A TWO–PARAMETER SYSTEM FOR LINEAR POLARIZATION OF SOME SOLAR SYSTEM OBJECTS

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A new two–parameter system is suggested to explain and predict linear polarization of asteroids, comets, and atmosphereless solar system objects in general. The system introduces a few trigonometric functions as a basis with two unknown parameters that are to be solved in the linear least–squares sense from the observations. The unknowns are the amplitude and inversion angle of polarization. The two–parameter system fits well all the existing observations and, at least for the Moon, is able to predict the whole positive branch from the negative branch only. However, the system still needs further theoretical development. No physical explanation for the system is offered at this point.

1. Introduction

The two–parameter H, G magnitude system for asteroids was adopted by IAU Commission 20 in 1985. By this system, it is possible to predict magnitudes of asteroids at phase angles for which there are no observations. The ubiquitous feature of practically all asteroids, and many other atmosphereless bodies as well, is the so–called opposition effect.

No analogous system for the linear polarization of asteroids, other atmosphereless solar system bodies, and cometary comae has been suggested. Also linear polarization exhibits a ubiquitous feature: the reversal of polarization from negative to positive near the solar phase angle of about 20°. A common result both for the magnitude and polarization is the lack of any major color dependence of the dominant features.

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There are several reasons why even a strictly empirical system to describe linear polarization is useful. In the first place, such a system would provide coherence between different data sets at varying phase angle ranges. Second, instead of defining several polarization descriptors, such as minimum polarization, maximum polarization, polarization slope, and inversion angle, one can introduce two quantities from which all the others can be deduced. Finally, if such diverse objects as asteroids, planetary satellites, comets and rings obey the same system with only two adjustable parameters, something fundamental is being revealed about the physical microstructure.

2. Polarimetric Observations

Currently, there are reliable polarization data (relevant for our studies) on asteroids, several planetary satellites, comets and zodiacal light. Especially, the asteroid observations by Zellner *et al.* (1974), Zellner and Gradie (1976), and the (1580) Betulia observations by Tedesco and Drummond (1978) are of very high quality. The UBV coverage in their work is important for the study of possible color dependence.

The old Lyot (1929) data of the Moon are still relevant, and offer the best phase angle coverage of all observed objects. Data on Saturn's rings by Lyot (1929) on the other hand provides coverage at very small phase angles. It would be important to extend the polarization data of some bright E–type asteroids, e.g. (44) Nysa and (64) Angelina, to small phase angles to verify whether these objects would also exhibit a sharp "polarization spike" as the Saturn's rings do. This would be consistent with the photometric behavior of these objects, though the widths of the negative polarization branches presumably differ for Saturn's rings and the E–type asteroids.

The cometary data consists of measurements of two comets: P/Levy, Renard *et al.* (1992) and P/Halley, Dollfus and Suchail (1987), and Eaton *et al.* (1988). Unlike asteroids and planetary satellites, comets are active, and it is quite possible that part of the scatter in the data is due to temporal variations.

In principle, the linear polarization of zodiacal light could also be analyzed similarly. Then the degree of polarization as a function of elongation should first be transferred to polarization as a function of phase angle, which involves a solution of a simple integral equation. However, in doing so we implicitly assume that the physical properties of interplanetary dust particles do not change as a function of solar distance, which we almost certainly know to be an invalid assumption.

3. A Two–Parameter Model

In order to construct an analytical expression for polarization, it is natural to seek a basis of a few trigonometric functions. Because the polarization vanishes at the phase angles of $\alpha = 0°$ and $\alpha = 180°$, we introduce the term $\sin^{c_1}\alpha$, where c_1 is an unspecified constant. To produce an asymmetric polarization with respect to $\alpha = 90°$, we include the term $\cos^{c_2}\frac{1}{2}\alpha$, where c_2 is another unspecified constant. Finally, the polarization vanishes at the inversion angle α_0, which suggests the term $\sin(\alpha - \alpha_0)$. Combining these functions, we obtain

$$P(\alpha) = b \sin^{c_1} \alpha \cos^{c_2} \tfrac{1}{2}\alpha \sin(\alpha - \alpha_0). \quad (1)$$

This can be written in the form

$$\begin{aligned} P(\alpha) &= b_1 \ \sin^{1+c_1}\alpha \ \cos^{c_2}\tfrac{1}{2}\alpha - b_2 \ \sin^{c_1}\alpha \ \cos^{c_2}\tfrac{1}{2}\alpha \ \cos\alpha, \\ b_1 &= b\cos\alpha_0 \\ b_2 &= b\sin\alpha_0. \end{aligned} \qquad (2)$$

The constants c_1 and c_2 can be solved by the "trial and error" procedure, requiring that the overall rms–error reaches a minimum. It turned out that slightly different values for c_1 and c_2 are needed for the asteroids and for the other objects:

Asteroids: $c_1 = 0.7$
$c_2 = 0.35$

Other Objects: $c_1 = 0.5$
$c_2 = 0.35$

At this point we do not speculate the reason for this.

Now Eq. (2) can be fitted to the data by a least–squares method, which yields b_1 and b_2 with their covariance matrix. From b_1 and b_2, we can solve for b and α_0 as

$$\begin{aligned} b &= \sqrt{b_1^2 + b_2^2} \\ \alpha_0 &= \arctan \frac{b_2}{b_1}. \end{aligned} \tag{3}$$

For any prediction purposes, we need the errors for b and α_0. Assuming that b_1 and b_2 follow bivariate normal statistics, we can solve the errors Δb and $\Delta \alpha_0$ as

$$\begin{aligned} \Delta b &= \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dxdy \; n_2(x,y;c) \; (\sqrt{x^2+y^2} - b)^2 \right]^{\frac{1}{2}} \\ \Delta \alpha_0 &= \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dxdy \; n_2(x,y;c) \; (\arctan \frac{y}{x} - \alpha_0)^2 \right]^{\frac{1}{2}} \\ n_2(x,y;c) &= \frac{1}{2\pi \Delta b_1 \Delta b_2 \sqrt{1-c^2}} \cdot \\ & \exp \left\{ -\frac{1}{2(1-c^2)} \left[\left(\frac{x-b_1}{\Delta b_1} \right)^2 + \left(\frac{y-b_2}{\Delta b_2} \right)^2 - 2c \frac{x-b_1}{\Delta b_1} \frac{y-b_2}{\Delta b_2} \right] \right\} \end{aligned} \tag{4}$$

Here Δb_1, Δb_2, and c are the errors and the correlation coefficient for b_1 and b_2, and can be directly obtained from the covariance matrix.

In Table 1 and Figures 1 and 2, we present the results for 18 asteroids and for some other objects. Without any doubt, all the fits are good. We were particularly pleased to realize that the lunar data for $\alpha > 25°$ can so well be predicted from the small phase angle data (negative branch) alone. Although the negative branches both for the increasing and decreasing lunar phases are fairly similar, their maximum polarization is different but still predictable.

Various quantities, which are often used to describe the linear polarization, can immediately be obtained from Eq. (1). The angles and amounts of minimum and maximum polarization, and the polarization slope, can be calculated from

Asteroids:

$$\alpha_{\rm min} \approx \frac{2}{5}\alpha_0$$
$$P_{\rm min} \approx -b\sin^{1+c_1}\tfrac{1}{2}\alpha_0$$

Other Objects:

$$\alpha_{\rm min} \approx \frac{1}{3}\alpha_0$$
$$P_{\rm min} \approx -2b\sin^{1+c_1}\tfrac{1}{3}\alpha_0$$

All Objects:

$$\alpha_{\rm max} \approx 85^\circ + \frac{1}{2}\alpha_0$$
$$P_{\rm max} \approx 2^{-\frac{1}{2}c_2}b\sin(85^\circ - \tfrac{1}{2}\alpha_0)$$
$$h = \frac{\partial P}{\partial \alpha}(\alpha = \alpha_0) \approx b\sin^{c_1}\alpha_0 \qquad (5)$$

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TABLE 1

Model parameters α_0 (inversion angle, a0) and b (amplitude) together with their errors for some solar system objects.

Object	rms	a0	da0	b	db
Moon increasing	0.17	24.98	0.96	8.69	1.00
Moon decreasing	0.19	23.97	0.92	10.91	1.20
Saturn rings	0.09	6.72	0.27	32.77	4.53
Comet inner coma	0.77	25.93	0.78	29.85	1.43
Comet outer coma	0.83	22.67	0.99	28.07	1.53
Comet jet	1.92	19.26	3.17	27.96	3.90

TABLE 2

Same as Table 1 for several asteroids in different colors

Object	rms	a0	da0	b	db
Ceres U	0.06	18.05	0.10	36.94	0.73
Ceres B	0.06	18.24	0.11	38.50	0.78
Ceres G	0.05	18.28	0.07	37.35	0.49
Juno G	0.04	20.68	0.20	12.56	0.48
Vesta U	0.05	22.19	0.23	10.51	0.35
Vesta B	0.03	21.66	0.27	7.24	0.31
Vesta G	0.04	22.33	0.23	7.40	0.33
Iris U	0.12	19.63	0.73	14.16	0.65
Iris B	0.06	21.25	0.33	12.12	0.33
Iris G	0.06	21.69	0.27	11.39	0.26
Flora B	0.06	19.88	0.26	11.98	0.39
Flora G	0.06	20.45	0.23	11.82	0.35
Metis B	0.04	21.29	0.15	12.43	0.41
Metis G	0.05	22.01	0.22	12.61	0.60
Psyche B	0.10	21.69	0.49	15.11	1.23
Psyche G	0.07	21.94	0.36	15.60	0.90
Fortuna B	0.24	21.43	0.38	30.44	1.66
Fortuna G	0.19	21.47	0.31	29.99	1.34
Lutetia B	0.05	24.68	0.12	18.22	0.42
Lutetia G	0.06	24.48	0.18	17.77	0.52
Laetitia B	0.06	20.50	0.21	12.55	0.49
Laetitia G	0.04	21.10	0.16	12.65	0.37
Nysa B	0.05	18.75	0.36	6.64	0.52
Nysa G	0.05	18.67	0.38	6.82	0.47
Alexandra B	0.07	21.80	0.13	31.93	0.58
Alexandra G	0.09	21.72	0.18	33.42	0.93
Angelina B	0.04	18.04	0.44	5.79	0.42
Angelina G	0.08	19.12	0.94	5.98	0.96
Klio B	0.18	20.20	0.52	28.41	1.47
Klio G	0.19	20.36	0.55	28.79	1.57
Melete B	0.11	19.54	0.19	32.53	1.00
Melete G	0.12	19.25	0.21	32.97	1.19
Lumen B	0.18	21.10	0.26	33.37	1.24
Lumen G	0.08	20.98	0.18	34.86	0.81
Eros U	0.13	19.39	1.30	12.34	0.31
Eros B	0.15	21.23	1.76	9.22	0.31
Eros V	0.09	22.36	0.94	8.81	0.21
Betulia B	0.20	20.73	1.03	25.86	1.03
Betulia G	0.17	20.72	0.95	25.00	0.92

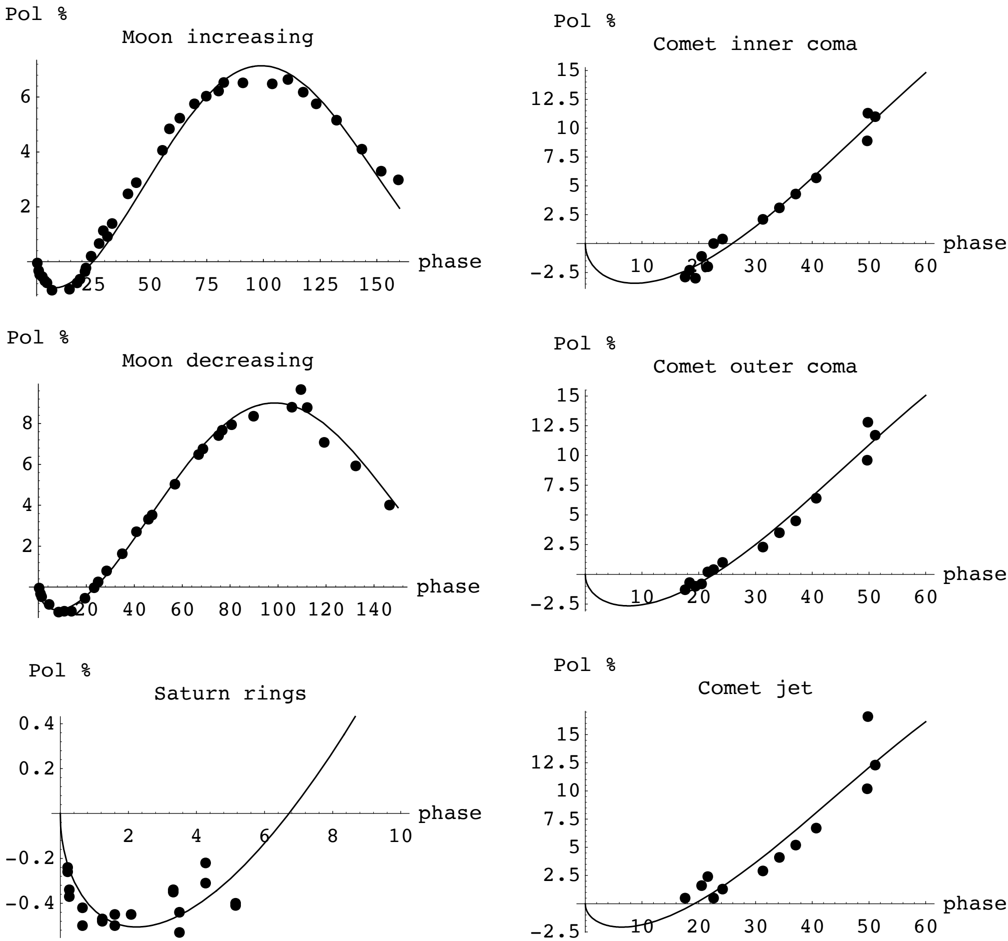

FIGURE 1.
Observed linear polarization of some solar system objects as compared with the two-parameter model. The model parameters are: $c_1 = 0.5$, $c_2=0.35$. For the increasing and decreasing phases of the Moon only the negative branch has been used in the least squares fit. It is remarkable how well also the whole positive branch can be predicted.

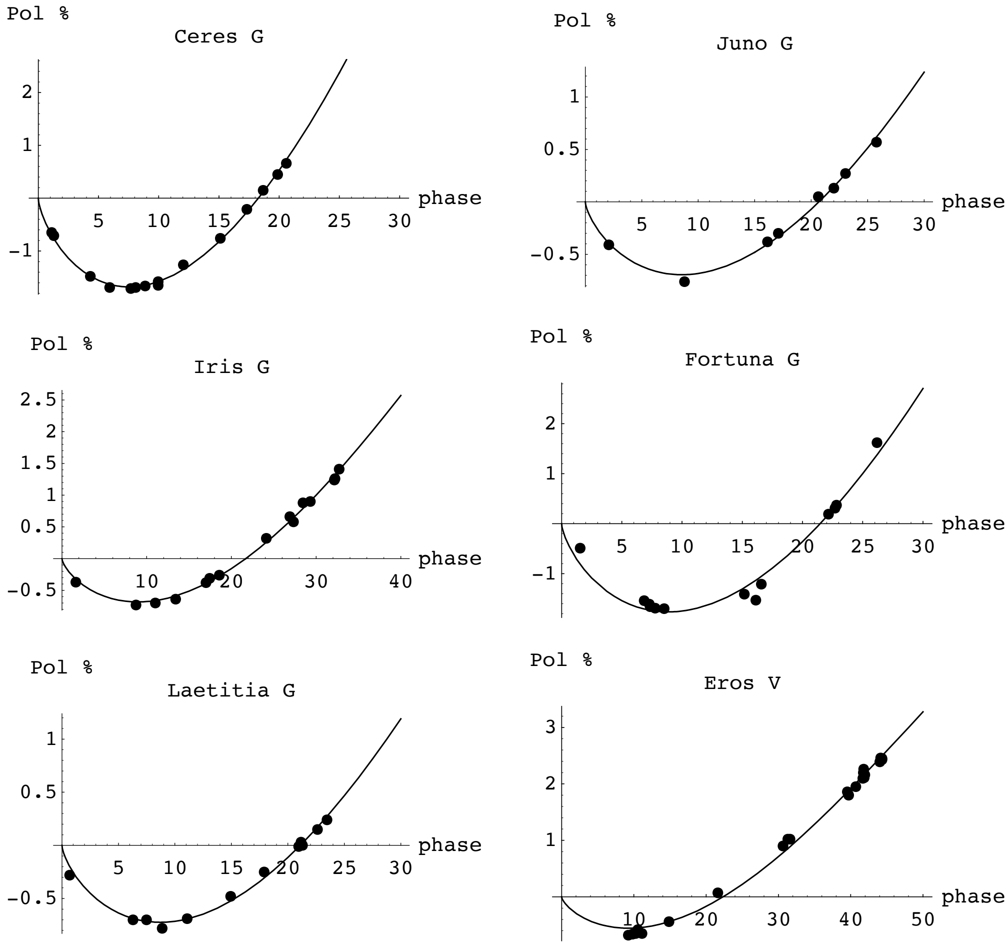

FIGURE 2.
Observed linear polarization of some asteroids in green color as compared with the two-parameter model. The model parameters are: $c_1 = 0.7$, $c_2=0.35$. There is not a slightest indication of any systematical difference between the model and the observations.