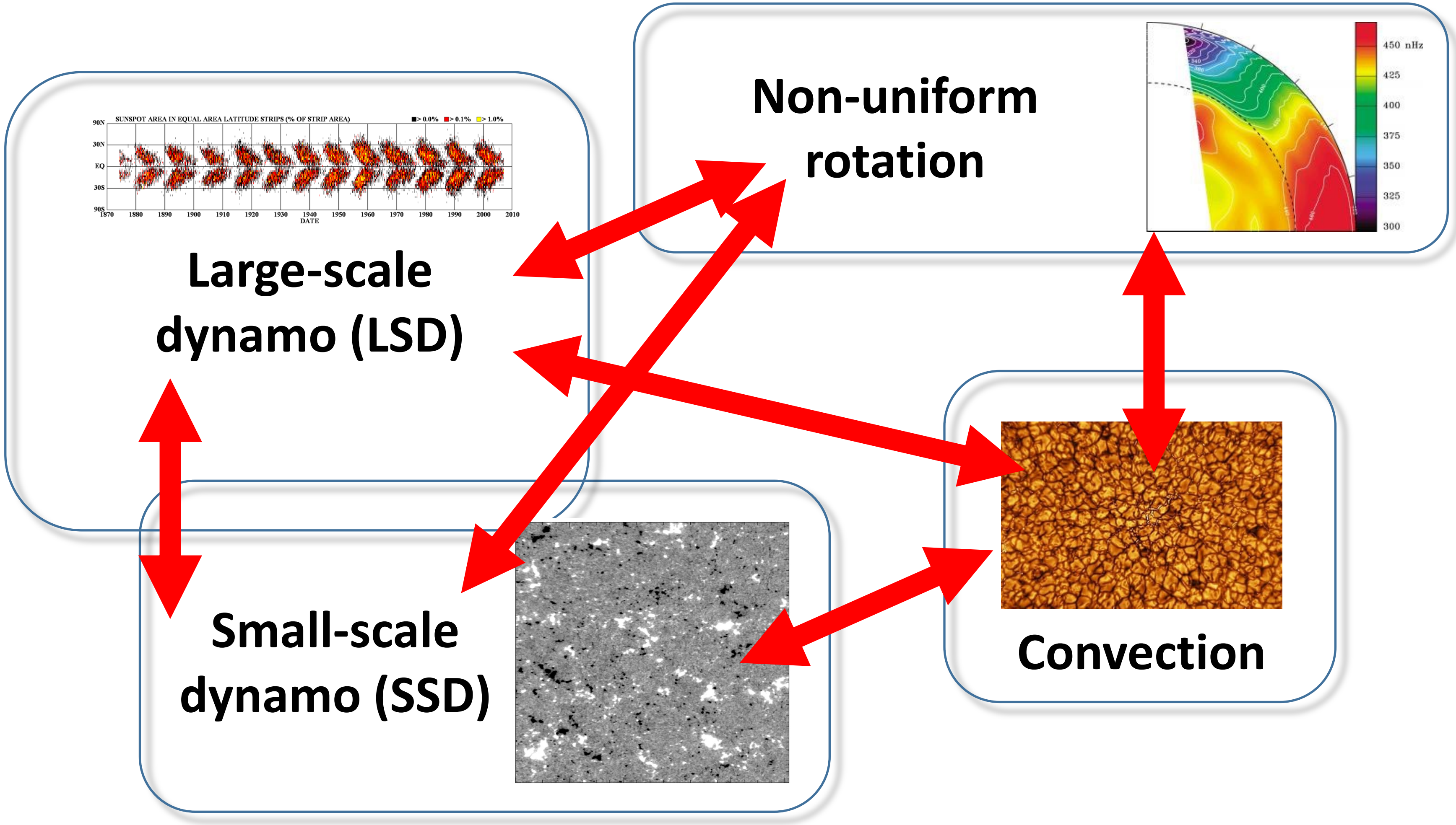
$$\mathrm{Pm} \equiv \frac{\mathrm{Rm}}{\mathrm{Re}} = \frac{\nu}{\eta}.$$

$$\mathrm{Pr} \equiv \frac{\nu}{\chi},$$

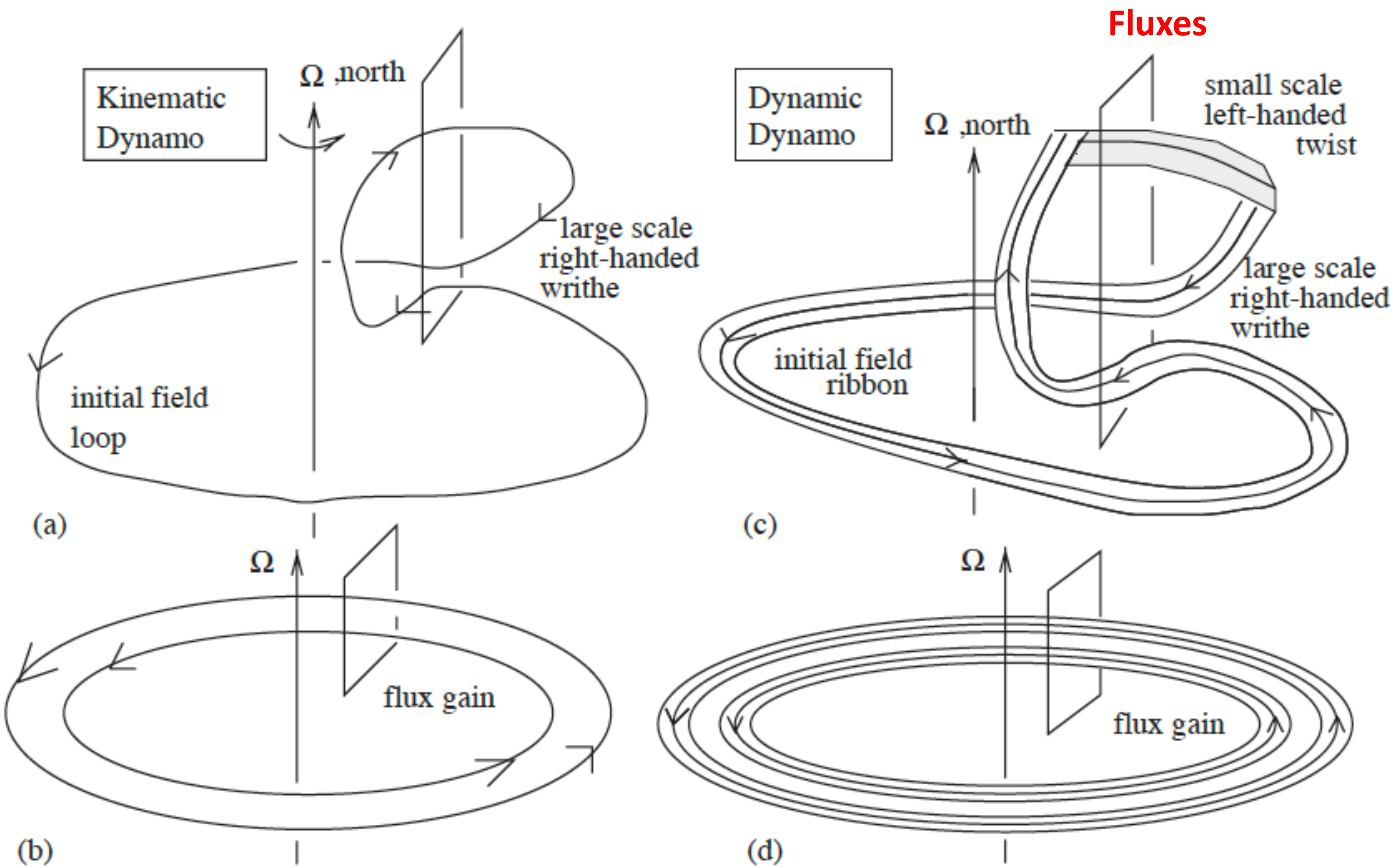
Big dynamic picture



Even larger DYNAMICAL picture



Schematic dynamic dynamo



Dynamic dynamo with equation

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \nabla \times [\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \alpha \overline{\boldsymbol{B}} - (\eta + \eta_t)\overline{\boldsymbol{J}}] \ ,$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = -2\eta_t k_f^2 \left(\frac{\alpha \langle \overline{\boldsymbol{B}}^2 \rangle - \eta_t \langle \overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} \rangle}{B_{eq}^2} + \frac{\alpha - \alpha_K}{\tilde{R}_m} \right)$$

No flux

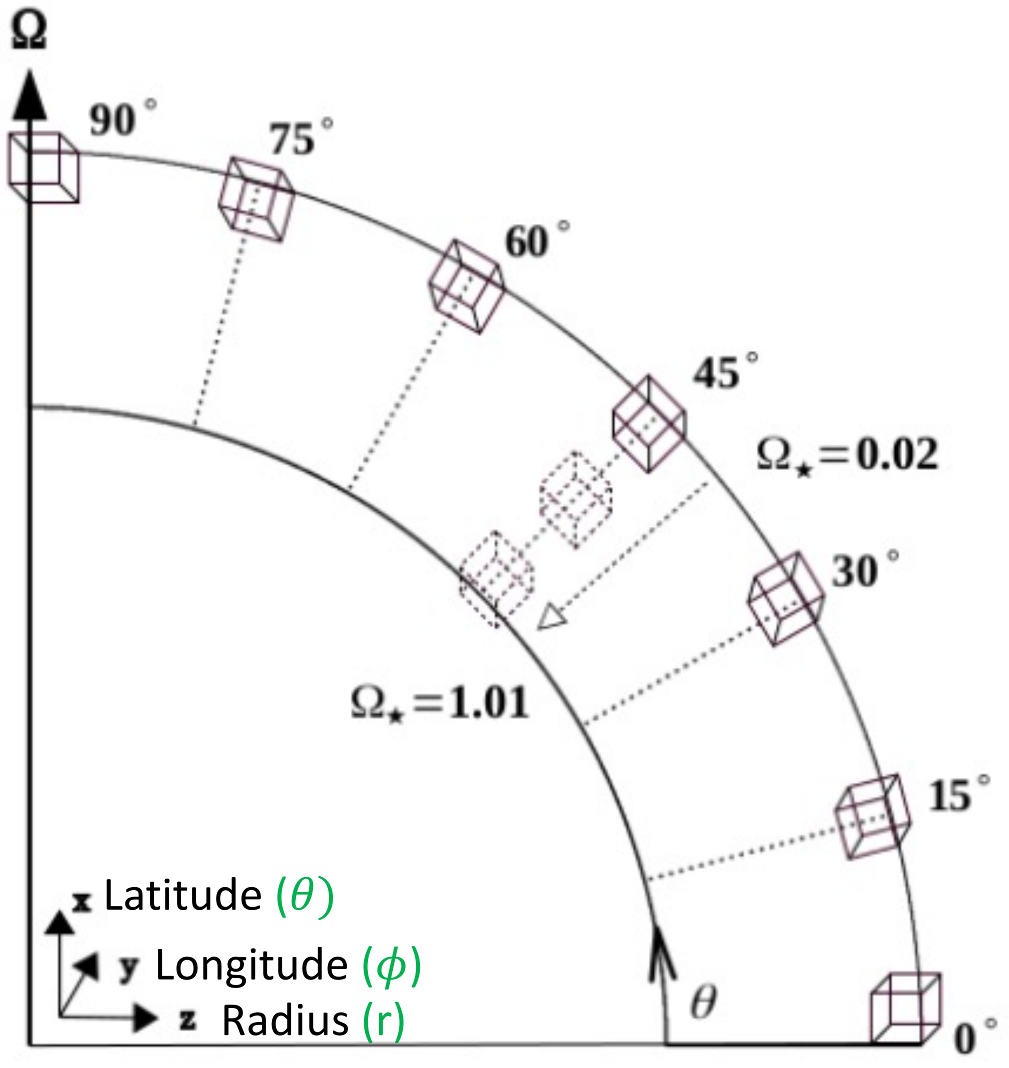
$$\frac{\partial \alpha}{\partial t} = -2\eta_t k_f^2 \left(\frac{\alpha \overline{\boldsymbol{B}}^2 - \eta_t \overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} + \frac{1}{2} k_f^{-2} \nabla \cdot \overline{\mathscr{F}}_C}{B_{eq}^2} + \frac{\alpha - \alpha_K}{R_m} \right)$$

Fluxes

In MF models all these effects need to be parameterized – in DNS they can arise self-consistently

Coordinate systems

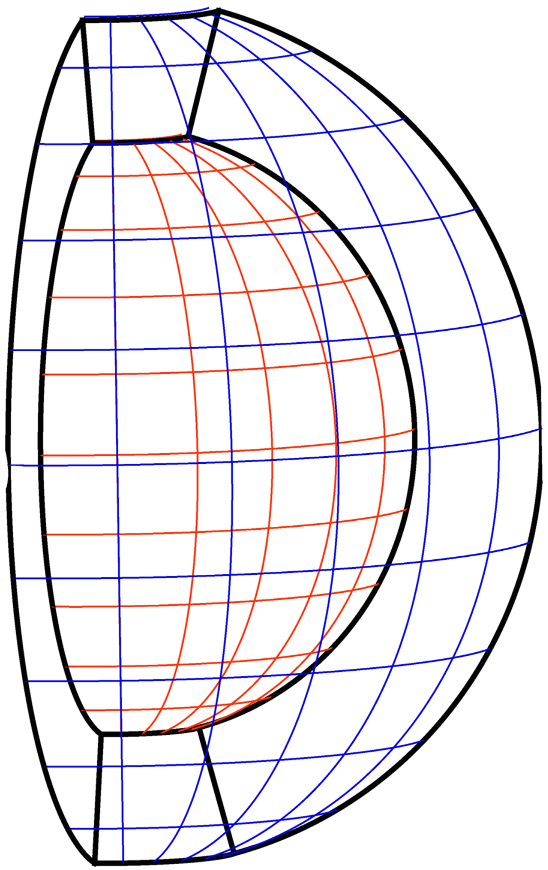
Poloidal field in the (r, θ)/(x,z)-plane
Toroidal field in the ϕ/y direction.



Spherical coordinate system counterparts

Coordinate systems

Wedges



Spherical coordinates, but both latitudinal and longitudinal directions are shrunk.
Longitude: for convenience, if the system is axisymmetric.
Latitude: to avoid polar singularity

Full MHD models: basic idea

Input

Output

TAR STARS

Discretize equations to the grid

Fill it with a stratified fluid

Heat if from below and cool from the top

Figure out other suitable bcs

Make it rotate

Convection

Turbulence

Twist

Differential rotation

Stretch

Meridional flow

Dynamo action

Self-consistent

Uncontrollable

Non-dimensionalization

$$C_\alpha = \frac{\alpha_0 R}{\eta_0}, \quad C_\Omega = \frac{\Omega_0 R^2}{\eta_0}, \quad \text{and} \quad C_U = \frac{u_0 R}{\eta_0}$$

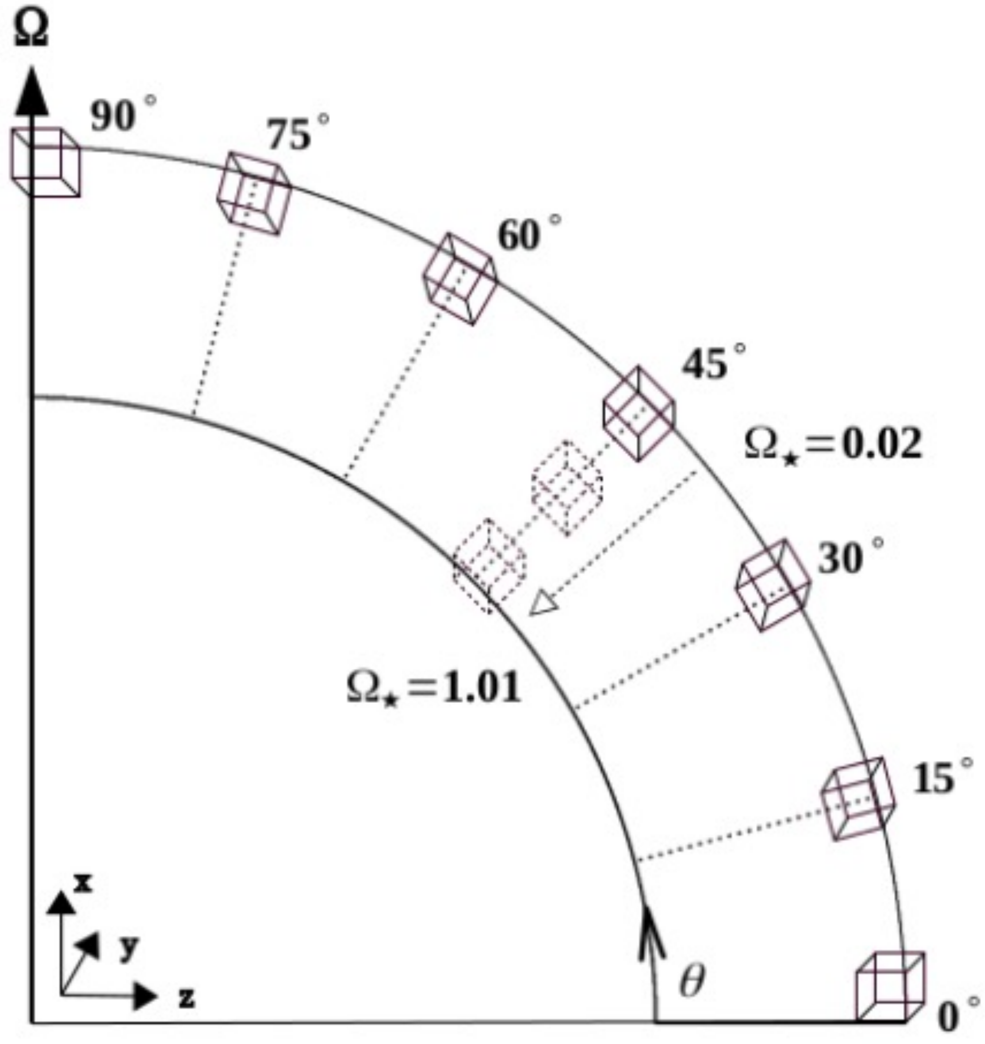
$$Re = \frac{u\,\ell}{\nu}, Rm = \frac{u\,\ell}{\eta}$$

- Computers deal with numbers
- They cannot keep track on units
- Before numerical solutions are attempted, the equations must be non-dimensionalised
- One ends up with non-dimensional control parameters…
- …which are the familiar Reynolds numbers for the full MHD equations
- …and dynamo numbers for the MF induction equation

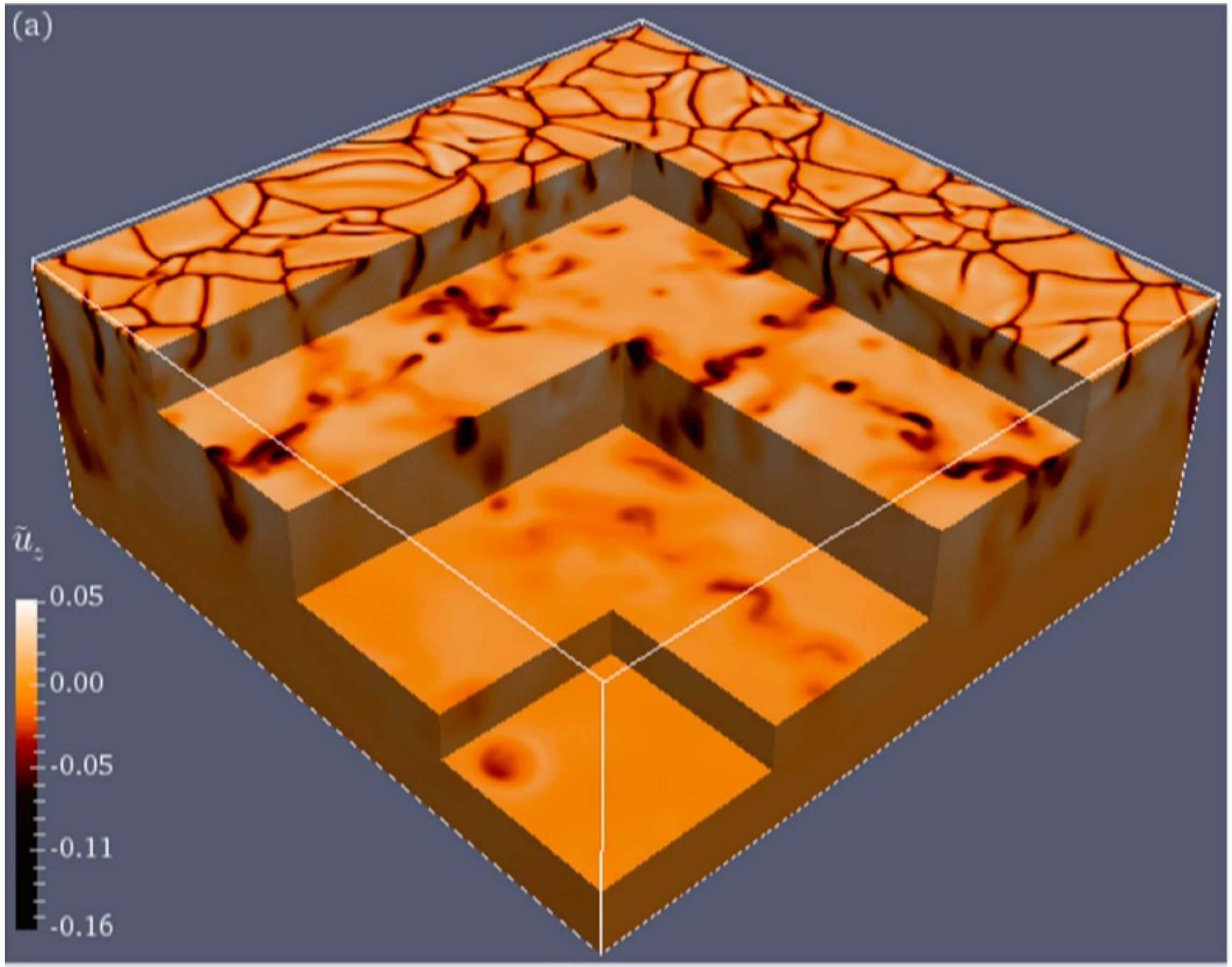
Local convection simulations

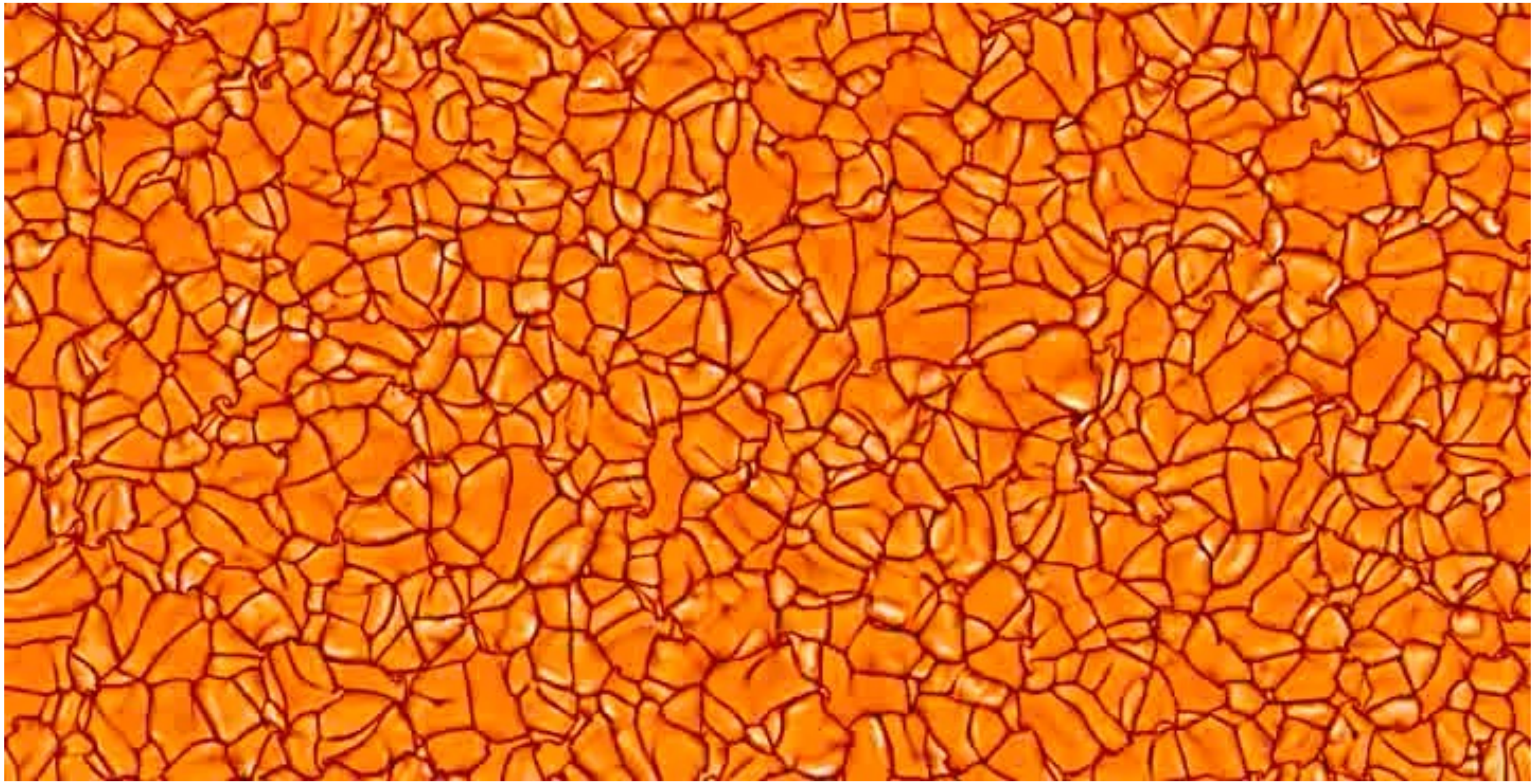
- Maximise resolution for accuracy
- Study key processes in isolation
- Reduce the complexity of the full system

- Do not tell the full story



Local models

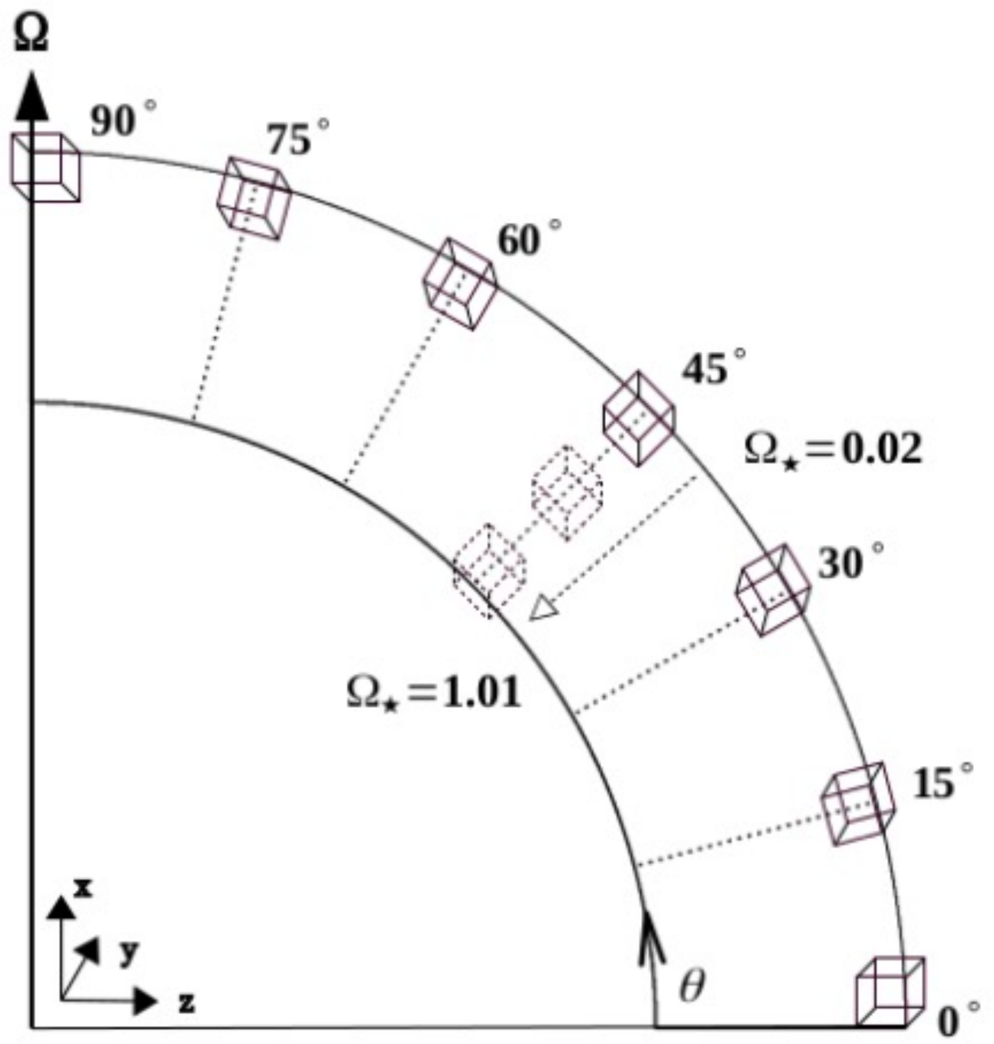




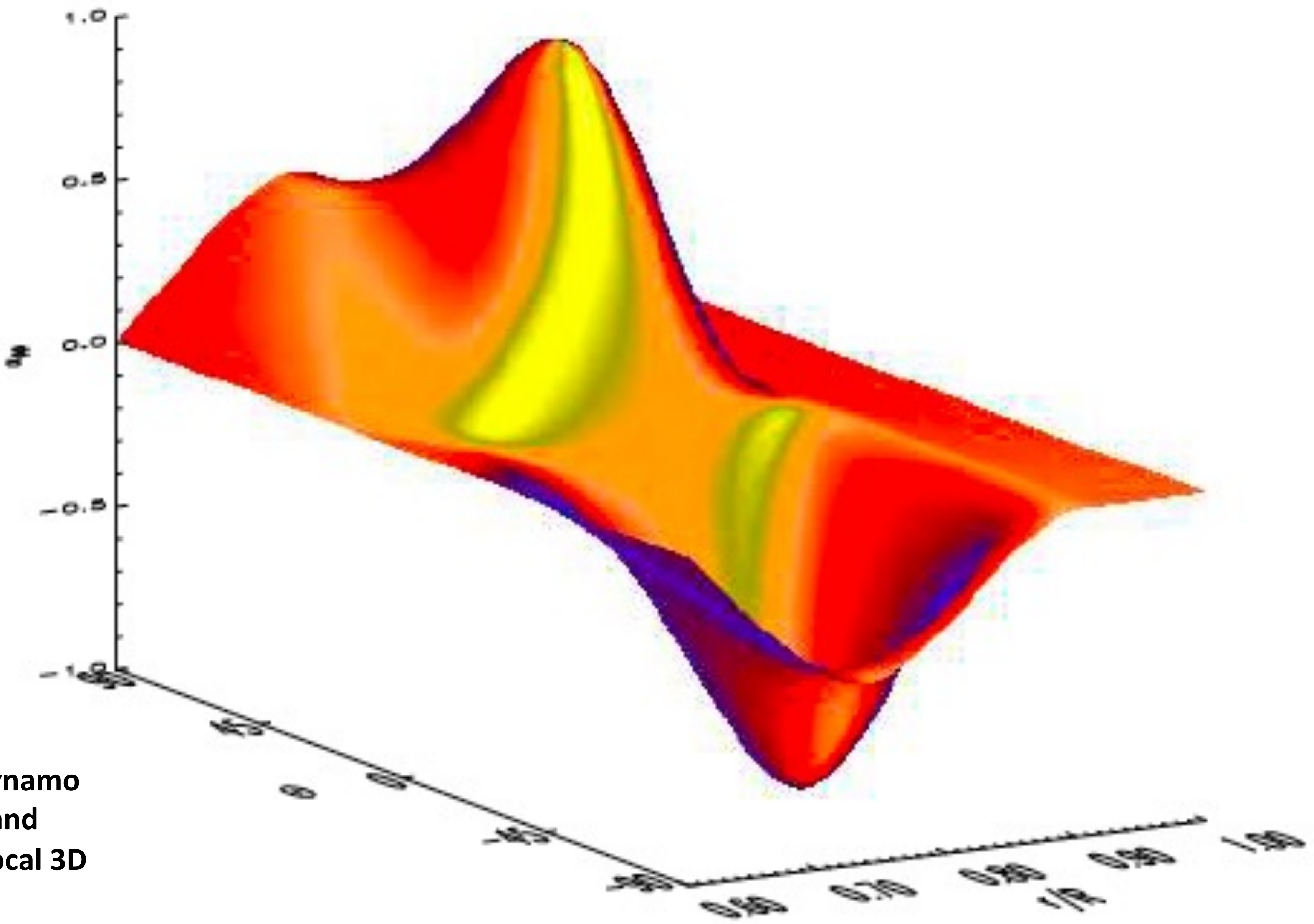
Movie from a local simulation near the surface

Local convection simulations

- Testing theories for convection
- Angular momentum transport
- Dynamo driving
  - Large-scale dynamo

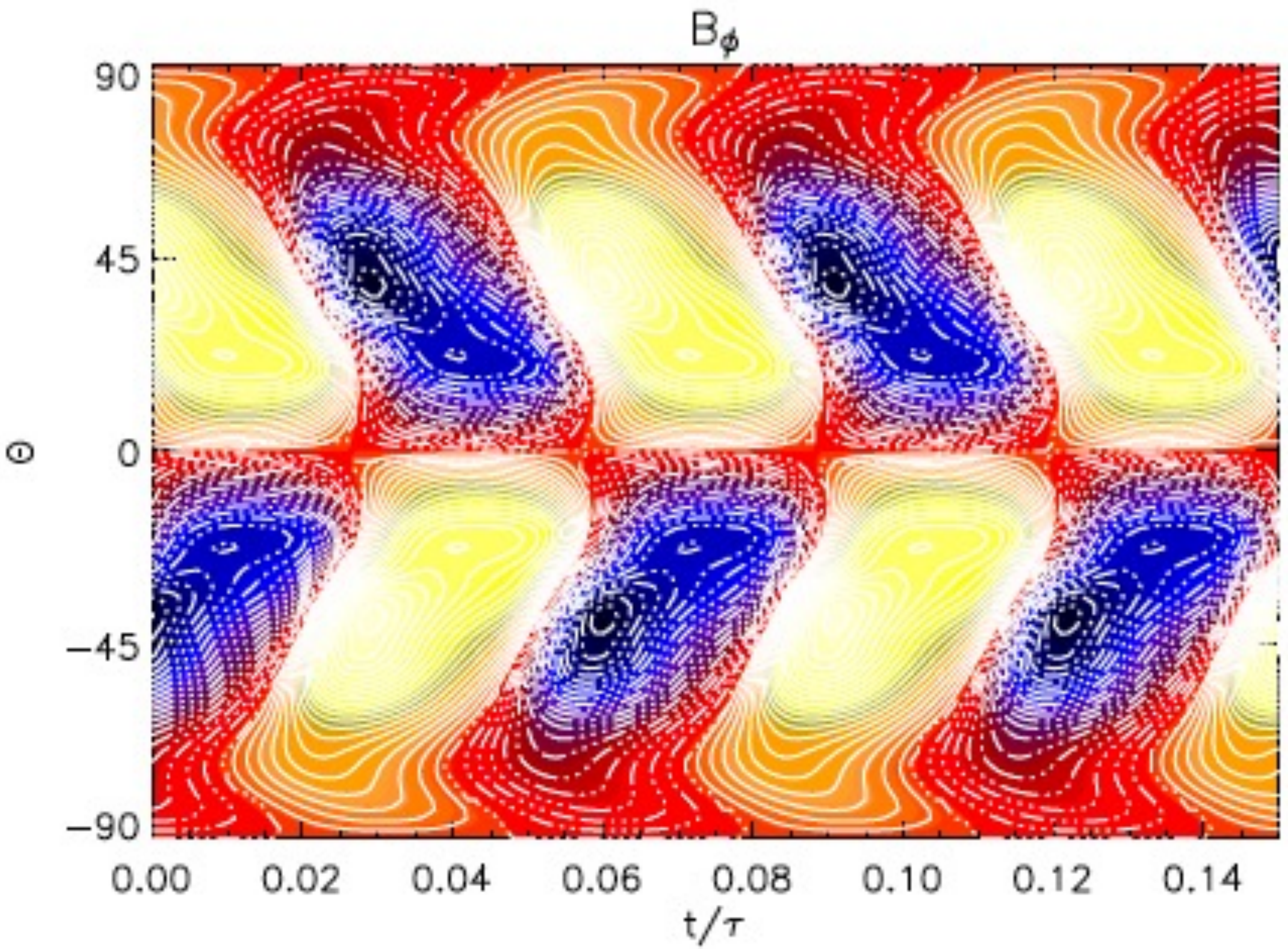


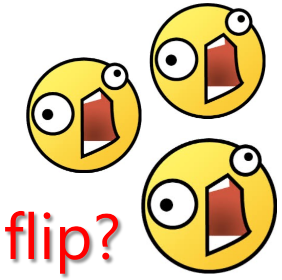
α effect as function of latitude and depth in the convection zone



Käpylä P. J., Korpi M. J., Tuominen I. 2006: Solar dynamo models with alpha-effect and turbulent pumping from local 3D calculations, AN, 327, 884

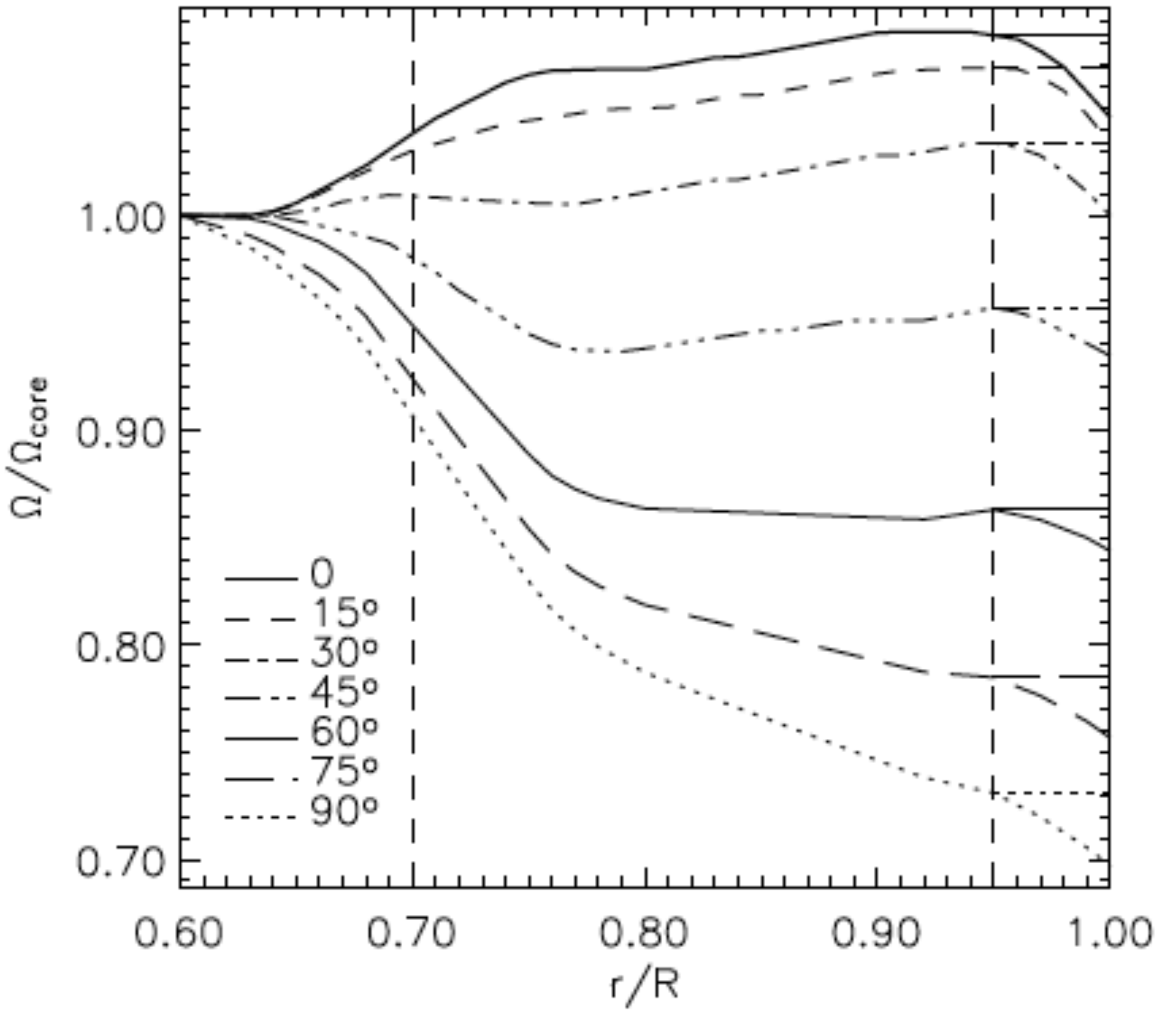
MF dynamo model with helioseismic shear

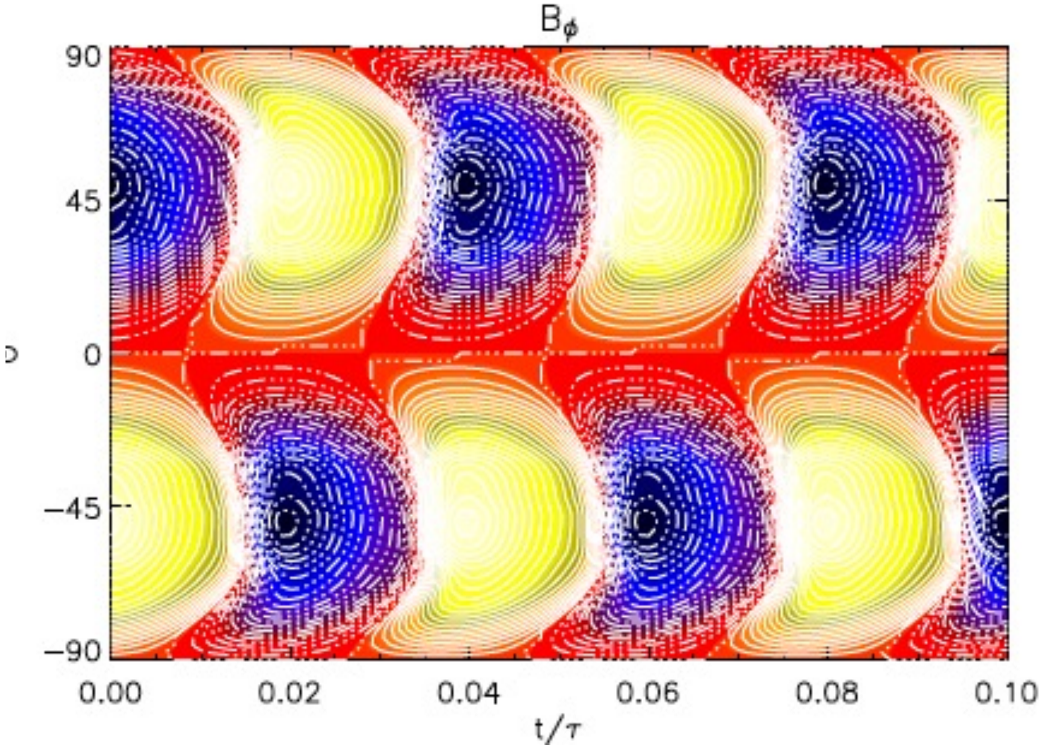




Whatta flip?

Leptocline and meridional flow required

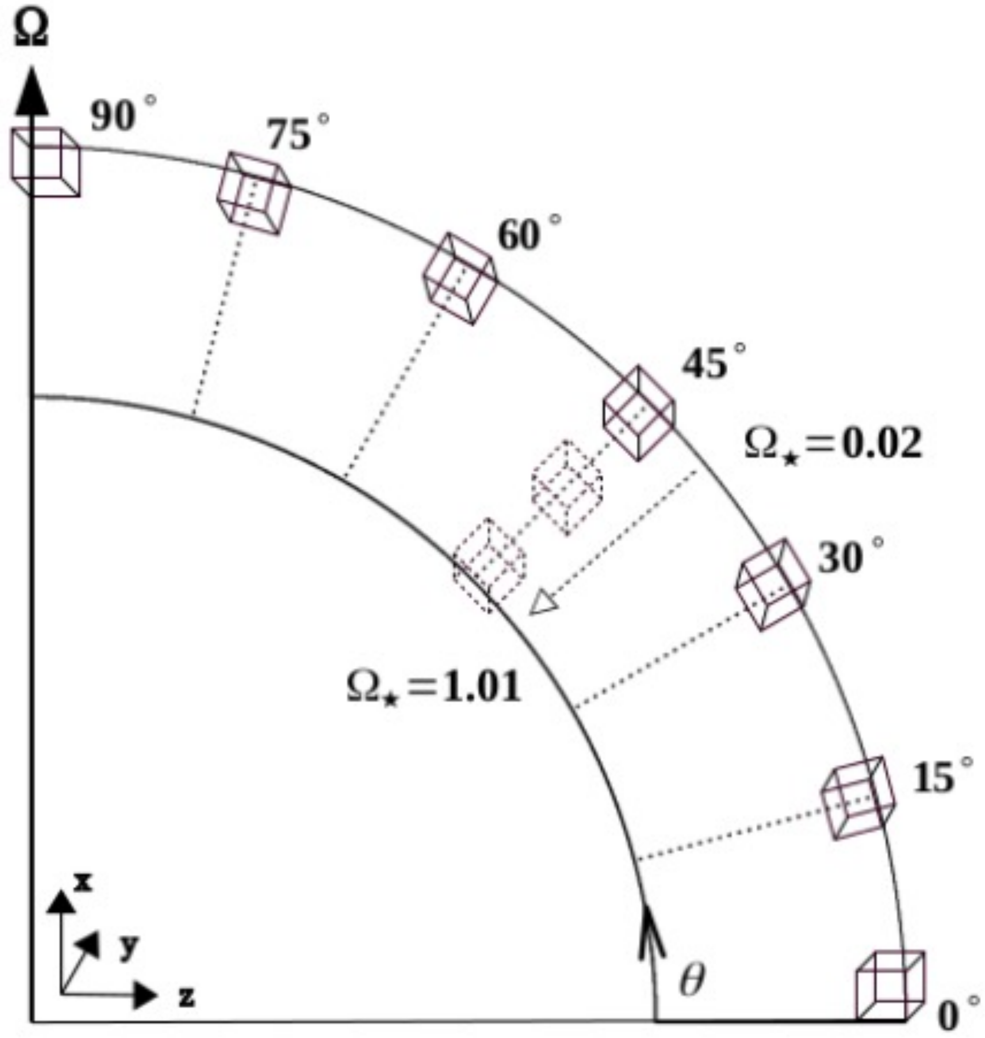




Butterfly without those effects

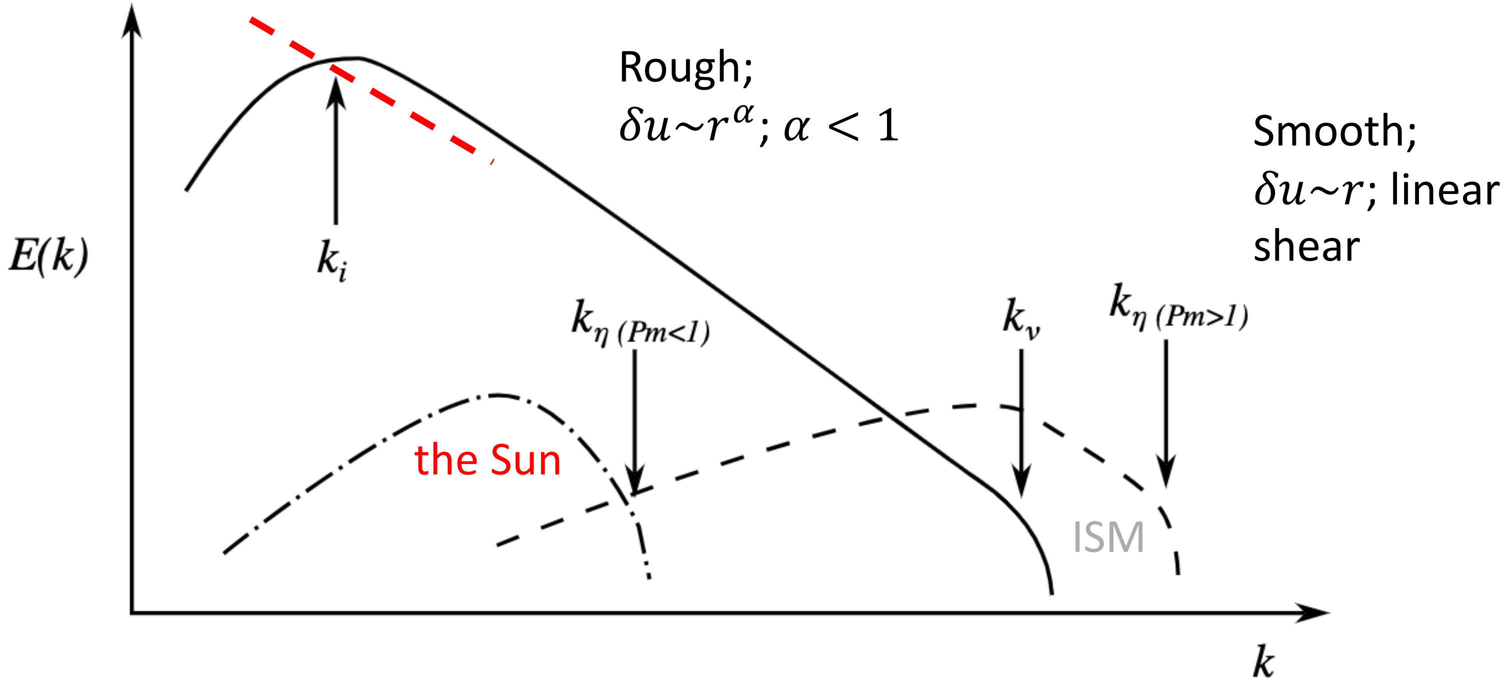
Local convection simulations

- Testing theories for convection
- Angular momentum transport
- Dynamo driving
  - Large-scale dynamo
  - Small-scale dynamo

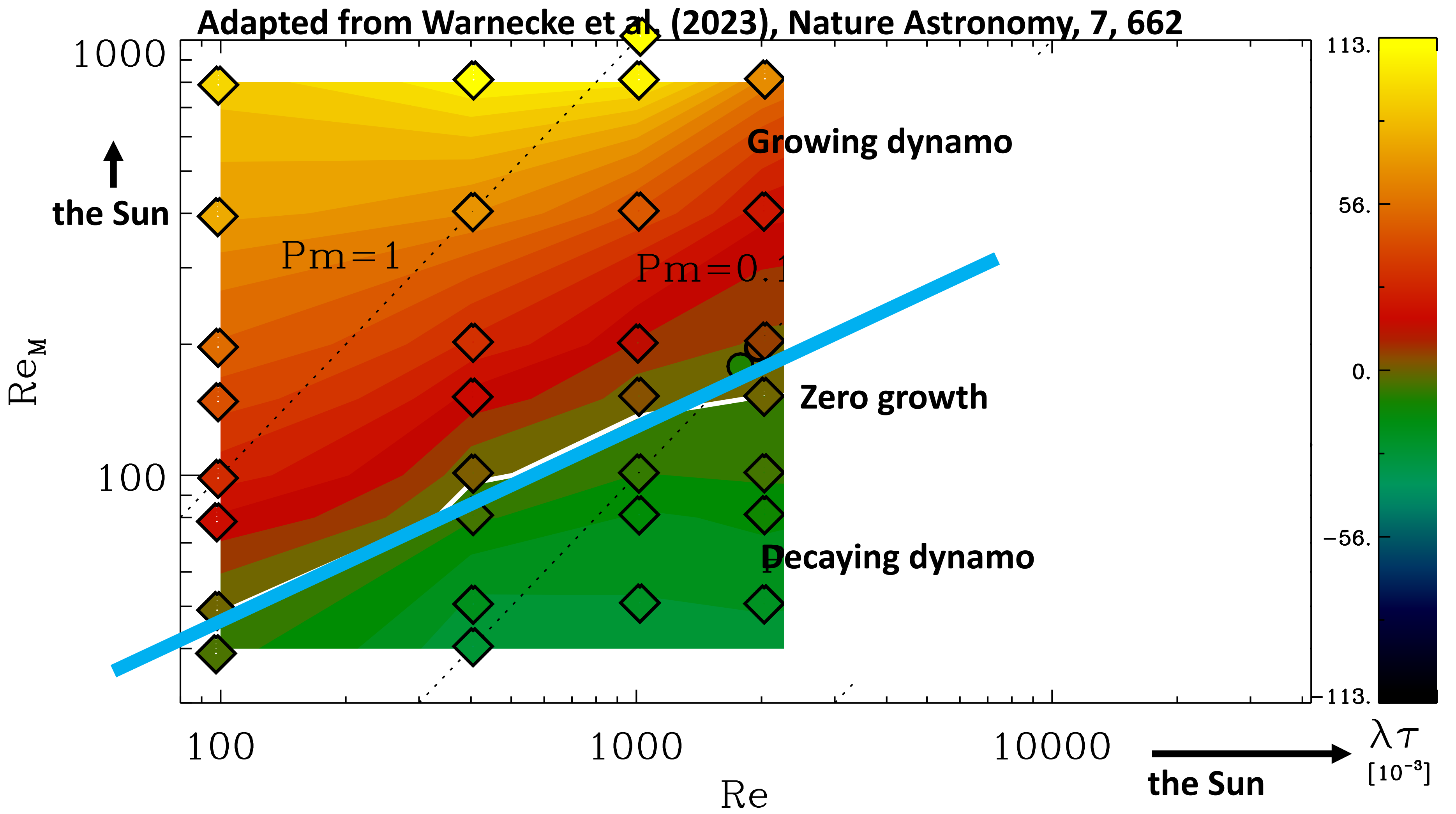


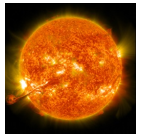
Stellar convection zones have small magnetic Prandtl numbers (*Pm*)

Schematic theoretical



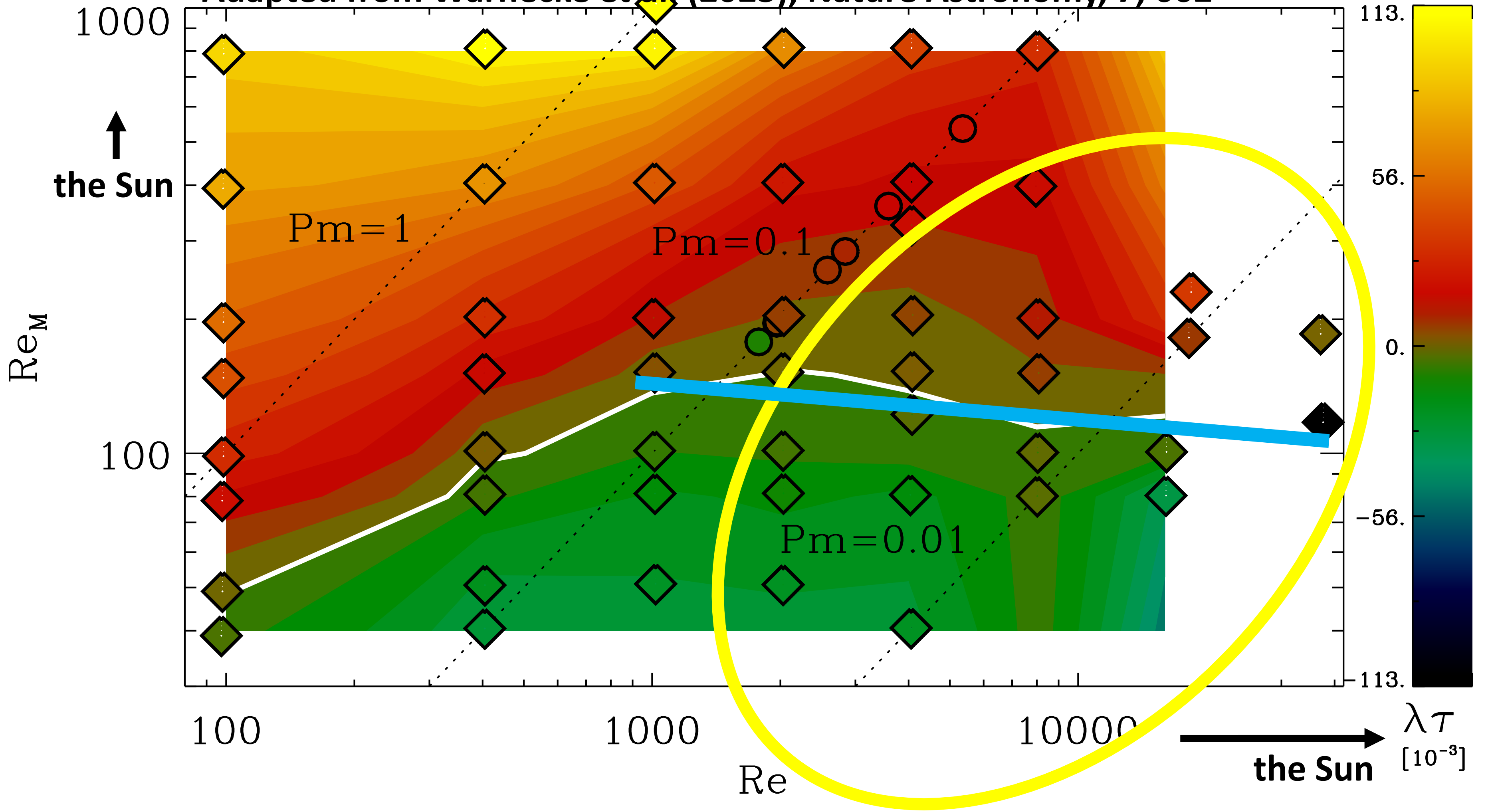
Few years ago SSD seemed to be impossible…

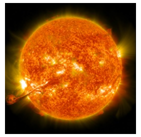


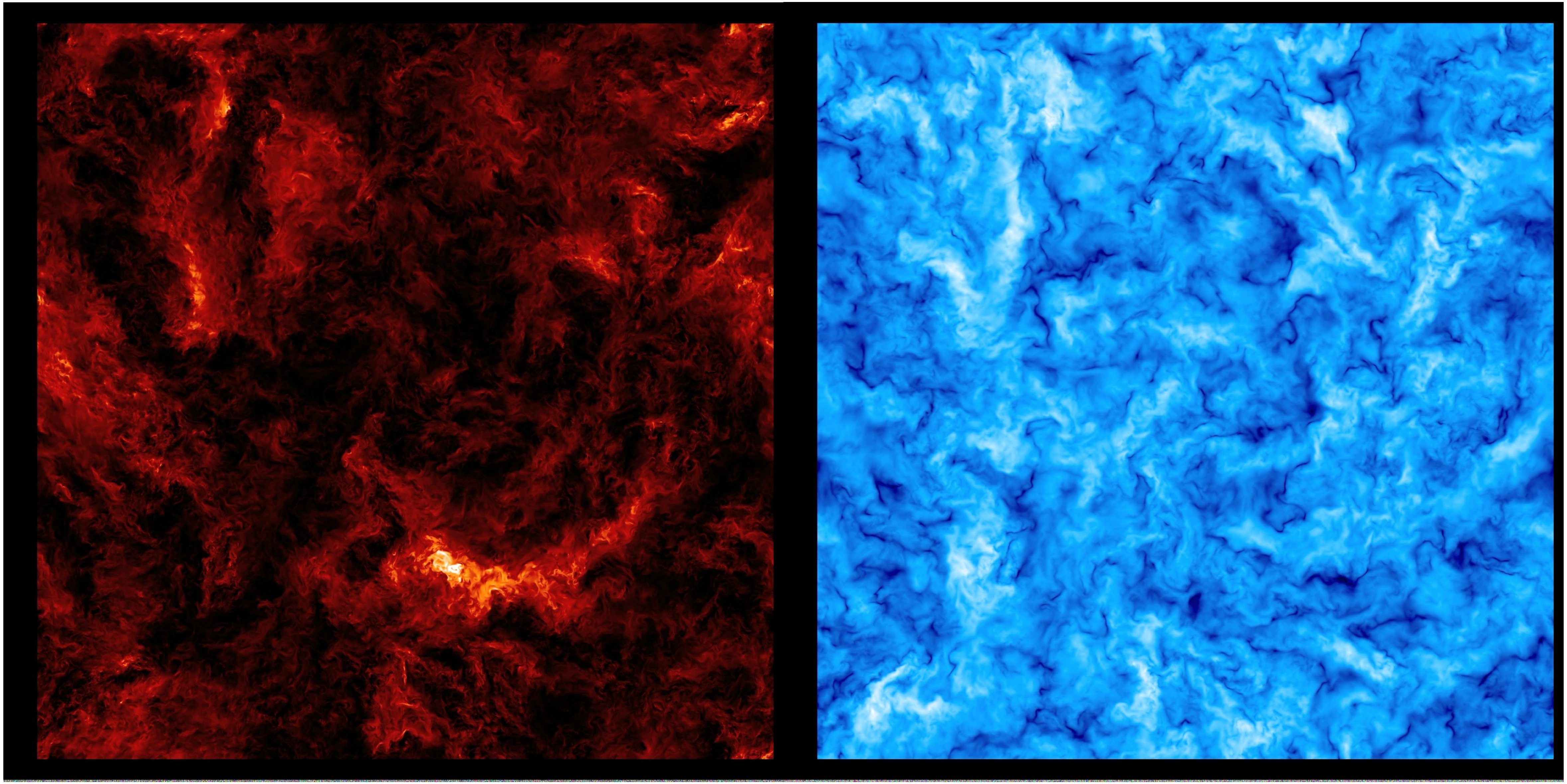


Improved computational resources changed the picture

Adapted from Warnecke et al. (2023), Nature Astronomy, 7, 662

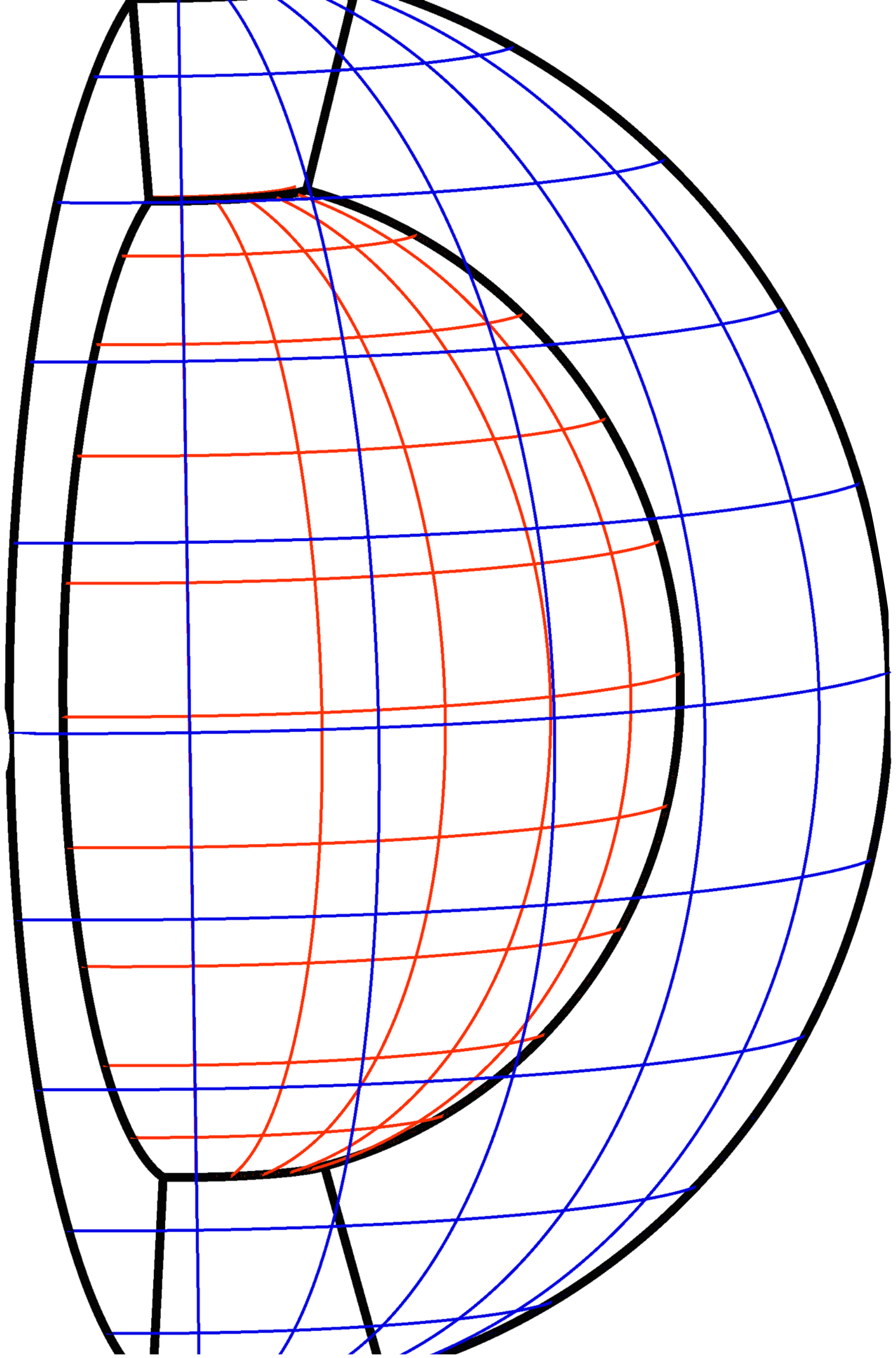






Speed

Magnetic energy



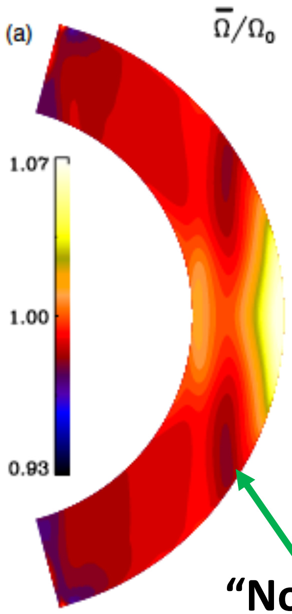
Global convection simulations

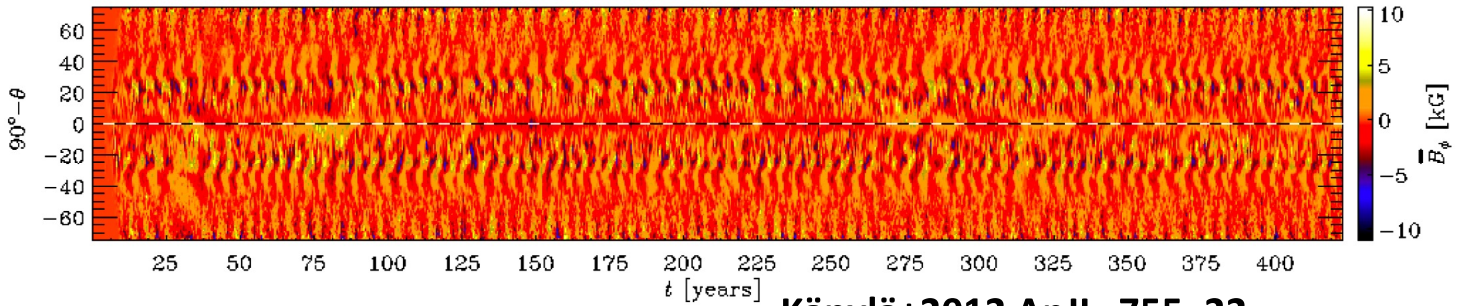
- Suffer in accuracy
- Allow studies on the full story

Can you yet trust the results?

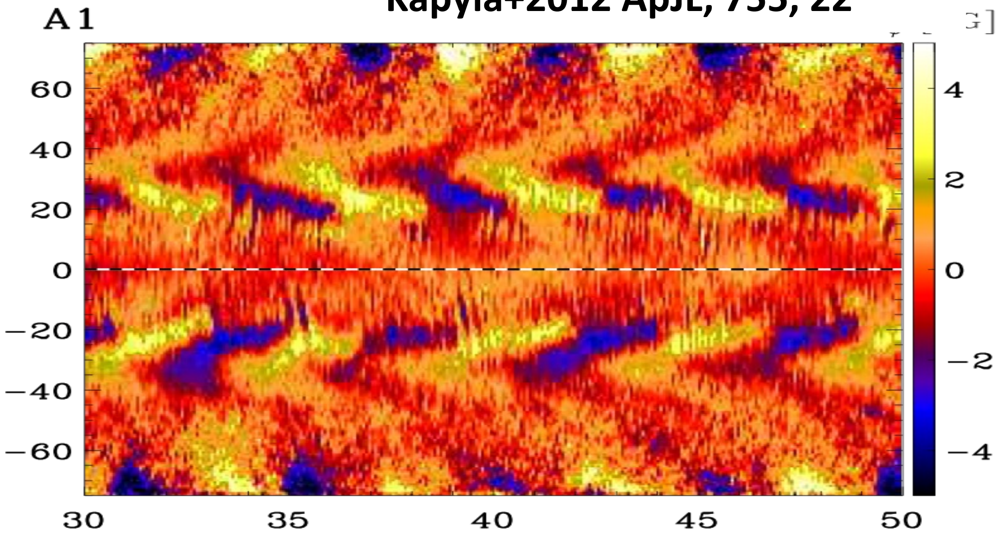
Solar-like cycles, but differential rotation profile is not matching

Käpylä+2016 A&A, 598, A56

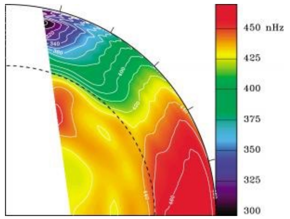


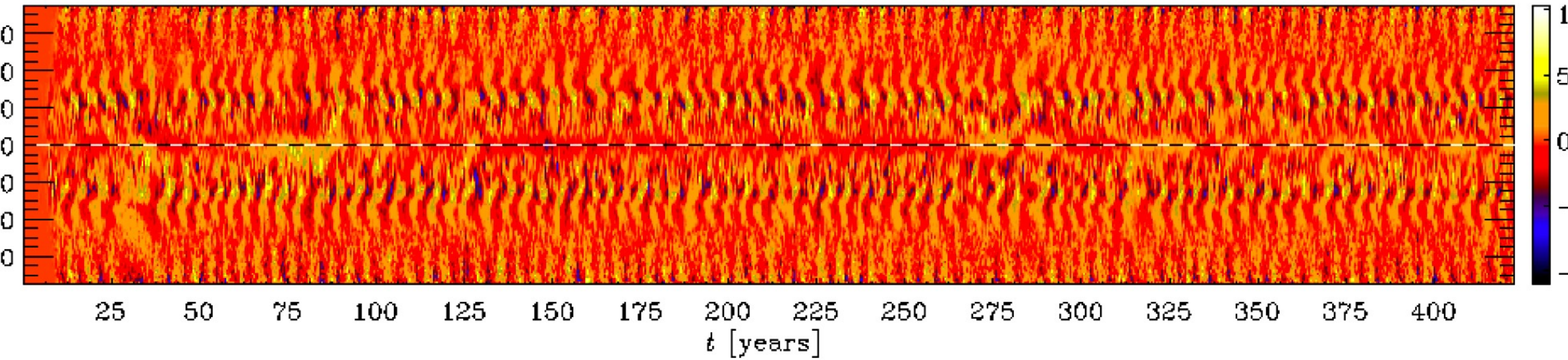


Käpylä+2012 ApJL, 755, 22



“Nose”
Too cylindrical

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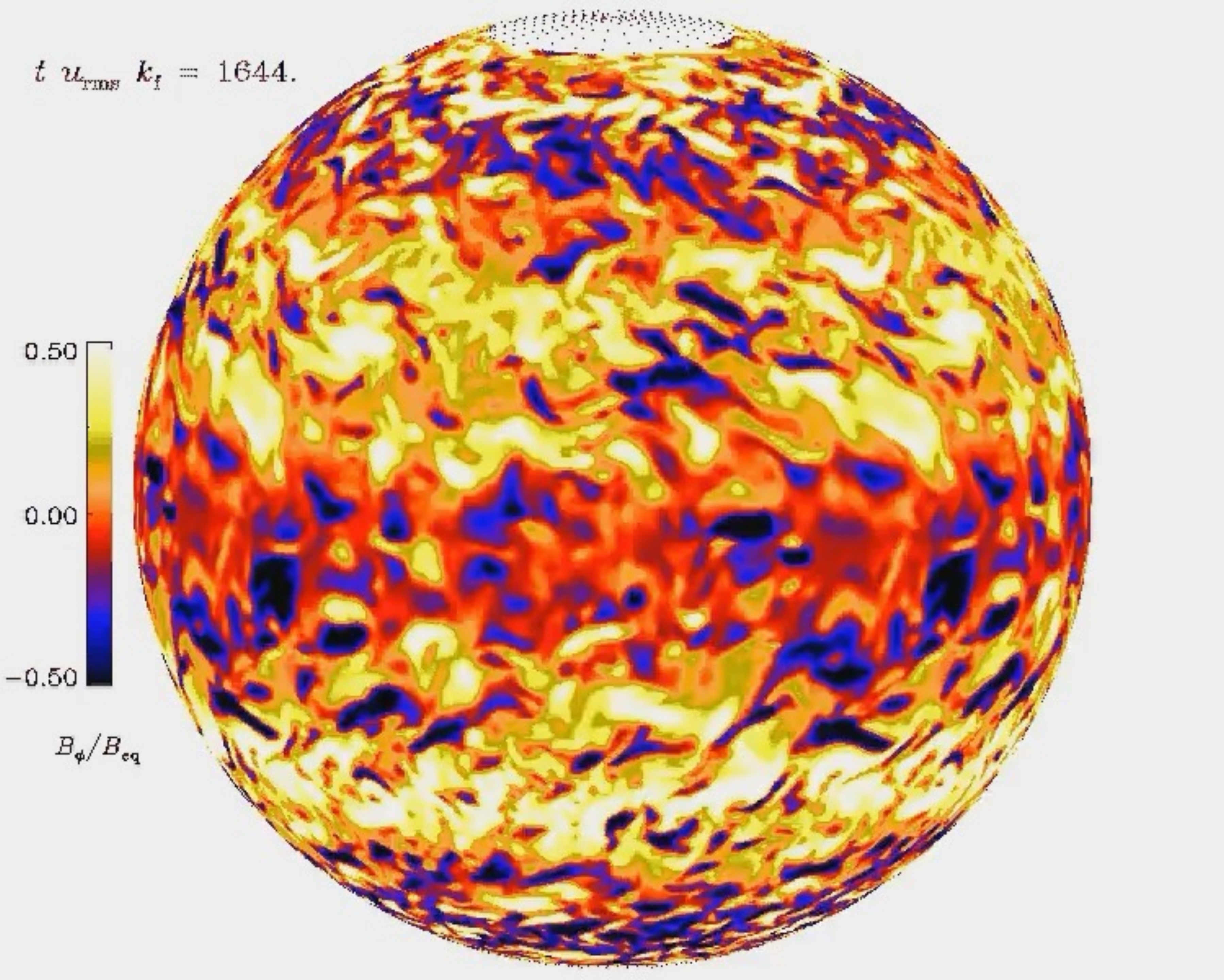
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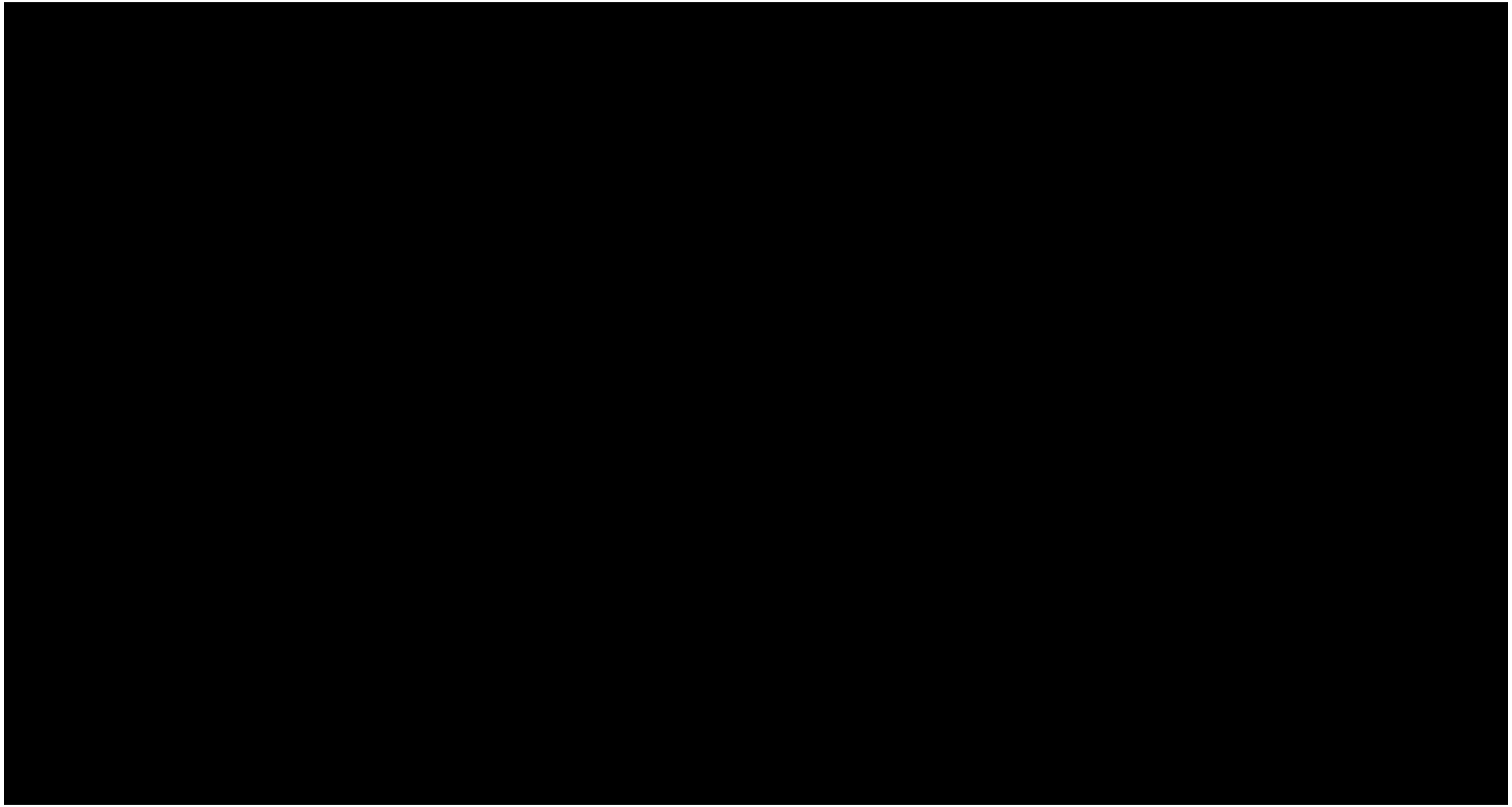
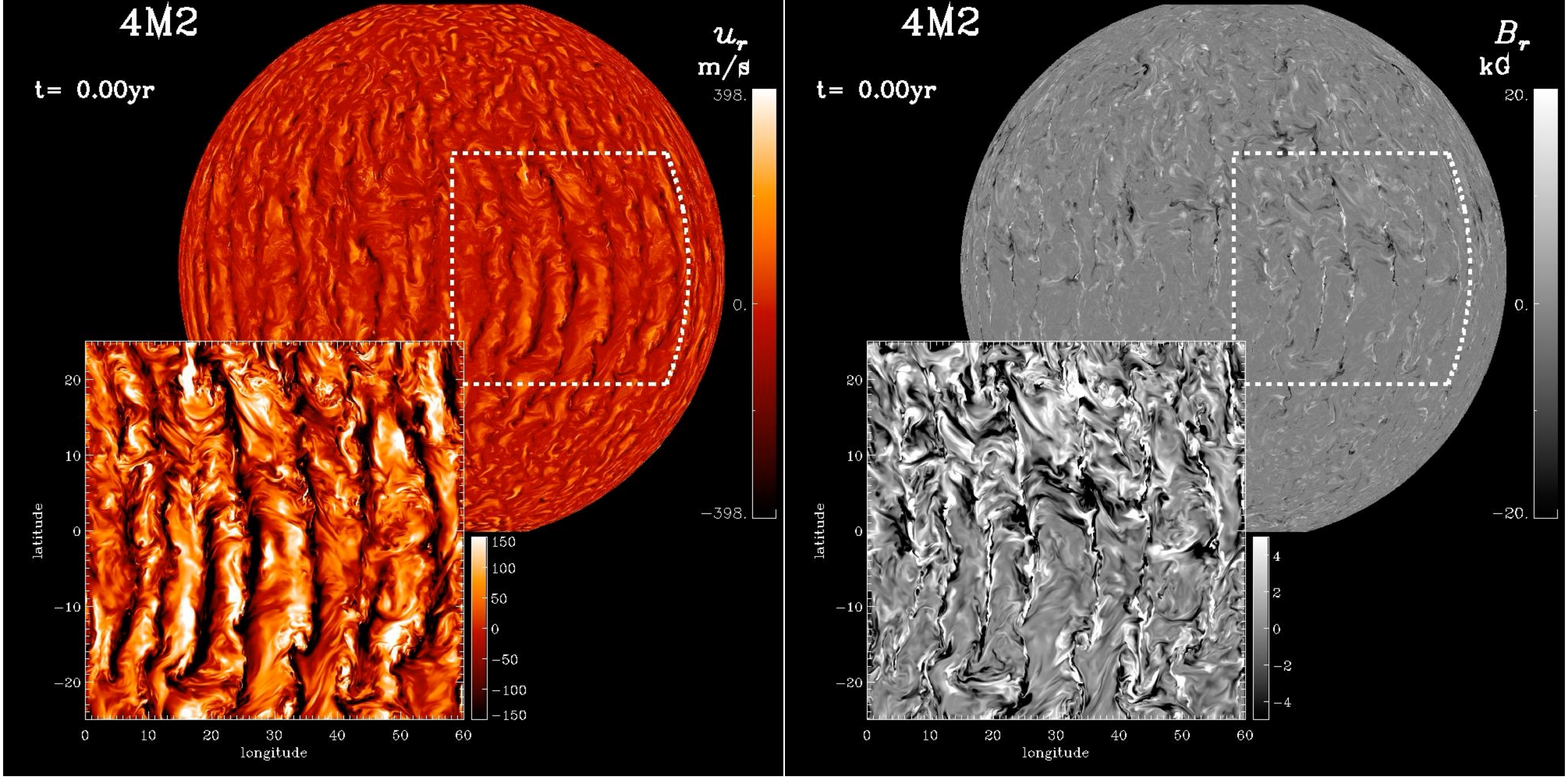
−0.50
$B_\phi/B_{\rm eq}$

Active rapid rotators



MF predictions and models agree very well



Highest resolution wedge models capture SSD and LSD at Pm=1

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- Understand how and why full MHD solvers can be useful to understand stellar magnetism
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- Most prominent results
- Recap of the whole contents of the MHD module

Most prominent results from full MHD

- Local models predict that
  - Parker dynamo is a functioning concept
  - SSD should exist in stellar convection zones
- Global models
  - Can produce solar-like cycles
  - Cannot produce the correct differential rotation in the Sun
  - When applied to active stars, reproduce theoretical (MF predictions)
  - Very recently capture SSD in Pm=1 fluids