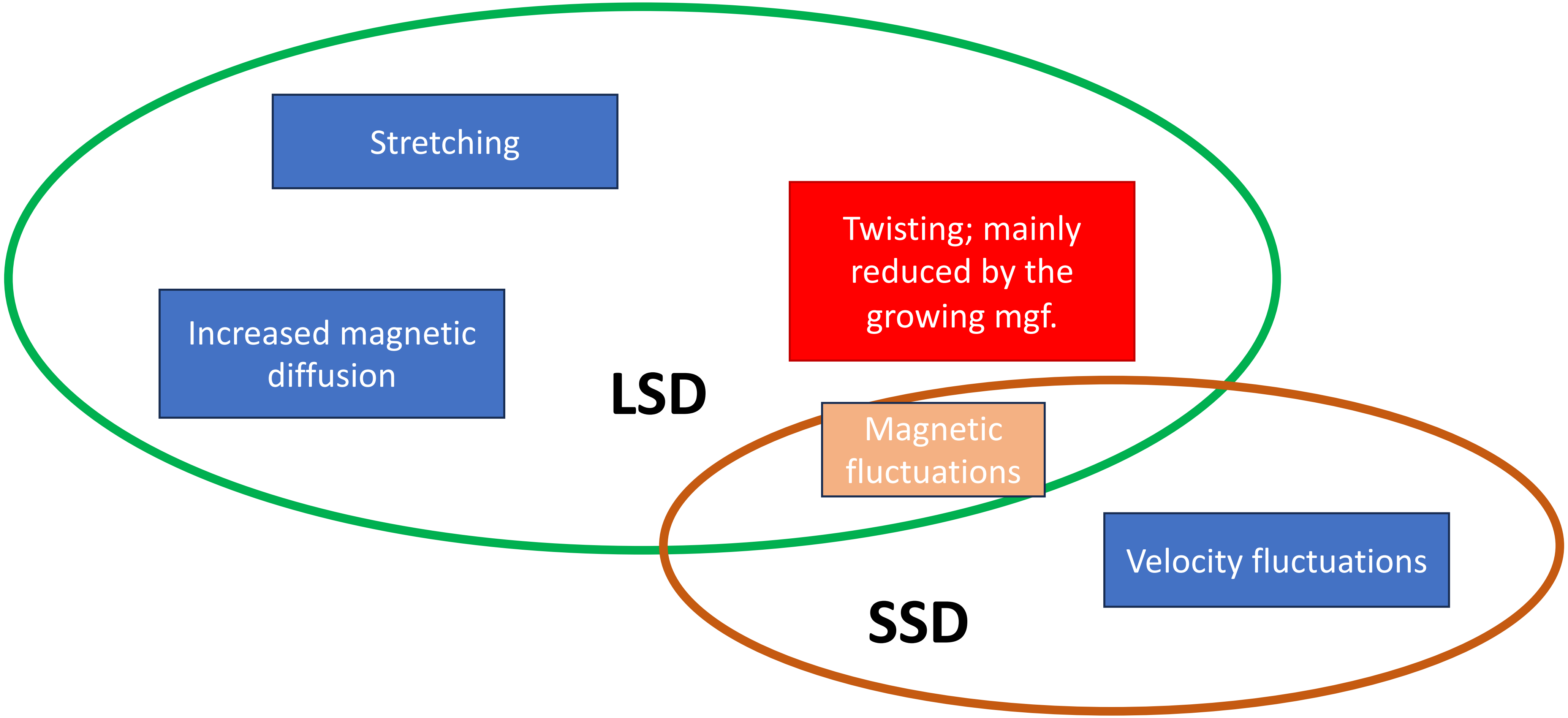
Mean field models for solar and stellar dynamos

Learning outcomes

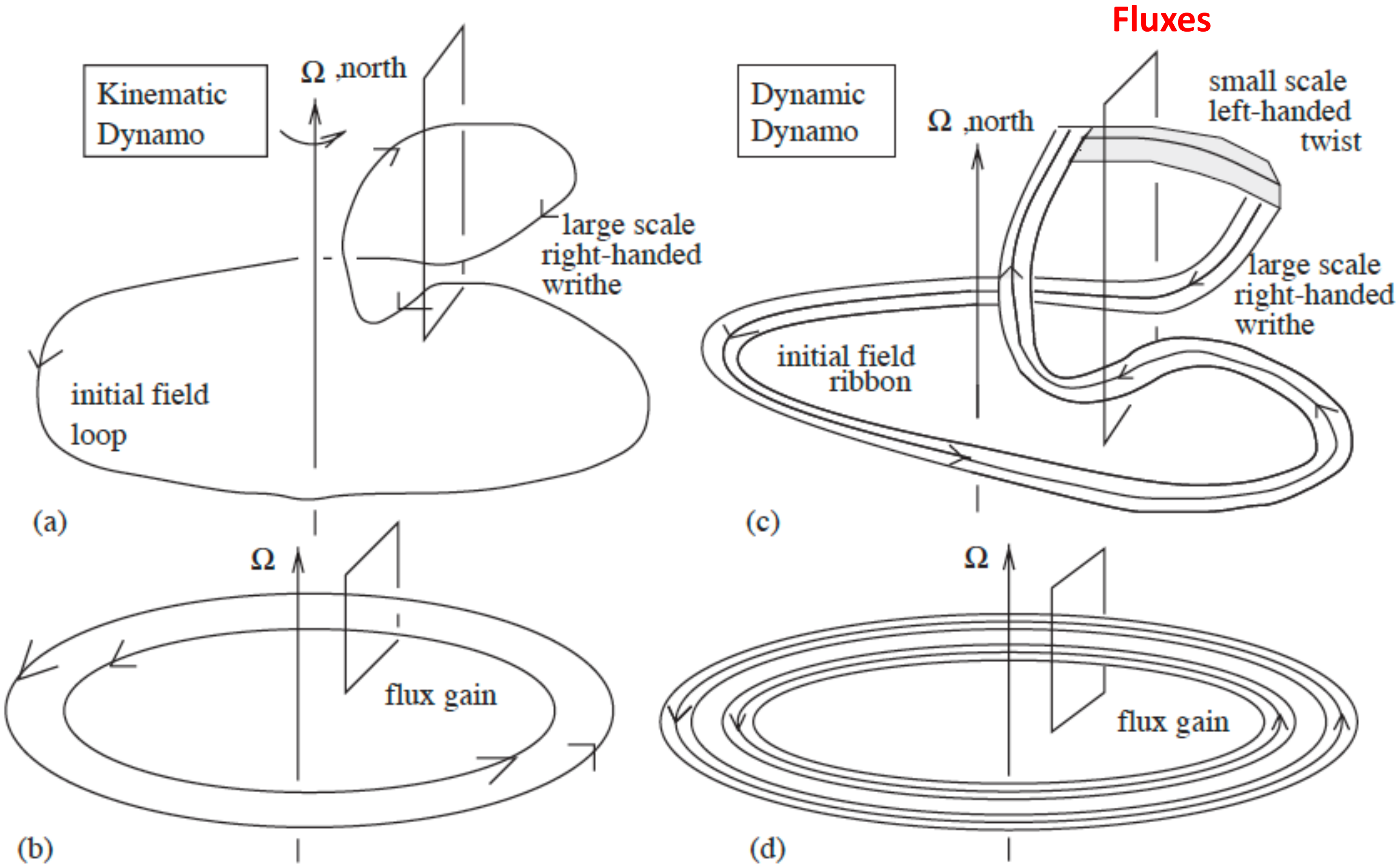
Continuing to learn to master but also to apply the concepts discussed last week

- we will extend the MF concept to first understand solar dynamo dichotomy and evidences against and for the two paradigms
- next to understand theory of stellar differential rotation
- finally to develop capability to extend the thinking from the solar dynamo to stellar dynamos

Big dynamic picture



Schematic dynamic dynamo



Dynamic dynamo with equation

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \nabla \times [\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \alpha \overline{\boldsymbol{B}} - (\eta + \eta_t)\overline{\boldsymbol{J}}] \ ,$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = -2\eta_t k_f^2 \left(\frac{\alpha \langle \overline{\boldsymbol{B}}^2 \rangle - \eta_t \langle \overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} \rangle}{B_{eq}^2} + \frac{\alpha - \alpha_K}{\tilde{R}_m} \right)$$

No flux

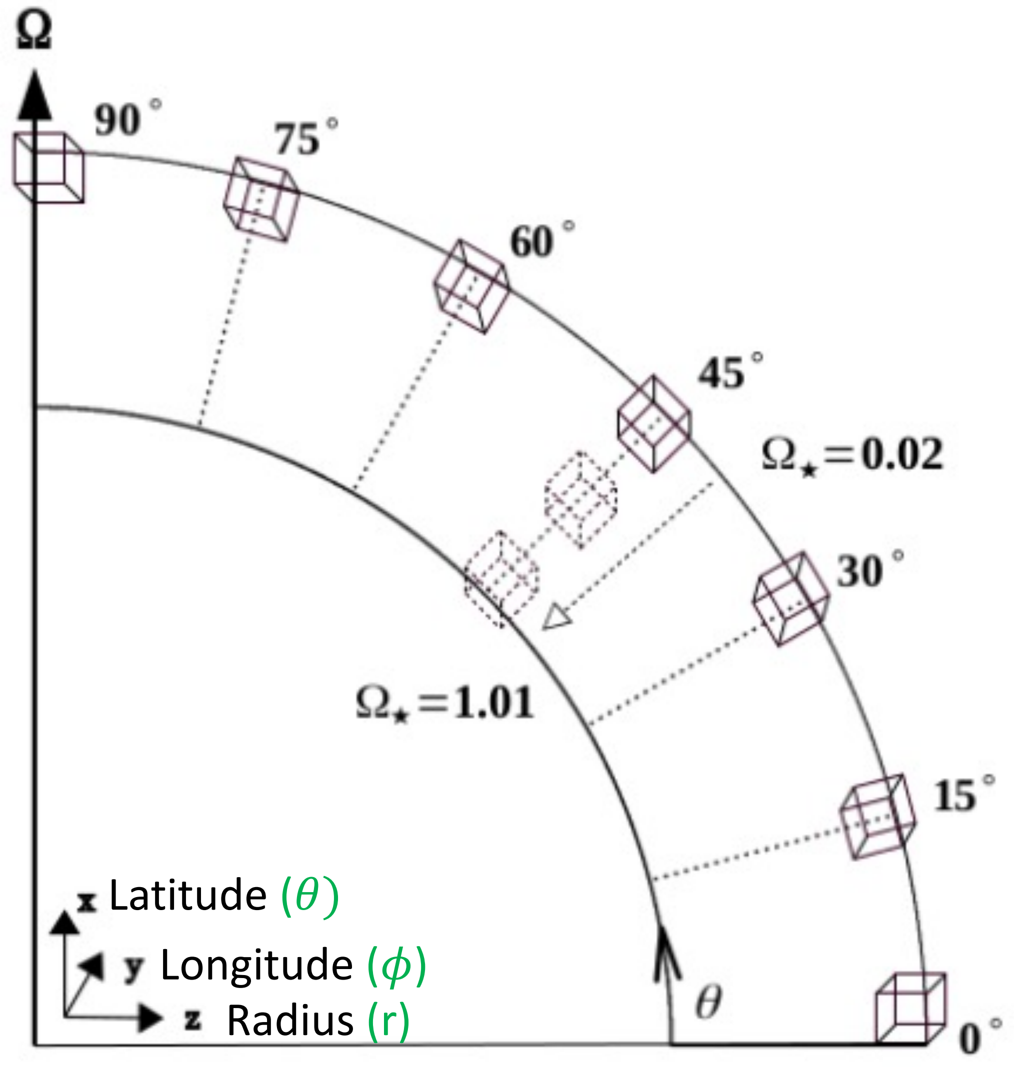
$$\frac{\partial \alpha}{\partial t} = -2\eta_t k_f^2 \left(\frac{\alpha \overline{\boldsymbol{B}}^2 - \eta_t \overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} + \frac{1}{2} k_f^{-2} \nabla \cdot \overline{\mathscr{F}}_C}{B_{eq}^2} + \frac{\alpha - \alpha_K}{R_m} \right)$$

Fluxes

Based on analytics, virtually all this quantities, especially the fluxes, remain unknown.

Coordinate systems

Poloidal field in the (r, θ)/(x,z)-plane
Toroidal field in the ϕ/y direction.



Spherical coordinate system counterparts

Further dynamo concepts

$\alpha_0 \approx 2-3\frac{m}{s}, \Omega_0 \approx 2.6\cdot 10^{-6}\frac{1}{s}, u_0 \approx 10\frac{m}{s}, \eta_0 \approx 5\cdot 10^{8}\frac{m^2}{s}$ for the Sun. What type of a dynamo is that?

- Axis- versus nonaxisymmetry $\frac{\partial}{\partial\phi} / \frac{\partial}{\partial y} = 0$
- Equatorial symmetry
- Poloidal-toroidal decomposition

$$\boldsymbol{B} = \boldsymbol{B_P} + \boldsymbol{B_T} = \nabla\times(A\hat{y}) + B\hat{y}$$

$$\frac{\partial A}{\partial t} = \alpha B + \eta_T\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial z^2}\right),$$

$$\frac{\partial B}{\partial t} = \alpha\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial z^2}\right) + B_P\cdot\nabla \boldsymbol{u}_y + \eta_T\left(\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial z^2}\right)$$

- Dynamo numbers

$$C_\alpha = \frac{\alpha_0 R}{\eta_0}, \quad C_\Omega = \frac{\Omega_0 R^2}{\eta_0}, \text{ and } C_U = \frac{u_0 R}{\eta_0}$$

Values with index zero are typical values of the quantities

Generic properties of the $\alpha\Omega$ dynamo

$$\frac{\partial A}{\partial t} = \alpha B + \eta_T \left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial z^2} \right),$$

$$\frac{\partial B}{\partial t} = \alpha \left(\cdots \right) + B_P \cdot \nabla u_y + \eta_T \left(\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial z^2} \right)$$

$\nabla\Omega$

Linear stability analysis and solving for the eigenvalue problem yields an oscillatory solution with

$$\omega_{cyc} = \pm \left| \frac{1}{2} \alpha \Omega k \right|^{1/2}$$

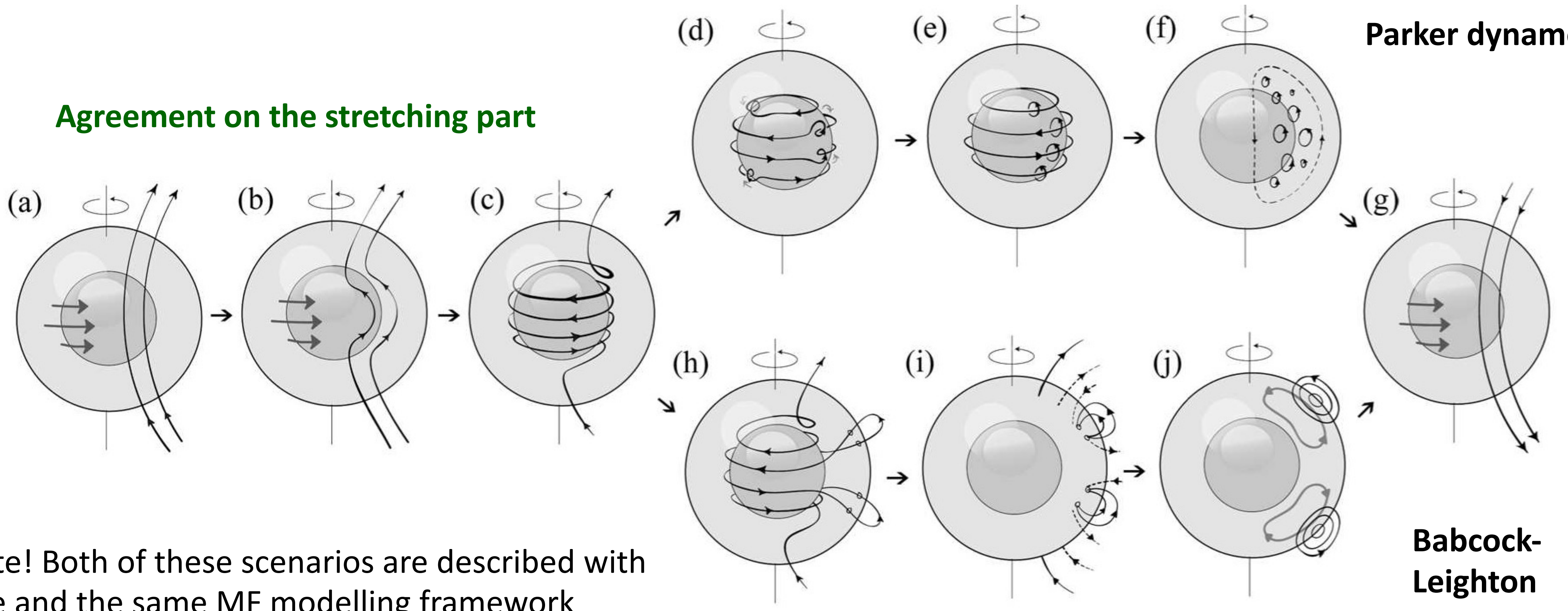
The magnetic field will migrate in the direction of

$$\boldsymbol{s} = \alpha \, \nabla \boldsymbol{\Omega} \times \hat{\boldsymbol{y}}$$

Current dichotomy of the solar dynamo

Agreement on the stretching part

Parker dynamo

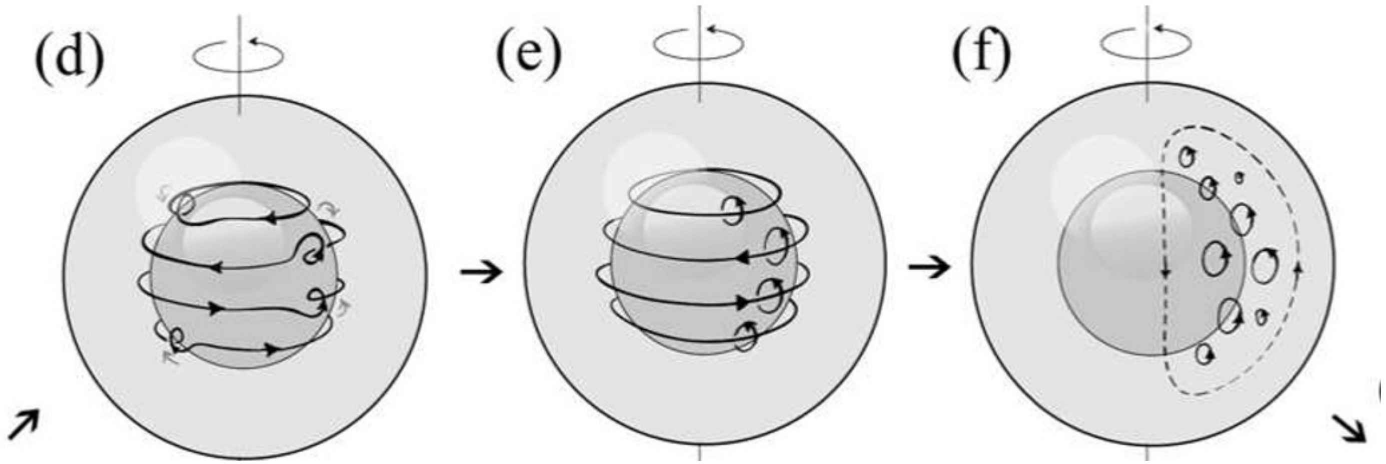


Babcock-Leighton dynamo

Disagreement on the twist part

Note! Both of these scenarios are described with one and the same MF modelling framework presented here. Only the region where the twist occurs is differently placed.

Parker dynamo



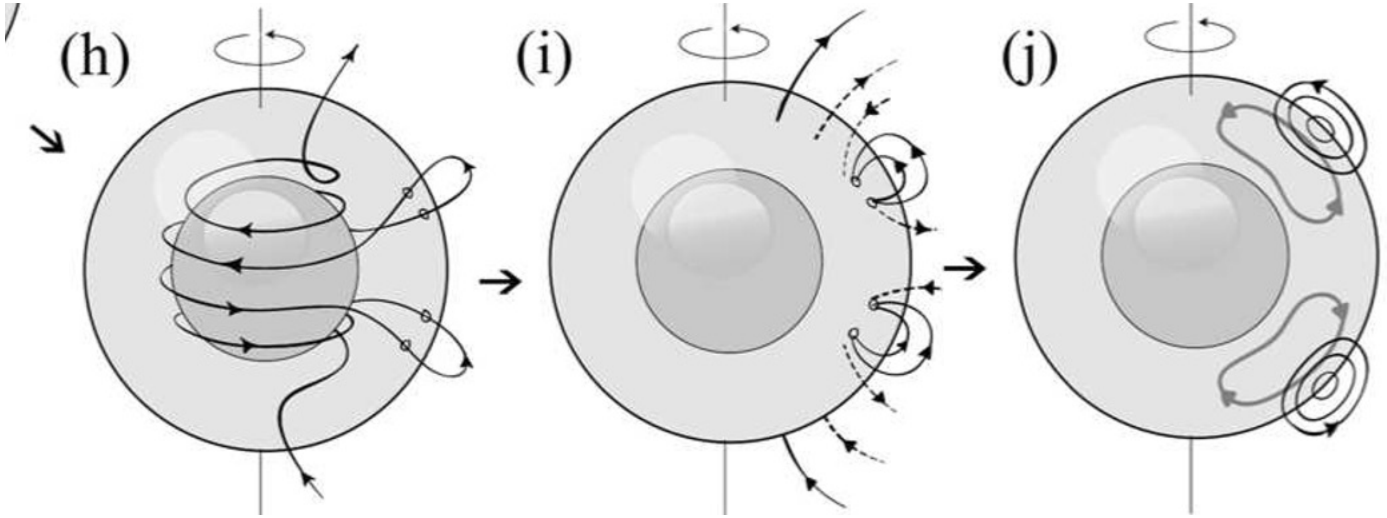
(d) Magnetic loops are formed by and frozen in to convective bubbles that move radially in the convection zone.

(e) Coriolis force twists the bubbles, creating helicity, and poloidal magnetic loops from the toroidal component

(f) Turbulent diffusivity is efficient, and makes the loops to reconnect to larger and larger entities, finally forming a large-scale poloidal field

Meridional flow is inefficient, as C_U is small due to large η_t. Subject to the magnetic helicity constraint.

Babcock-Leighton dynamo



(h) Magnetic loops are formed in the shear layer in the bottom of the convection zone. They get unstable and rise very fast to the surface, still being anchored to the bottom toroidal field. Coriolis force twists the loops, and create poloidal field before the loops emerge at the surface.

(i) The tips of the loops form bipolar active regions wich a tilt. The trailing polarity is advected towards the pole by the meridional flow. The polarity of the field changes thanks to the opposite polarity cancelling out at the poles.

(j) Meridional flow transports field from the poles to the bottom, to be acted on by the stretching again. No dynamo wave occurs, but the meridional flow advects the magnetic fields towards the equator near the bottom of the convection zone.

“Magnetic” origin – magnetic field is not acted on by convection, and therefore the dynamo is not subject to the helicity constraint. Meridional flow needs to be emphasized by decreasing the value of η_t.

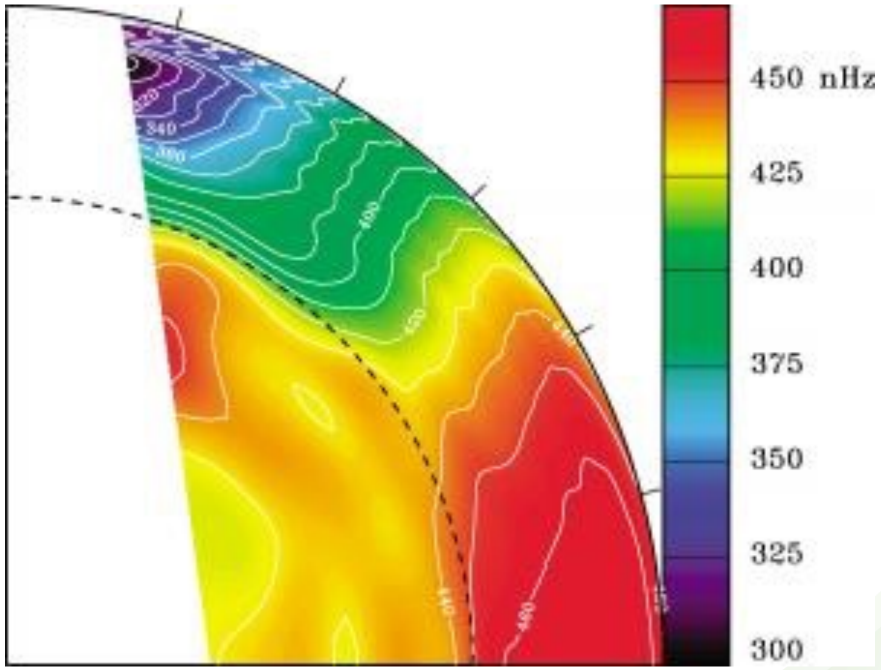
Why such a dichotomy?

Catastrophic quenching problem

- Closed systems without helicity fluxes studied with numerical methods in the beginning of 1990's
- Dynamo action not found as expected
- The alpha effect was judged to be catastrophically quenched due to the strong magnetic fluctuations
- It took more than 10 years to rectify the incorrect conclusion

Helioseismic rotation profile

- Was measured in the turnpoint of 1990’s.
- Revealed two narrow shear layers
  - Tachocline
  - Leptocline
- In the bulk of the convection zone the shear is weaker and mainly increasing outwards.
  - Dynamo wave migration crisis (exercises)



Consequences

IN LATE 1990’S THE ALMOST FORGOTTEN BL DYNAMO RESURRECTED

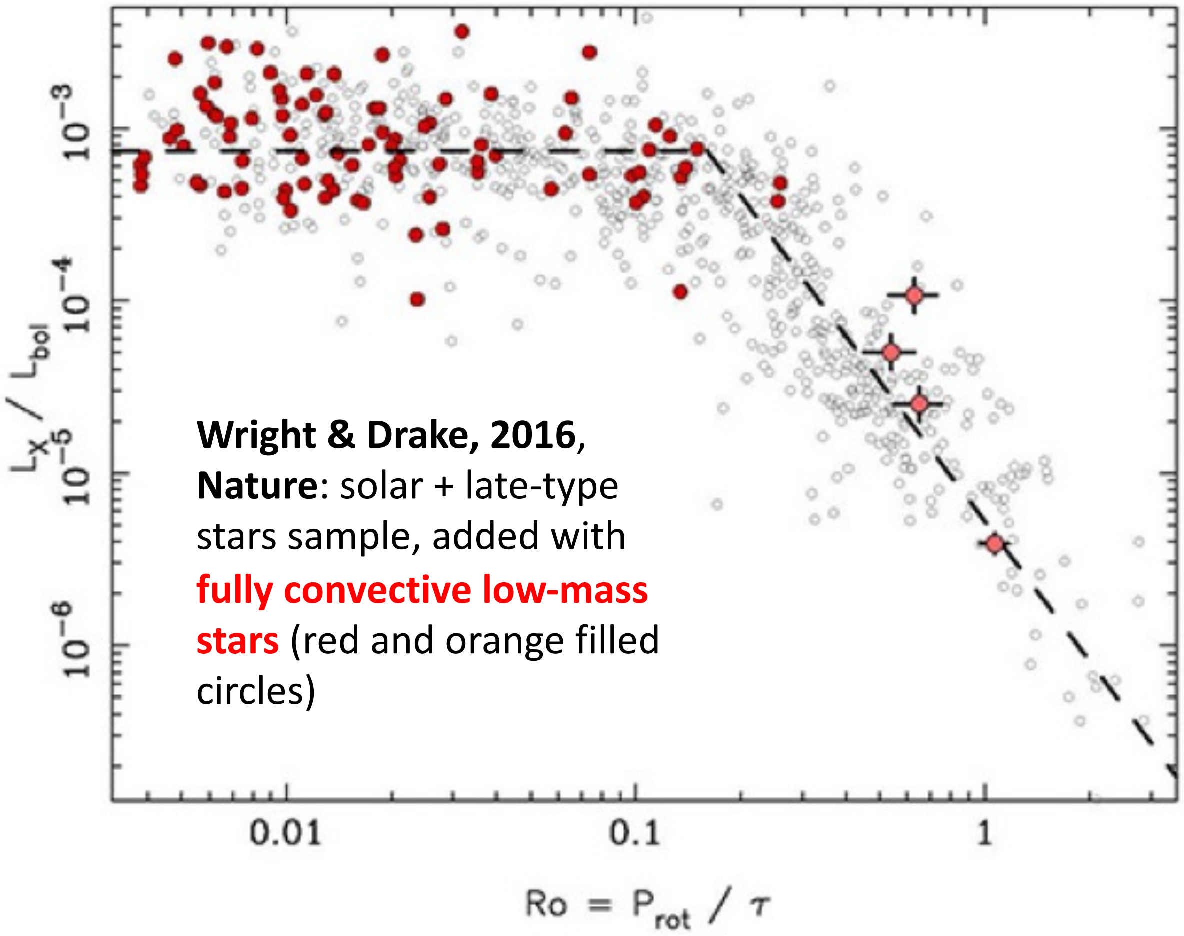
AND PARKER DYNAMO STARTED LOSING ITS FAME

SCIENTISTS WERE SURE THAT **TACHOCLINES** WERE THE KEY ELEMENT OF STELLAR DYNAMOS IN GENERAL

But then stellar observations changed the picture…

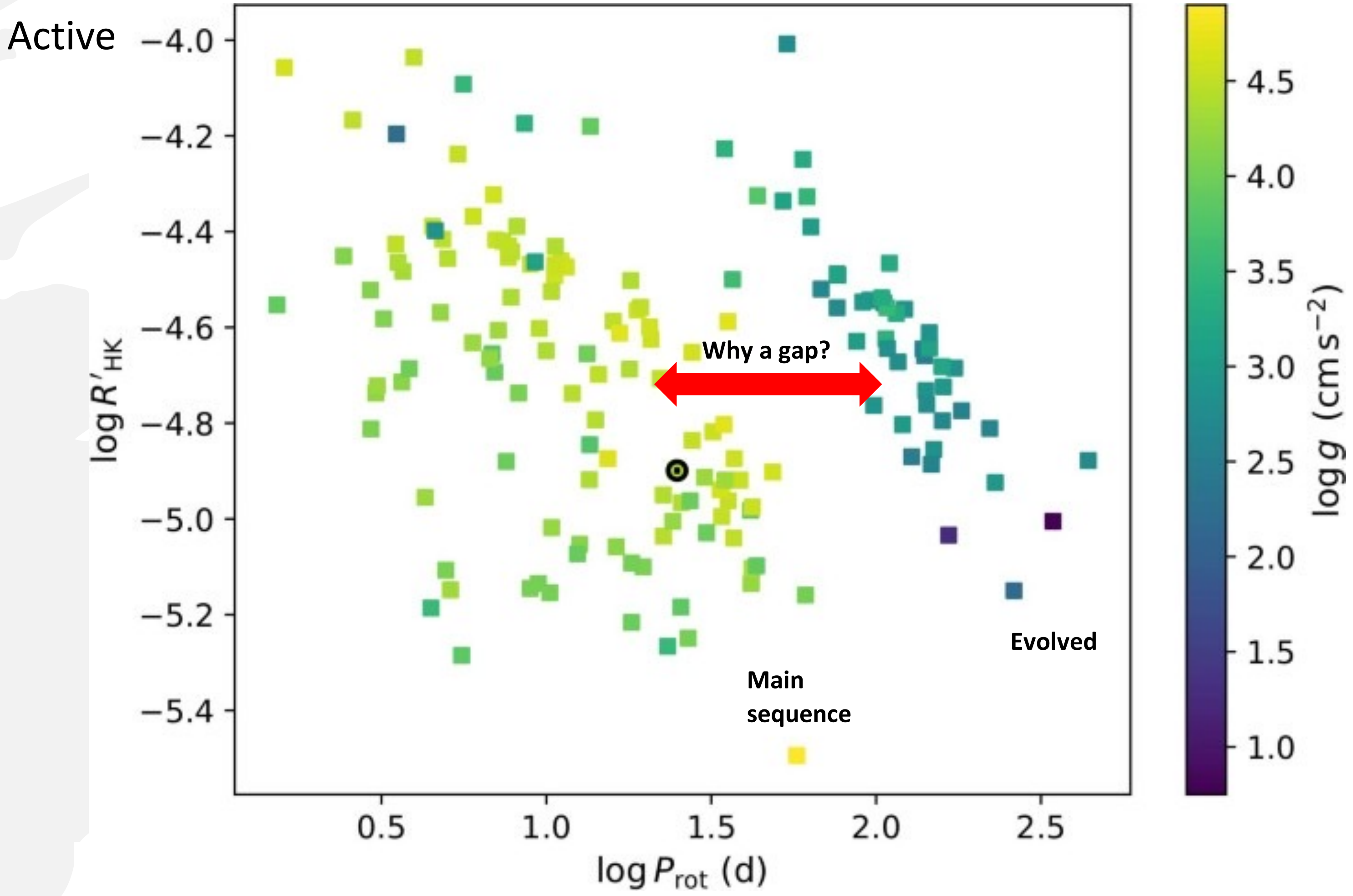
Fully convective stars

- These stars cannot develop a solar-like tachocline
- But still they show similar level of X-ray luminosity as solar-like stars with outer convection zones.
- Strong evidence against the BL dynamo
- This dynamo concept has then been modified to work outside the tachocline, but the question mark of operability remains



Evolved stars

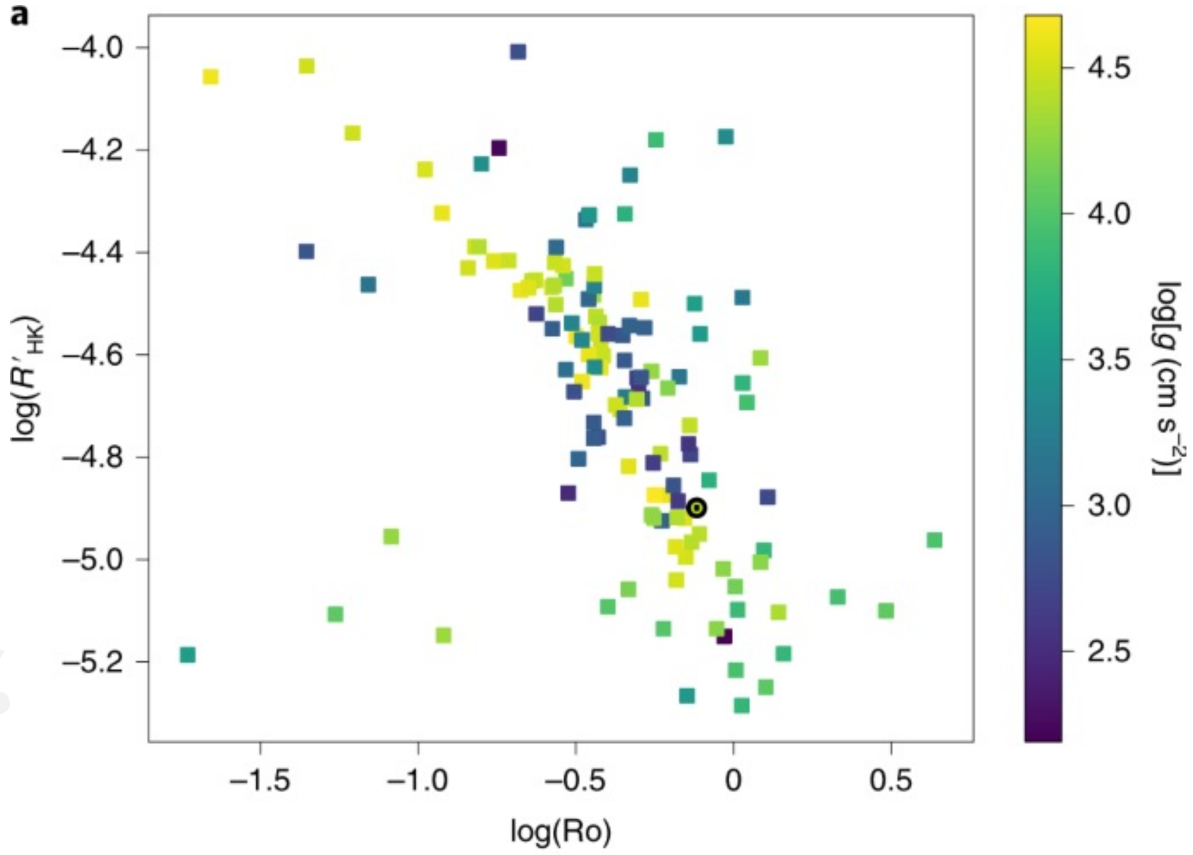
- BL dynamos assume that convection is decoupled from magnetic activity
- Then one would not expect the magnetic activity level to depend on quantities like Rossby/Coriolis number, as they contain convective velocity as a parameter
- Rather, a dependence on rotation period alone is expected
- This issue can be studied by comparing main sequence and evolved stars



Lehtinen, J.J., Spada, F., Käpylä, M.J. *et al.* Common dynamo scaling in slowly rotating young and evolved stars. *Nat Astron* 4, 658–662 (2020). https://doi.org/10.1038/s41550-020-1039-x

Evolved stars

- Indeed, the gap vanishes when Rossby number is used.
- Strong observational evidence for Parker dynamo that is directly related to convection.



Dichotomy persists, but Parker dynamo is currently winning again

Very short summary of stellar differential rotation

MF approach to Navier-Stokes equation

Eq. for differential rotation

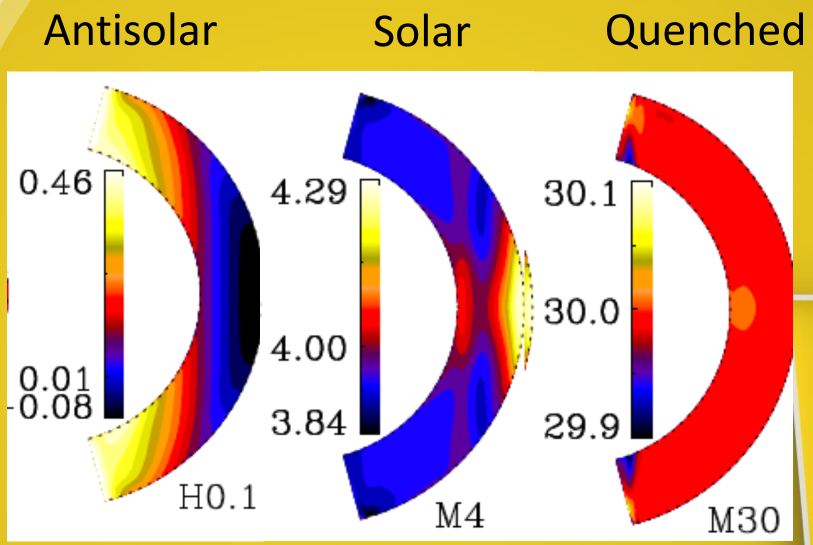
$$\frac{\partial}{\partial t}(\overline{\rho}\varpi^2\Omega) = -\nabla \cdot \{\varpi[\varpi\overline{\rho \boldsymbol{U}^m}\,\Omega + \overline{\rho}\,\overline{u_\phi \boldsymbol{u}} - 2\nu\overline{\rho}\overline{\boldsymbol{S}} \cdot \hat{\boldsymbol{\phi}} - (\overline{B}_\phi\overline{\boldsymbol{B}}/\mu_0 + \overline{b_\phi \boldsymbol{b}})]\},$$

$$\frac{\partial \overline{w}_\phi}{\partial t} = \varpi\frac{\partial \Omega^2}{\partial z} + (\nabla\overline{s} \times \nabla\overline{T})_\phi - \left[\nabla \times \frac{1}{\overline{\rho}}[\nabla \cdot (\overline{\rho}\boldsymbol{Q} - 2\nu\overline{\rho}\overline{\boldsymbol{S}})]\right]_\phi + [\nabla \times \nabla \cdot (\overline{\boldsymbol{B}}\overline{\boldsymbol{B}}^T + \boldsymbol{M})],$$

Eq. for meridional circulation

- **New terms arising**, corresponding to the emf, are the **Reynolds and Maxwell stresses**
- **Closures** to describe those terms are required; Λ effect theory
- Extensive literature exist on the topic; here a very concise summary

MF theory gives the following predictions



- When Ro > 1 and Co < 1 differential rotation profiles have a accelerated pole and a decelerated equator (antisolar)
- Transition occurs at Ro, Co=1, when accelerated equators and decelerated poles occur
- Meridional circulation is strong in the antisolar regime, elsewhere weak
- Differential rotation is quenched at rotationally dominated regime, when cylindrical rotation profiles appear.
- Stars with antisolar profiles are very rare observationally; stellar dynamos are supposed to all take place in the solarlike regime.

Dynamos in rapid rotators are most likely α^2 dynamos

$$
\begin{aligned}
\frac{\partial A}{\partial t} &= \alpha B + \eta_{\rm T}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial z^2}\right), \\
\frac{\partial B}{\partial t} &= \alpha\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial z^2}\right) + B \cdots + \eta_{\rm T}\left(\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial z^2}\right)
\end{aligned}
$$

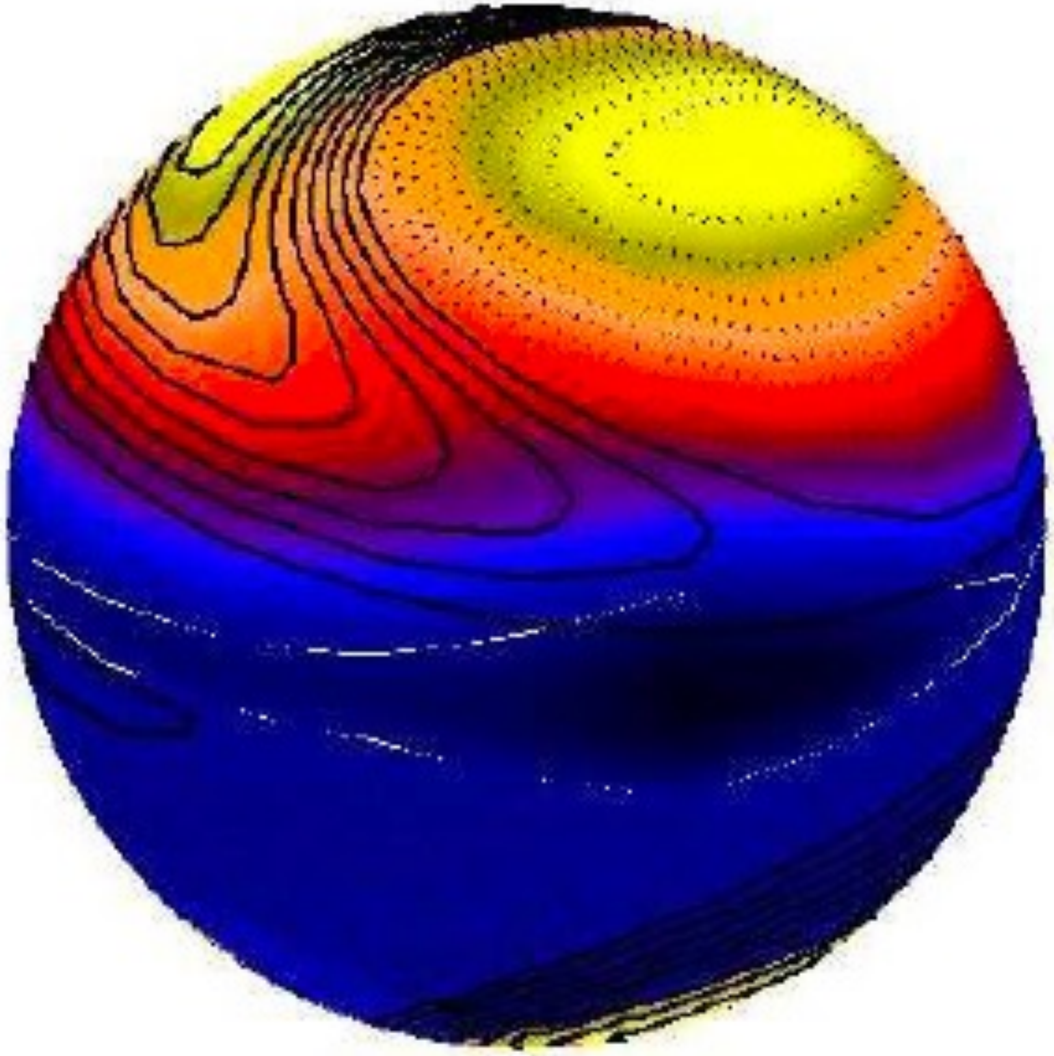
$\nabla\Omega$

Linear stability analysis and solving for the eigenvalue problem yields a *non-oscillatory* solution.

The magnetic fields excited are usually *non-axisymmetric*, the m=1 Fourier mode dominating.

The phase speed of the magnetic field is not necessarily the same as the rotation rate *(azimuthal dynamo waves).*

Dynamos in rapid rotators are most likely α^2 dynamos



Learning outcomes

We continued to learn to master but also to apply the concepts discussed last week

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Next week

We move back to full MHD equations

- Basics about how to solve them numerically
- Introduce the most typical modelling frameworks
- Present most relevant results obtained so far

Recap the most important learning outcomes of the MHD lecture series