Astrophysical light scattering problems, spring 2023 (PAP316, 5 cr)
Exercise 1/3

Please note that your answers are due by **April 6, 2023**. Send the answers in pdf form by email to the course assistant `mikko.vuori@helsinki.fi`.
Background literature: C. F. Bohren & D. R. Huffman, Absorption and Scattering of Light by Small Particles, Wiley, 2008, Sect. 2.11 (B&H).

1. Consider a beam of light with Stokes parameters $(I, Q, U, V)^T$ and $(I', Q', U', V')^T$ expressed using the basis vectors $\hat{\mathbf{e}}_\perp$, $\hat{\mathbf{e}}_\parallel$ and $\hat{\mathbf{e}}'_\perp$, $\hat{\mathbf{e}}'_\parallel$, respectively. The angle between $\hat{\mathbf{e}}_\perp$ and $\hat{\mathbf{e}}'_\perp$ is ψ. Show that

$$\begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\psi & \sin 2\psi & 0 \\ 0 & -\sin 2\psi & \cos 2\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}.$$

(3 points)

2. Consider an ideal linear polarizer that transmits, without the change of the electric field amplitude, only the field components parallel to a particular axis called the transmission axis. Let ξ is the smallest angle between the basis vector $\hat{\mathbf{e}}_\parallel$ and the transmission axis direction $\hat{\mathbf{e}}_t$. Show that the Mueller matrix M for the ideal linear polarizer is

$$M = \frac{1}{2} \begin{pmatrix} 1 & \cos 2\xi & \sin 2\xi & 0 \\ \cos 2\xi & \cos^2 2\xi & \cos 2\xi \sin 2\xi & 0 \\ \sin 2\xi & \sin 2\xi \cos 2\xi & \sin^2 2\xi & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(3 points)

3-4. Consider an ideal linear retarder that divides a given incident electric field vector into two linearly polarized, mutually orthogonal components E_1 and E_2 in the coordinate system $\hat{\mathbf{e}}_1$, $\hat{\mathbf{e}}_2$ and introduces an electromagnetic phase difference $\delta_1 - \delta_2$ between the components. The irradiance is conserved. Show that the Mueller matrix M for the ideal linear retarder is

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & C^2 + S^2 \cos\delta & SC(1-\cos\delta) & -S\sin\delta \\ 0 & SC(1-\cos\delta) & S^2 + C^2\cos\delta & C\sin\delta \\ 0 & S\sin\delta & -C\sin\delta & \cos\delta \end{pmatrix}$$

where $C = \cos 2\beta$, $S = \sin 2\beta$, β denotes the angle between $\hat{\mathbf{e}}_\parallel$ and $\hat{\mathbf{e}}_1$ (notice the handedness in Fig. 2.17 of B&H), and the retardance is $\delta = \delta_1 - \delta_2$.
(6 points)

5. Devise an ideal circular polarizer with the help of the ideal linear polarizer and the ideal linear retarder.
(3 points)