Astrophysical light scattering problems, spring 2023 (PAP316, 5 cr)
Exercise 3/3

Please note that your answers are due by **April 20, 2023**. Send the answers in pdf form by email to the course assistant `mikko.vuori@helsinki.fi`.
Background literature: C. F. Bohren & D. R. Huffman, Absorption and Scattering of Light by Small Particles, Wiley, 2008, Sects. 4 and 5, and Appendix A (B&H).

1. Consider light scattering by dust particles small compared the wavelength, that is, Rayleigh scattering. The scattering phase matrix is

$$\mathbf{P} = \begin{pmatrix} \frac{3}{4}(1+\cos^2\theta) & \frac{3}{4}\sin^2\theta & 0 & 0 \\ \frac{3}{4}\sin^2\theta & \frac{3}{4}(1+\cos^2\theta) & 0 & 0 \\ 0 & 0 & \frac{3}{2}\cos\theta & 0 \\ 0 & 0 & 0 & \frac{3}{2}\cos\theta \end{pmatrix},$$

where θ is the scattering angle. Show that

$$\frac{1}{4\pi}\int_{4\pi} d\Omega P_{11} = 1,$$

and that the phase matrix is a pure Mueller matrix. Derive the amplitude scattering matrix elements with the help of Question 1 in Exercise 2. How can you make the result agree with the B&H Rayleigh amplitude scattering matrix derived from the Maxwell equations?
(3 points)

2-3. Consider light scattering by spherical particles (Mie scattering) with the help of suitable software (from the online ScattPort hub or B&H software provided on the course home page) based on the exact solution of the vector wave equation derived from the Maxwell equations.

a. Describe concisely the BHMIE algorithm of B&H required for the computation of scattering, absorption, and extinction cross sections as well as scattering matrices for spherical particles.

b. Compute the scattering characteristics S_{11} and $-S_{21}/S_{11}$ for a size parameter $x = 2\pi a/\lambda = 0.1$ (a is the particle radius and λ is the wavelength) and refractive index $m = 1.33 + i10^{-5}$ (mimicking water). Compare to Rayleigh scattering in Question 1.

c. Compute S_{11} and $-S_{21}/S_{11}$ for the size parameters $x = 5$ and 10. How do the interference patterns compare to x for $m = 1.33 + i10^{-5}$? What happens if you change to $m = 1.6 + i10^{-3}$ (mimicking certain silicates)?

d. Continue to increase the size parameter to $x = 10$ for the two refractive indices. What happens to the forward lobe in S_{11}?

e. Further increase the size parameter to $x = 100 - 1000$ to find clues of the rainbows for the two refractive indices in S_{11} and $-S_{21}/S_{11}$.

f. From the computations in a–e, can you find clues of the glory phenomenon in the bacward scattering direction (backscattering peak)?

In order to more clearly detect the rainbow and glory phenomena above, one would need to carry out particle size averaging to smoothen out the alternating constructive and destructive interference patterns.
(6 points)

4-5. Consider ensemble-averaged light scattering by Gaussian random particles in the geometric optics approximation using the SIRIS software provided on the course home page.

a. Describe concisely the SIRIS algorithm required for tracing rays within nonspherical particles.

b. Compute the scattering characteristics S_{11} and $-S_{21}/S_{11}$ for a size parameter $x = 2\pi a/\lambda = 100$ (a is the particle mean radial distance), relative standard deviation of radial distance $\sigma = 0.1$, and power law index of correlation function for logarithmic radial distances $\nu = 4$ by assuming again a refractive index of $m = 1.33 + i10^{-5}$. Compare the scattering matrix of such a deformed droplet to that for Mie scattering in Question 2-3e. Analyze the differences.

c. What happens if you assume $m = 1.6 + i10^{-3}$ instead?

d. If you now reduce absorption by assuming $m = 1.6 + i10^{-5}$, how do S_{11} and $-S_{21}/S_{11}$ change?

e. Reduce the standard deviation to $\sigma = 0.001$ and repeat the computations for $m = 1.33 + i10^{-5}$ and $x = 100$. Do the rainbows show up? How do the results compare to those in Question 2-3e. Can you reproduce the glory phenomenon in the geometric optics approximation?

f. Repeat the previous compuation with $m = 1.6 + i10^{-5}$. What happens near backscattering and why?

(6 points)