Comets, exercise IV

We will continue the work started in previous exercises…

The final goal:
(a) Create your own model particle for cometary dust.
(b) Compute light scattering properties (linear polarization as a function of phase/scattering angle, C_{ext}, C_{sca}, C_{abs} and asymmetry parameter g) for selected wavelengths from solar radiation spectra (0.25 – 2.5 µm) (see Fig. 1).
(c) Simulate the trajectory of your particle exiting from the nucleus with different speeds (1, 10 and 100 m/s)

Solar radiation spectrum

The solar radiation spectrum follows very closely the blackbody spectrum at 5250°C, which is given by Planck's law

$$I(\lambda \text{ [in } m], T \text{ [in } Kelvin]) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp(hc/(\lambda kT)) - 1}$$

(see Appendix for physical constants). For integration you might need Planck's law normalized to unity, in that case use $nI(\lambda, 5250°C) = I(\lambda, 5250°C) / 1.58331*10^7$.

Simulate dust particle trajectory

There are two tasks in here: to simulate particle trajectory in two-body gravity field, and to compute μ-factor (radiation pressure factor) from scattering data which will alter the Sun gravitation force. You can team up to 2-3 groups if you like.

Trajectory in two-body 2D problem

(This is not my expertise, so be critical to my advices here). We consider only a 2D-problem where Sun, comet and particle are all in the same plane. During simulation time the comet is practically at constant distance from Sun, say 1 AU, so only the particle is moving in the simulation. The distance to Sun is so large, that the gravitational force from Sun is constant during simulation.

Gravitational force is given by Newton law

$$F = G\frac{m_1 m_2}{r^2}$$

where m's are masses and r is distance. Mass for the comet is for you to decide, mass for e.g. Halley is $2.2*10^{14}$ kg. Mass for dust particle depends on the volume and the density. Bulk density of cometary (if not water ice) might be a bit uncertain, density for silicate material could be e.g. ~3000 kg/m^3. The net gravitational force vector is the sum of the Sun and comet gravitation force vectors.

Gravitation force vector can be divided by particle mass to get the acceleration vector. You can start the simulation by giving the particle an initial velocity (1-100 m/s) from some point at comets surface (e.g. radius for Halley ~5000 m) and by letting constant acceleration change the velocity and place in e.g. one second time steps.

Radiation pressure and µ-factor

According to Fulle (*Motion of Cometary Dust* in *Comets II*) the effect of solar radiation pressure can be computed via µ-factor times the Suns gravitational force. The µ is

$$\beta = 1 - \mu = \frac{3E_{\odot}Q_{pr}}{8\pi c G M_{\odot}\rho_d d} = \frac{C_{pr}Q_{pr}}{\rho_d d}$$

where C_{pr} is 0.00115168 kg / m^2, ρ_d is particle density and d is particle diameter. The Q_{pr} is the radiation pressure efficiency that can be computed from radiation pressure cross-section C_{pr}, by dividing C_{pr} with particle cross-section area (e.g. equal-volume sphere area).

The C_{pr} is $C_{pr} = C_{ext} - g*C_{sca}$. The C_{ext} is computed by ADDA and reported in CrossSec-file. Scattering cross-section can be computed from Mueller matrix element P_{11} by

$$C_{\mathrm{sca}} = \frac{\lambda^2}{2\pi}\int_0^{\pi} \sin(\theta)P_{11}(\theta)\,d\theta$$

and asymmetry parameter g as

$$g = \frac{\lambda^2}{2\pi\, C_{sca}}\int_0^{\pi} \cos(\theta)\sin(\theta)P_{11}(\theta)\,d\theta$$

The total Q_{pr} over solar radiation spectrum can be integrated by

$$total\ Q_{pr} = \int_{0.25*10^{-6} m}^{2.25*10^{-6}\ m} nI(\lambda, 5250°C)\ Q_{pr}(\lambda)d\lambda$$

where nI is the normalized Planck's law.

Finally, when µ-factor is ready, plug it in to gravity simulation model by changing the Suns gravitational force vector F_{Sun} to $\mu*F_{Sun}$.

Appendix: physical constants

Planck constant *h*	6.62607*10^{-34} Joule s
Speed of light *c*	299792458 m / s
Boltzmann constant *k*	1.38065*10^{-23} Joule / Kelvin
Gravitational constant *G*	6.67428*10^{-11} m^2 Newton / kg^2
Sun mass	1.9891*10^{30} kg
Sun-Earth distance (1 AU)	1.49598*10^{11} m