Coherent backscattering with ensemble-averaged scattering matrices

Karri Muinonen[a,*]

[a] *Department of Physics, University of Helsinki, P.O. Box 64, Gustaf Hällströmin katu 2, FI-00014 U. Helsinki, Finland*

Abstract

The decomposition of ensemble-averaged scattering matrices into pure Mueller matrices allows for radiative-transfer coherent-backscattering (RT-CB) computations for discrete random media of nonspherical particles. In particular, RT-CB computations for media composed of a size distribution of spherical particles can be treated in two ways. First, the computations can be run by incorporating the speficic set of spherical particles composing the media. Second, the computations can be run by incorporating the decomposition of the ensemble-averaged scattering matrix into two pure Mueller matrices. Comparisons of the two approaches are provided for example ensembles of spherical particles. Finally, first results are shown for RT-CB computations for discrete random media of nonspherical particles.

Keywords: light scattering, scattering phase matrix, Mueller matrix, Jones matrix, decomposition, coherency matrix

2021 MSC: 00-01, 99-00

*Corresponding author

Email address: karri.muinonen@helsinki.fi (Karri Muinonen)

URL: https://wiki.helsinki.fi/display/PSR/Karri+Muinonen (Karri Muinonen)

1. Introduction

2. Scattering phase matrix

Consider the 4×4 block-diagonal, ensemble-averaged scattering phase matrix $\mathbf{P}_0 = \mathbf{P}_0(\theta)$, where θ is the scattering angle:

$$\mathbf{P}_0 = \begin{pmatrix} a_1 & b_1 & 0 & 0 \\ b_1 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & b_2 \\ 0 & 0 & -b_2 & a_4 \end{pmatrix}, \tag{1}$$

where we assume the normalization of

$$\int_{(4\pi)} \frac{d\Omega}{4\pi}\, a_1(\theta) = 1, \tag{2}$$

that is,

$$\frac{1}{2}\int_0^\pi d\theta \sin\theta\, a_1(\theta) = 1. \tag{3}$$

The scattering phase matrix is subject to a number of symmetry relations (Hovenier & van der Mee 2000). For arbitrary scattering angles θ,

$$\begin{aligned} |a_j| &\leq a_1, \quad j = 2,3,4, \\ |b_j| &\leq a_1, \quad j = 1,2, \\ (a_3 + a_4)^2 + 4b_2^2 &\leq (a_1 + a_2)^2 - 4b_1^2, \\ |a_3 - a_4| &\leq a_1 - a_2, \\ |a_2 - b_1| &\leq a_1 - b_1, \\ |a_2 + b_1| &\leq a_1 + b_1. \end{aligned} \tag{4}$$

In the forward ($\theta = 0$) and backward scattering directions ($\theta = \pi$),

$$\begin{aligned} a_2(0) &= a_3(0), \\ a_2(\pi) &= -a_3(\pi), \\ a_4(\pi) &= a_1(\pi) - 2a_2(\pi), \\ b_1(0) &= b_2(0) = b_1(\pi) = b_2(\pi) = 0. \end{aligned} \tag{5}$$

We model the observed matrix $\mathbf{P}_0$ in full detail with a scattering phase matrix $\mathbf{P} = \mathbf{P}(\theta)$ that is reconstructed from four matrices $\mathbf{U} = \mathbf{U}(\theta)$, $\mathbf{V} = \mathbf{V}(\theta)$, $\mathbf{W} = \mathbf{W}(\theta)$, and $\mathbf{Z} = \mathbf{Z}(\theta)$ (Cloude, Savenkov et al.):

$$\begin{aligned}
\mathbf{P} &= w_U\ \mathbf{U} + w_V\ \mathbf{V} + w_W\ \mathbf{W} + w_Z\ \mathbf{Z}, \\
& \quad 0 \le w_U \le 1, \quad 0 \le w_V \le 1, \quad 0 \le w_W \le 1, \quad 0 \le w_Z \le 1, \\
& \quad w_U + w_V + w_W + w_Z = 1,
\end{aligned} \tag{6}$$

where w_U, w_V, w_W, and w_Z are the normalized weights and where the common normalization condition holds for the phase functions:

$$\begin{aligned}
\int_{(4\pi)} \frac{d\Omega}{4\pi}\, U_{11}(\theta) &= \int_{(4\pi)} \frac{d\Omega}{4\pi}\, V_{11}(\theta) = \\
\int_{(4\pi)} \frac{d\Omega}{4\pi}\, W_{11}(\theta) &= \int_{(4\pi)} \frac{d\Omega}{4\pi}\, Z_{11}(\theta) = 1.
\end{aligned} \tag{7}$$

The eigenproblem can be solved analytically (Muinonen 2023, in preparation). In the notation of Eq. 6, we obtain

$$\begin{aligned}
\mathbf{U} &= \begin{pmatrix} U_{11} & U_{12} & 0 & 0 \\ U_{12} & U_{11} & 0 & 0 \\ 0 & 0 & U_{33} & U_{34} \\ 0 & 0 & -U_{34} & U_{33} \end{pmatrix}, \\
\mathbf{V} &= \begin{pmatrix} V_{11} & V_{12} & 0 & 0 \\ V_{12} & V_{11} & 0 & 0 \\ 0 & 0 & V_{33} & V_{34} \\ 0 & 0 & -V_{34} & V_{33} \end{pmatrix}, \\
\mathbf{W} &= \begin{pmatrix} W_{11} & 0 & 0 & 0 \\ 0 & -W_{11} & 0 & 0 \\ 0 & 0 & -W_{11} & 0 \\ 0 & 0 & 0 & W_{11} \end{pmatrix}, \\
\mathbf{Z} &= \begin{pmatrix} Z_{11} & 0 & 0 & 0 \\ 0 & -Z_{11} & 0 & 0 \\ 0 & 0 & Z_{11} & 0 \\ 0 & 0 & 0 & -Z_{11} \end{pmatrix},
\end{aligned} \tag{8}$$

where

$$
\begin{aligned}
U_{11} &= \frac{1}{2w_U}\left[\frac{1}{2}(a_1+a_2)+\sqrt{b_1^2+b_2^2+\frac{1}{4}(a_3+a_4)^2}\right],\\
V_{11} &= \frac{1}{2w_V}\left[\frac{1}{2}(a_1+a_2)-\sqrt{b_1^2+b_2^2+\frac{1}{4}(a_3+a_4)^2}\right],\\
\frac{U_{12}}{U_{11}} &= -\frac{V_{12}}{V_{11}} = \frac{b_1}{\sqrt{b_1^2+b_2^2+\frac{1}{4}(a_3+a_4)^2}},\\
\frac{U_{33}}{U_{11}} &= -\frac{V_{33}}{V_{11}} = \frac{\frac{1}{2}(a_3+a_4)}{\sqrt{b_1^2+b_2^2+\frac{1}{4}(a_3+a_4)^2}},\\
\frac{U_{34}}{U_{11}} &= -\frac{V_{34}}{V_{11}} = \frac{b_2}{\sqrt{b_1^2+b_2^2+\frac{1}{4}(a_3+a_4)^2}},\\
W_{11} &= \frac{1}{2w_W}\,(a_1-a_2-a_3+a_4),\\
Z_{11} &= \frac{1}{2w_Z}\,(a_1-a_2+a_3-a_4),
\end{aligned}
\tag{9}
$$

with

$$
\begin{aligned}
w_U &= \frac{1}{4}+\frac{1}{8}\int_0^\pi d\theta\sin\theta\,\left[a_2(\theta)+2\sqrt{b_1^2+b_2^2+\frac{1}{4}(a_3+a_4)^2}\right],\\
w_V &= \frac{1}{4}+\frac{1}{8}\int_0^\pi d\theta\sin\theta\,\left[a_2(\theta)-2\sqrt{b_1^2+b_2^2+\frac{1}{4}(a_3+a_4)^2}\right],\\
w_W &= \frac{1}{4}+\frac{1}{8}\int_0^\pi d\theta\sin\theta\,\left[-a_2(\theta)-a_3(\theta)+a_4(\theta)\right],\\
w_Z &= \frac{1}{4}+\frac{1}{8}\int_0^\pi d\theta\sin\theta\,\left[-a_2(\theta)+a_3(\theta)-a_4(\theta)\right].
\end{aligned}
\tag{10}
$$

3. Amplitude matrices

Consider a 2×2 Jones amplitude scattering matrix with nonzero diagonal elements S_1, S_2 and vanishing off-diagonal elements S_3, S_4. The elements are functions of the scattering angle θ. Consequently, the nonzero 4×4 Mueller scattering matrix elements are

$$
\begin{aligned}
S_{11} &= \frac{1}{2}\left(|S_1|^2+|S_2|^2\right) = S_{22},\\
S_{12} &= \frac{1}{2}\left(-|S_1|^2+|S_2|^2\right) = S_{21},
\end{aligned}
$$

Figure 1: Mie.

Figure 2: Mie.

$$\begin{aligned} S_{33} &= \mathrm{Re}S_1 S_2^* = S_{44}, \\ S_{34} &= -\mathrm{Im}S_1 S_2^* = -S_{43}, \end{aligned} \tag{11}$$

which is the form of the common Mie scattering matrix (Bohren & Huffmann 2008).

If only the relative electromagnetic phase between S_1, S_2 at each scattering angle is required, S_1 can be assumed real-valued. The inverse relation of the Jones matrix elements as a function of the Mueller matrix elements can then be written as

$$\begin{aligned} S_1 &= \sqrt{S_{11} - S_{12}}, \\ S_2 &= \frac{1}{S_1}(S_{33} + \mathrm{i}S_{34}). \end{aligned} \tag{12}$$

The matrices **U** and **V** have their corresponding amplitude matrices of the form in Eq. 12. The Mueller matrix **V** (Eq. 9) violates the symmetry relations required for scattering matrices in the backscattering direction (Hovenier & van der Mee 2000).

4. Results and discussion

4.1. Phase matrices of spherical particles

4.2. Phase matrices of Gaussian-random-sphere particles

4.3. Measured phase matrices of feldspar particles

5. Conclusion

References

Figure 3: GRS ice.

Figure 4: GRS ice.

Figure 5: Feldspar.

Figure 6: Feldspar.