

# **Astrophysical light scattering problems (PAP316)**

## **Lecture 3b**

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# Introduction

- Physical characterization of **asteroids**
- **Forward** and **inverse** scattering problems concerning **asteroid spins, shapes, and surface scattering properties**
- Plane of scattering, scattering angle, solar phase angle
- Disk-integrated brightness  $L$  (often in magnitude scale,  $-2.5 \log_{10} L$ )
- Degree of linear polarization  $(L_r - L_l)/(L_r + L_l)$
- Multiple scattering models, including **single-particle, coherent backscattering, and shadowing effects**

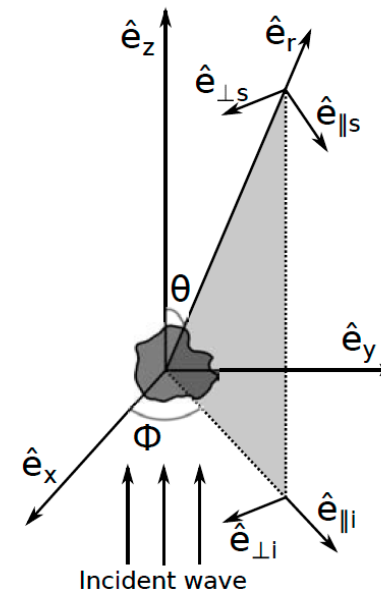
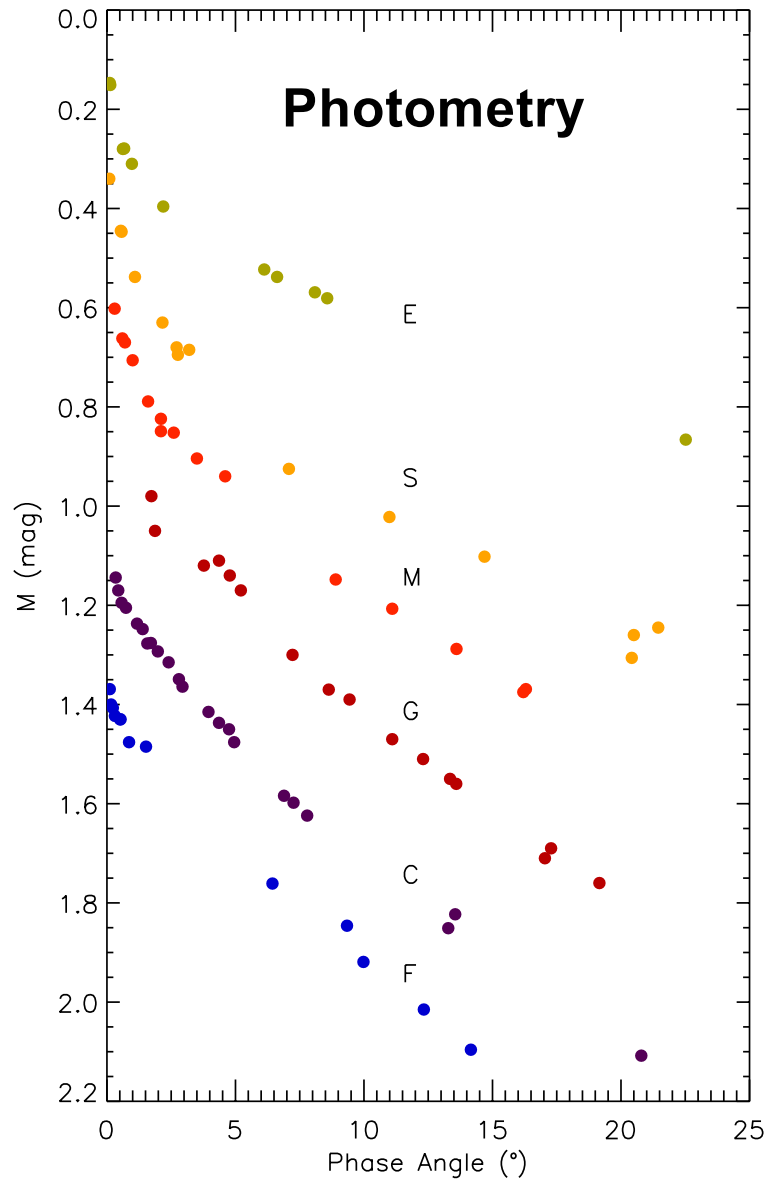
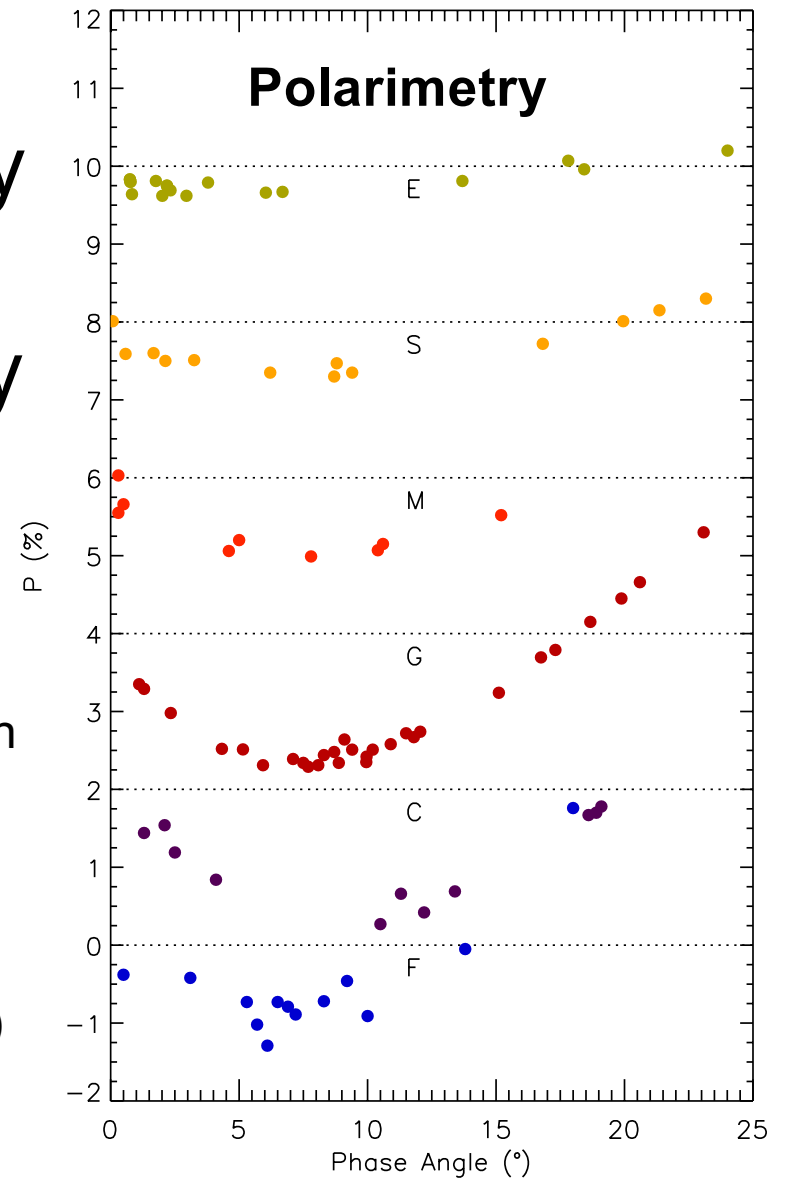


Figure 2.2: Illustration of the incident wave and the scattering plane. (Bohren and Huffman, 1983)

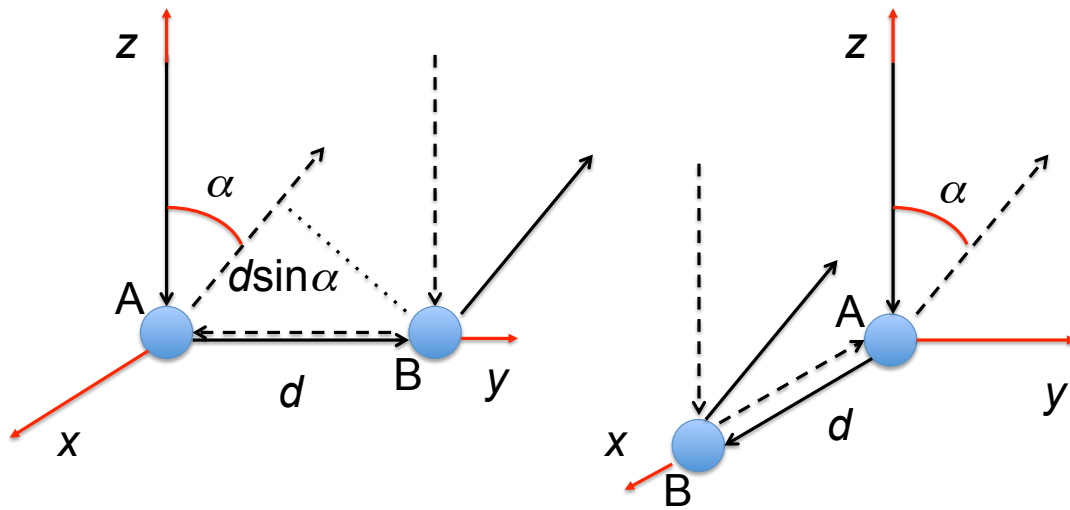


# Asteroid photometry and polarimetry

Muinonen et al., in  
Polarimetry of  
Stars and  
Planetary  
Systems, 2016  
(obs. ref. therein)



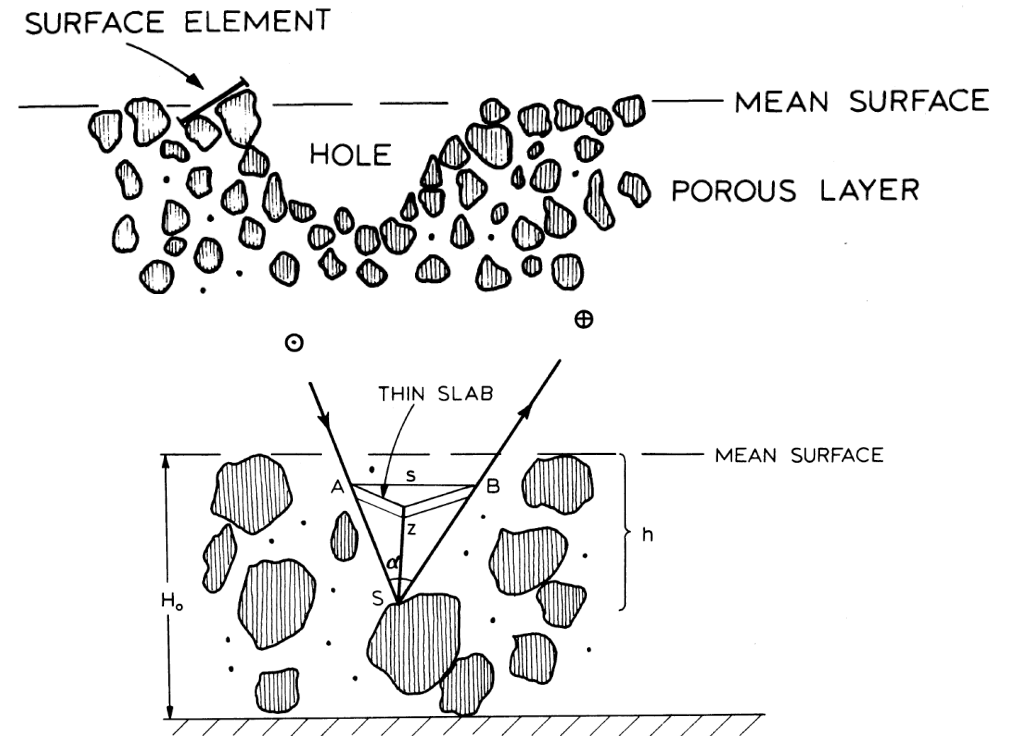
## Coherent backscattering mechanism (CBM)



Muinonen (1989, 1990)

Shkuratov (1985, 1988, 1989)

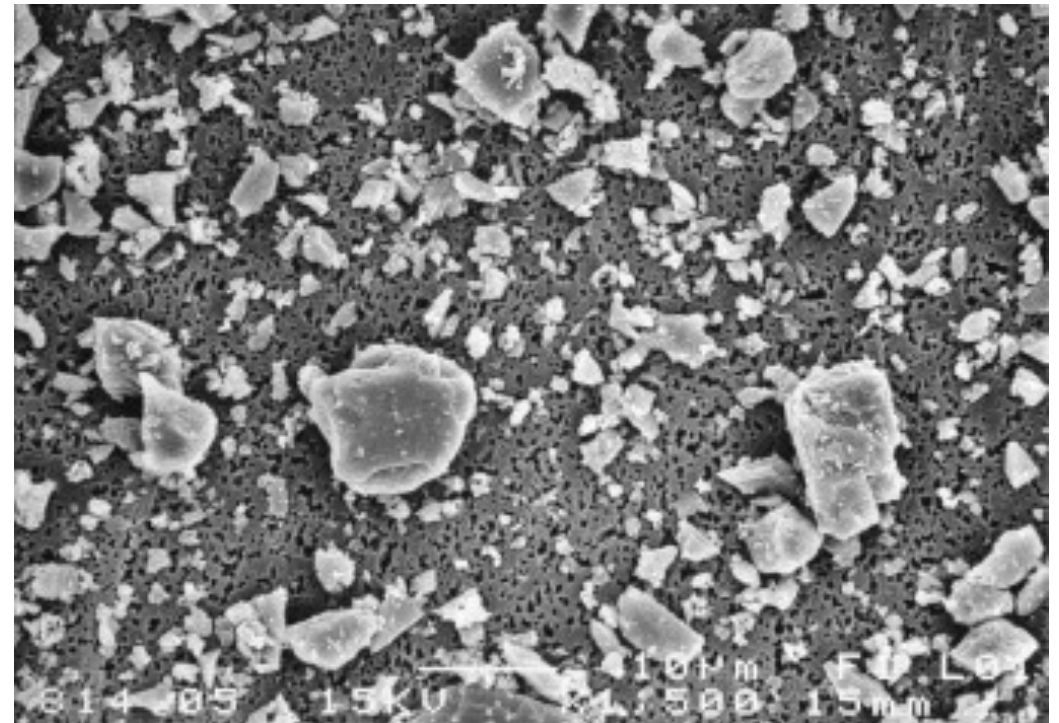
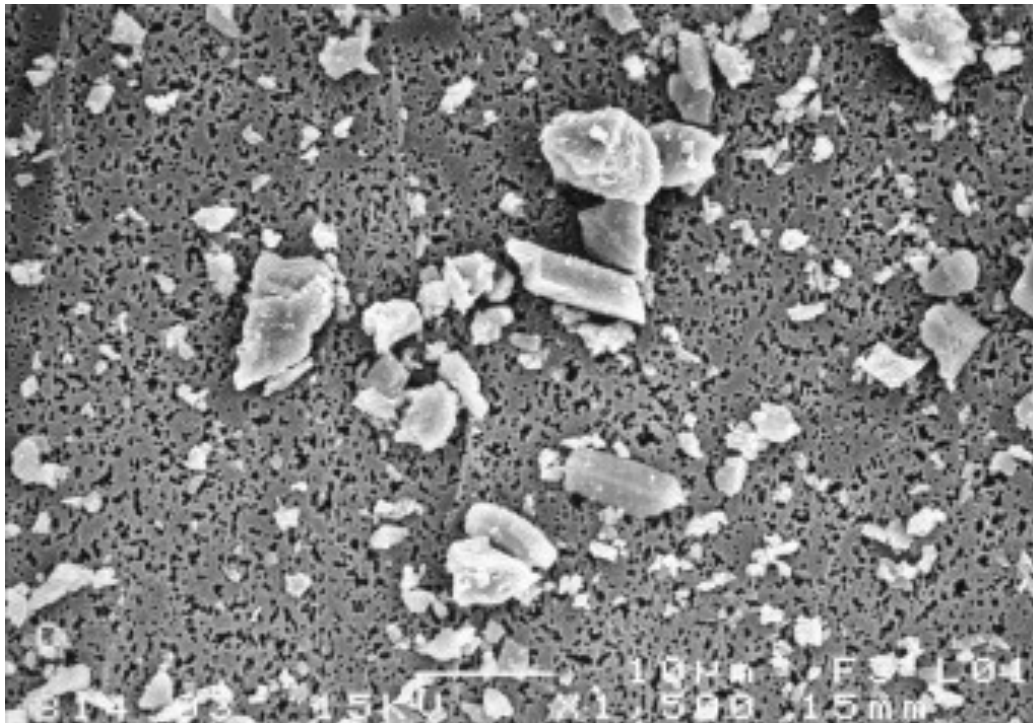
## Shadowing mechanism (SM)

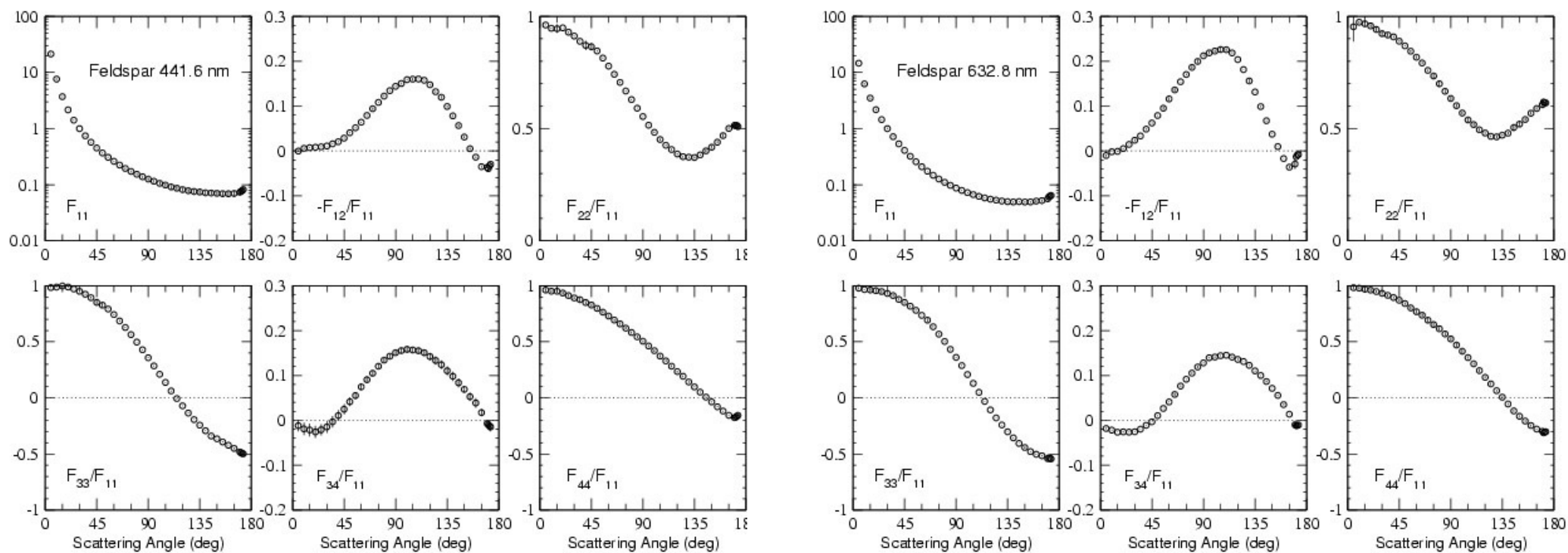


e.g., Lumme & Bowell (1981), Hapke (1963),  
Seeliger (1887), Russell (1916)

# Small-particle mechanism (SPM)

Feldspar samples from Granada-Amsterdam Light Scattering Database  
(Volten et al. 2001, Muñoz et al. 2012)

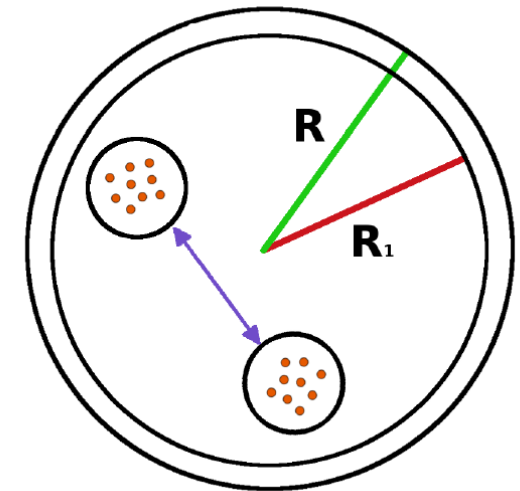
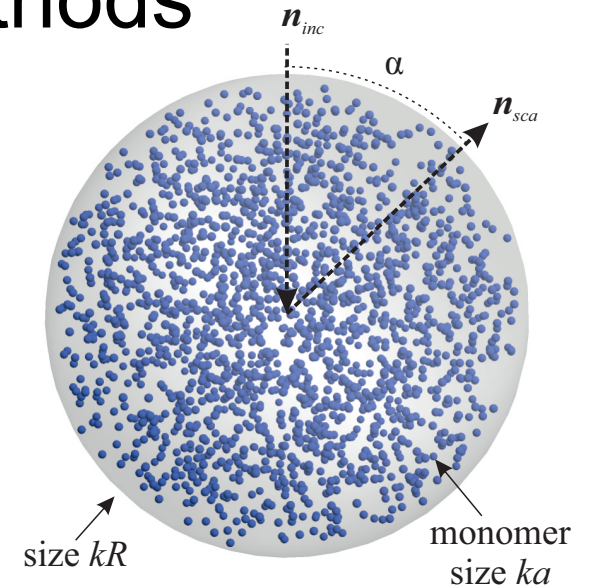




SPM can be related, for example, to standing electromagnetic waves induced inside the particles (Muinonen et al. 2011)

# Theory and numerical methods

- Maxwell equations & Radiative transfer equation
- Radiative transfer and coherent backscattering (RT-CB; Muinonen et al., ApJ 2012; Muinonen, WRM 2004 and URSI EMTS 1989)
- Superposition T-Matrix Method (STMM or MSTM; Mackowski & Mishchenko, JQSRT 2011; FaSTMM, Markkanen & Yuffa JQSRT 2017)
- Electric Current Volume Integral Equation Method (JVIE; Markkanen & Yuffa, JQSRT 2017, Markkanen et al., IEEE-TAP 2012)
- Radiative transfer with reciprocal transactions ( $R^2T^2$ ; Muinonen et al., URSI EMTS 2016ab, RS 2017, OL 2018, JoVE 2019; Markkanen et al., OL 2018, Markkanen et al., ApJ 2018, Väisänen et al., PLoS ONE 2019, OL 2020)
- RT-CB for nonspherical particles using decomposition of ensemble-averaged scattering matrices into pure Mueller matrices (Muinonen et al., in preparation, cf. Cloude, Proc. SPIE 1990 and Savenkov et al., JQSRT 2021)





# Scattering matrix decomposition

## Ensemble-averaged scattering phase matrix

$$\mathbf{P}_0 = \begin{pmatrix} P_{11}^{(0)} & P_{12}^{(0)} & 0 & 0 \\ P_{21}^{(0)} & P_{22}^{(0)} & 0 & 0 \\ 0 & 0 & P_{33}^{(0)} & P_{34}^{(0)} \\ 0 & 0 & P_{43}^{(0)} & P_{44}^{(0)} \end{pmatrix}$$

$$a_1 = P_{11}^{(0)}, \quad a_2 = P_{22}^{(0)},$$

$$a_3 = P_{33}^{(0)}, \quad a_4 = P_{44}^{(0)},$$

$$b_1 = P_{12}^{(0)}, \quad b_2 = P_{34}^{(0)}.$$

$$P_{21}^{(0)} = P_{12}^{(0)},$$

$$P_{43}^{(0)} = -P_{34}^{(0)}.$$

$$\int_{(4\pi)} d\Omega a_1(\theta) = 4\pi,$$

$$\int_0^\pi d\theta \sin \theta a_1(\theta) = 2.$$



Decomposition into four pure matrices (cf. eigenproblem for the Cloude coherency matrix):

$$\mathbf{P} = w_U \mathbf{U} + w_V \mathbf{V} + w_W \mathbf{W} + w_Z \mathbf{Z},$$

$$0 \leq w_U \leq 1, \quad 0 \leq w_V \leq 1, \quad 0 \leq w_W \leq 1, \quad 0 \leq w_Z \leq 1,$$

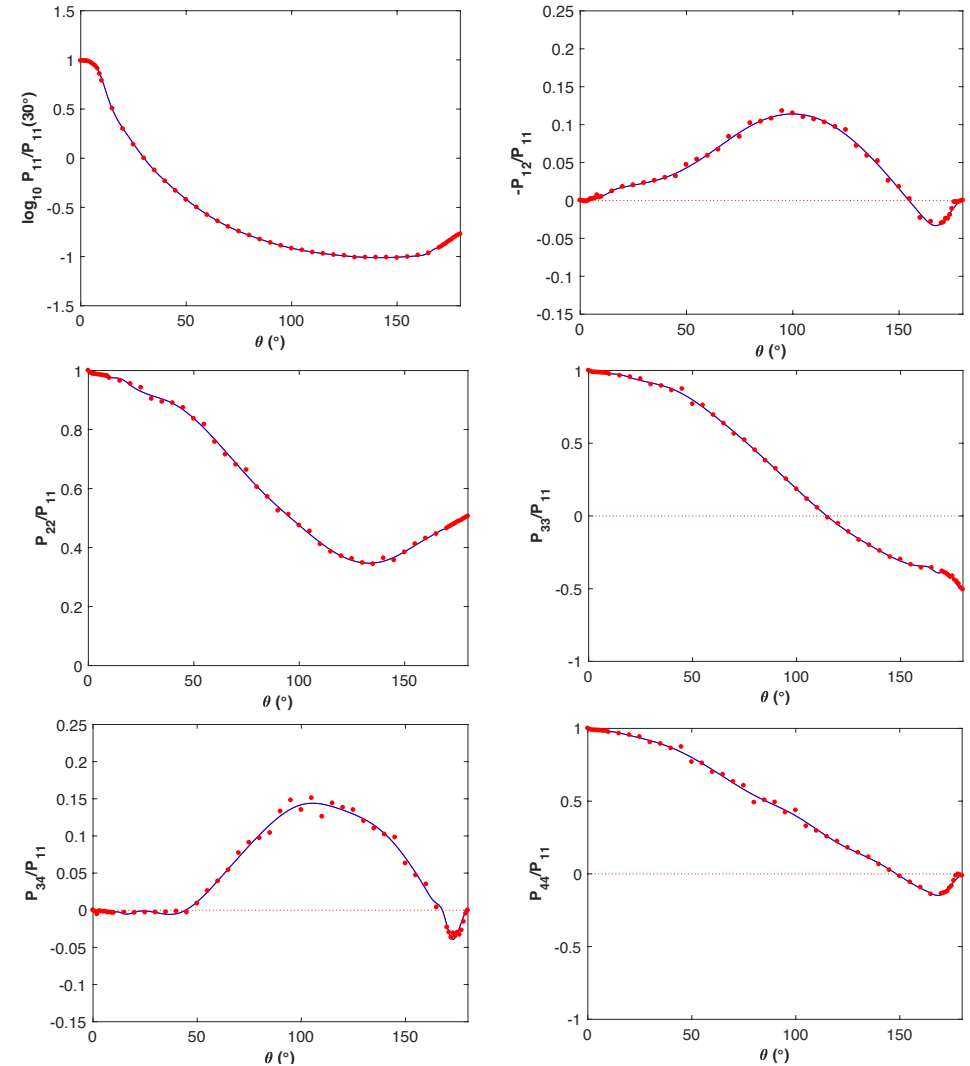
$$w_U + w_V + w_W + w_Z = 1,$$

$$\mathbf{U} = \begin{pmatrix} U_{11} & U_{12} & 0 & 0 \\ U_{12} & U_{11} & 0 & 0 \\ 0 & 0 & U_{33} & U_{34} \\ 0 & 0 & -U_{34} & U_{33} \end{pmatrix}, \quad \mathbf{W} = \begin{pmatrix} W_{11} & 0 & 0 & 0 \\ 0 & -W_{11} & 0 & 0 \\ 0 & 0 & -W_{11} & 0 \\ 0 & 0 & 0 & W_{11} \end{pmatrix},$$

$$\mathbf{V} = \begin{pmatrix} V_{11} & V_{12} & 0 & 0 \\ V_{12} & V_{11} & 0 & 0 \\ 0 & 0 & V_{33} & V_{34} \\ 0 & 0 & -V_{34} & V_{33} \end{pmatrix}, \quad \mathbf{Z} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 \\ 0 & -Z_{11} & 0 & 0 \\ 0 & 0 & Z_{11} & 0 \\ 0 & 0 & 0 & -Z_{11} \end{pmatrix},$$

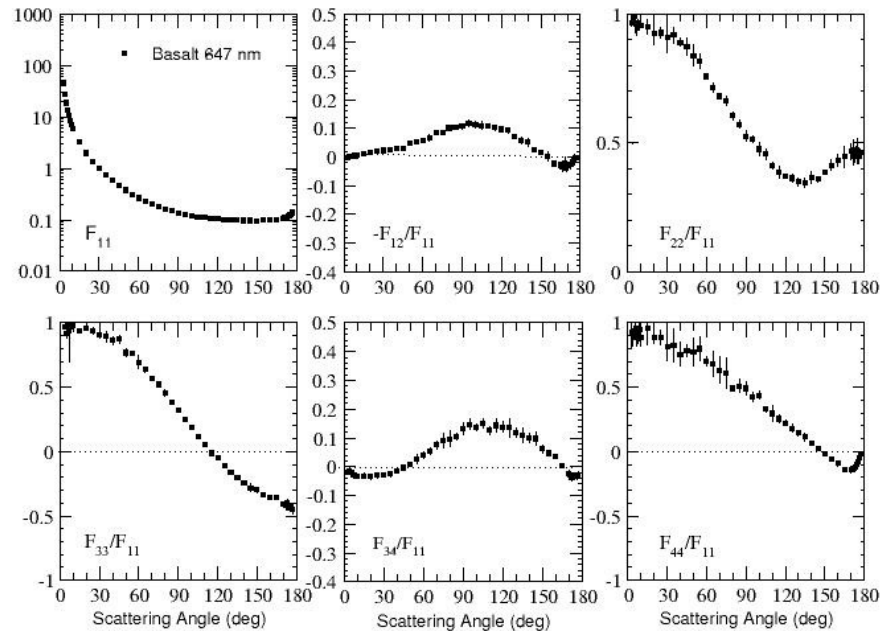
# Demonstration: Lunar photometry and polarimetry (1/2)

- Granada-Amsterdam [basalt measurements](#) modified for input incoherent scattering matrix (Dabrowska et al. 2015, Muñoz et al. 2012)
- Single-scattering albedo tuned to  $\omega=0.723$  to result in lunar geometric albedo of  $p=0.136$
- Mean-free-path length tuned to  $kl = 60$  to result in correct photometric and polarimetric opposition surges
- Spline representation of input  $-P_{12}/P_{11}$  tuned to result in a match with the observed polarization for the waxing (increasing) Moon
- Procedure repeated with [approximate](#) and [full decomposition](#) of the input scattering matrix for [RT-CB](#)

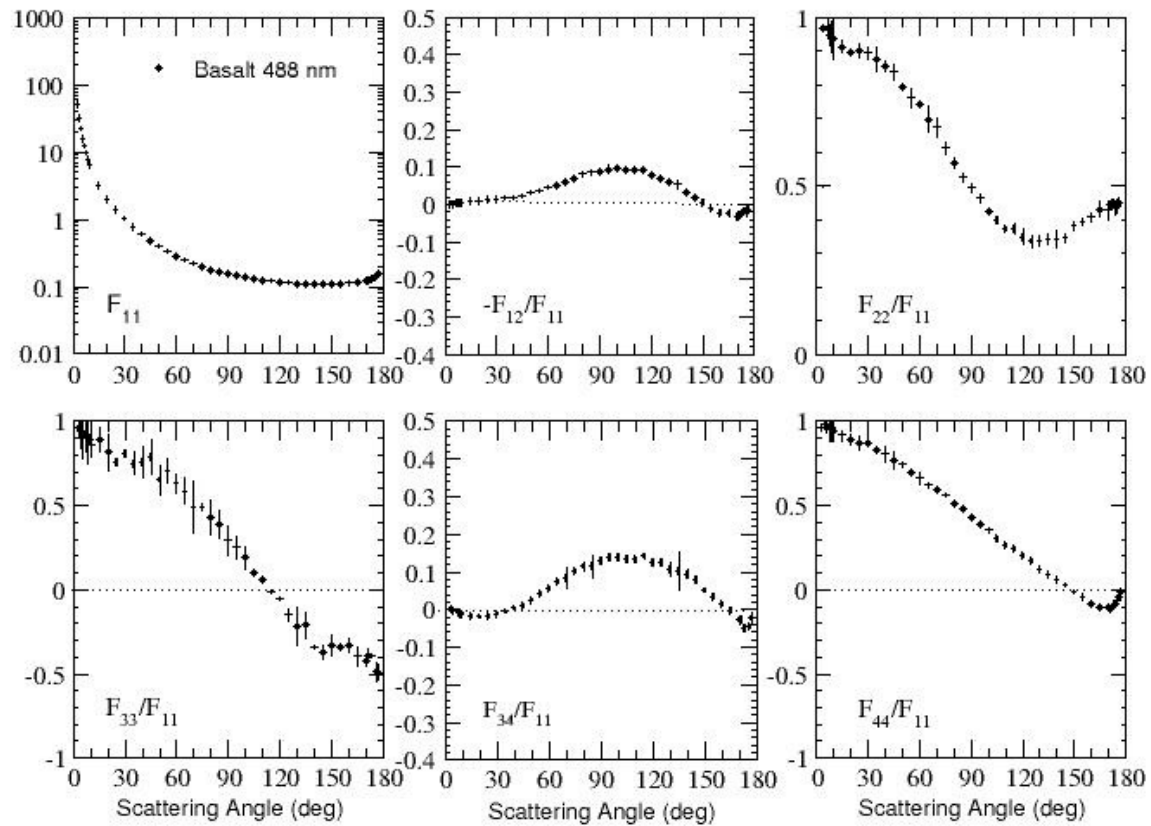


# Basalt particles, 647 nm

- Forward-peaked phase function
- Notice the shallow degree of linear polarization for unpolarized incident light
- Polarization maximum towards backscattering hemisphere
- Strong depolarization towards backward scattering



# Basalt particles, 488 nm



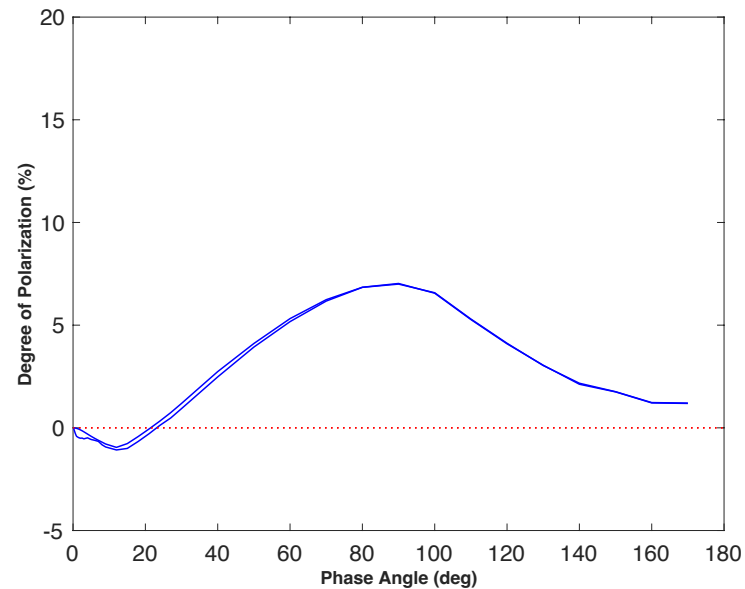
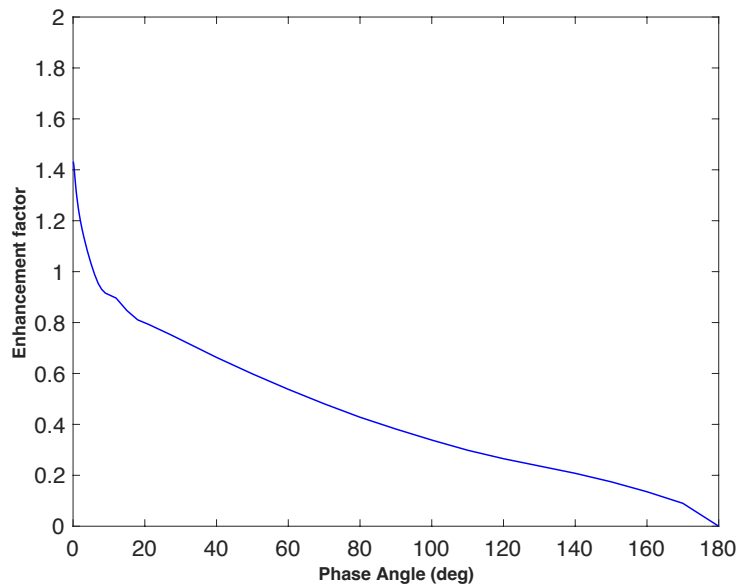
# Software pieces

- Scattering phase matrix decomposition
  - pmdec.f
- Geometric optics ray tracer
  - SIRIS (originally grayopt.f)
- Multiple scattering by a spherical medium using the decomposition
  - cbsdec.f
  - radiative transfer with coherent backscattering (RT-CB)
- Trigonometric polarization model
  - ptrig.f

# Pre-processing

- Experimental scattering matrices need to be extrapolated near forward and backward scattering angles
- Symmetry relations must be enforced in the forward and backward scattering directions
- Near forward scattering, diffraction must be smoothed out to produce so-called incoherent scattering matrices

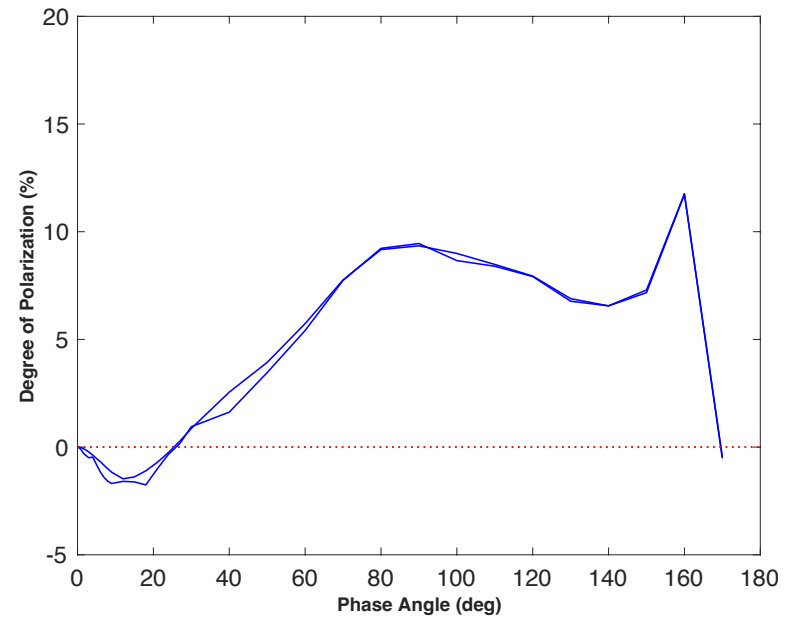
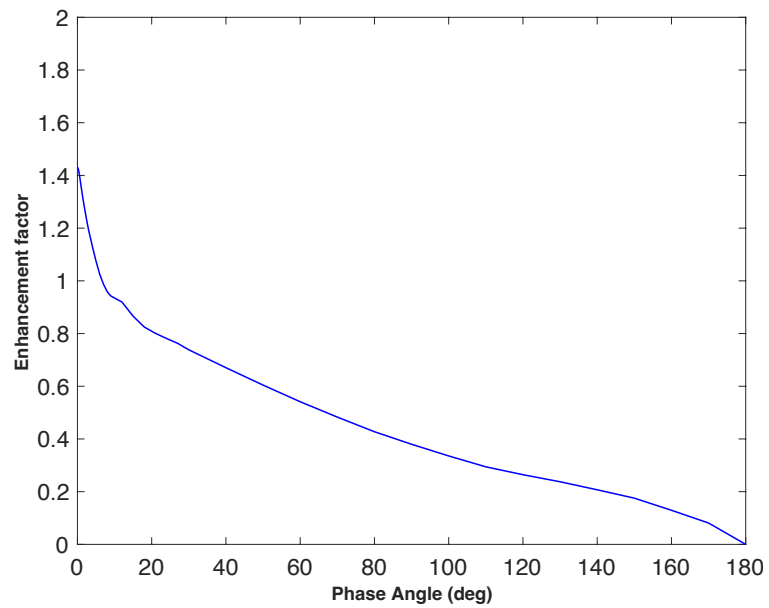
# Iteration 1



- Consider 647-nm small-particle scattering matrix
- Start with an educated guess for the single-scattering albedo and mean free path length
- $\omega = 0.70$ ,  $l = 7 \mu\text{m}$
- Geometric albedo below the lunar value of  $\sim 0.14$

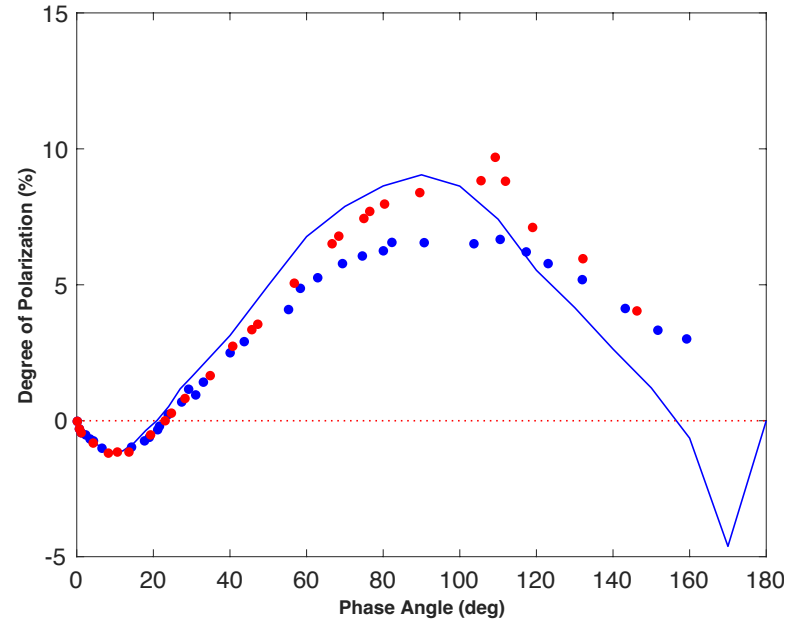
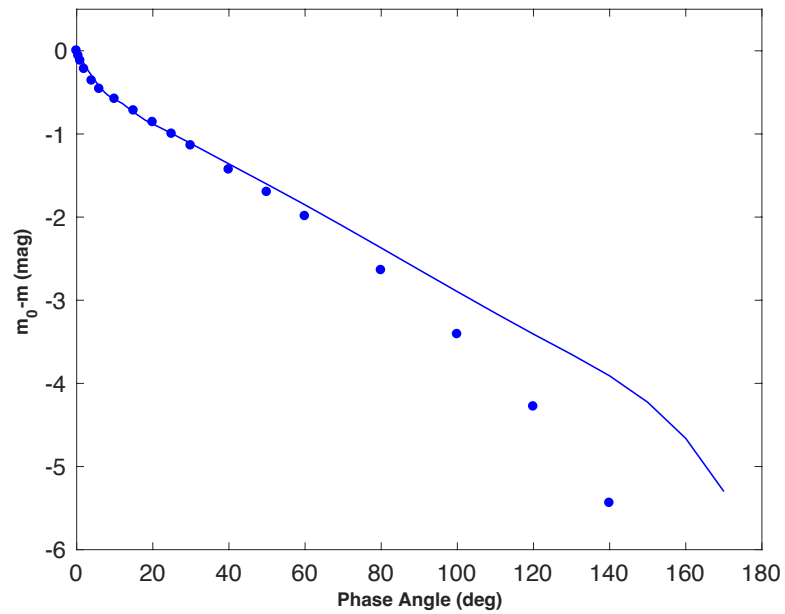


# Iteration 2



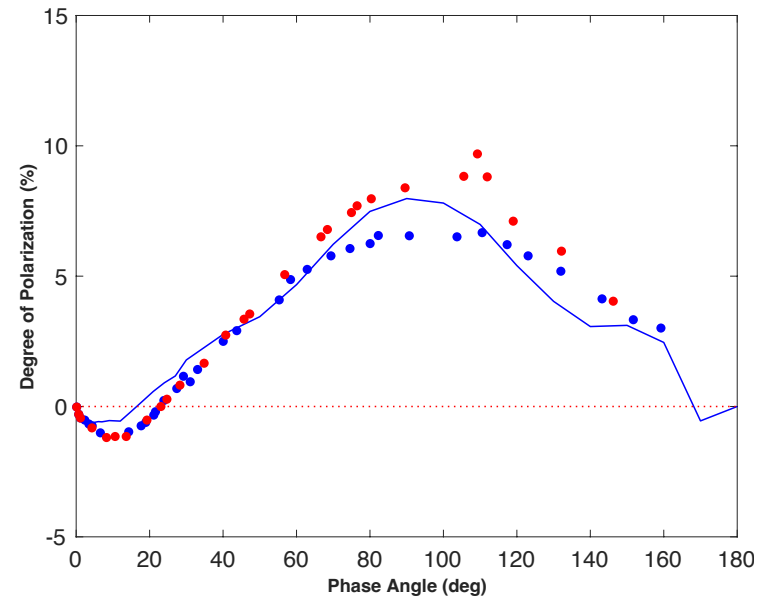
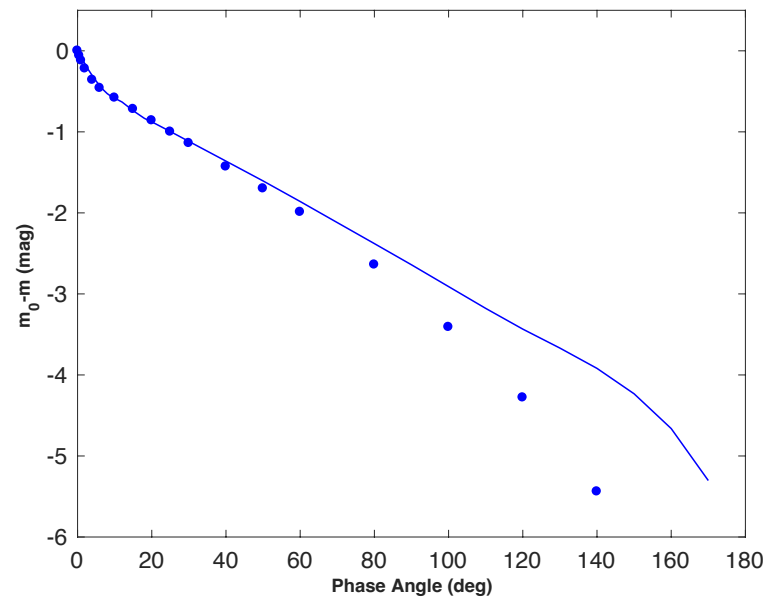
- Introduce geometric optics component,  $w = 0.1$
- Increase  $\omega$ , decrease  $l$
- $\omega = 0.75$ ,  $l = 4 \mu\text{m}$

# Iteration 3



- Keep  $\omega$  and  $I$  unchanged
- Decrease  $w$ :  $w = 0.07$

# Iteration 4



- Keep  $\omega$ ,  $l$ , and  $w$  unchanged
- Scale original small-particle  $-S_{12}/S_{11}$  by  $s=0.8$

# Iteration 5

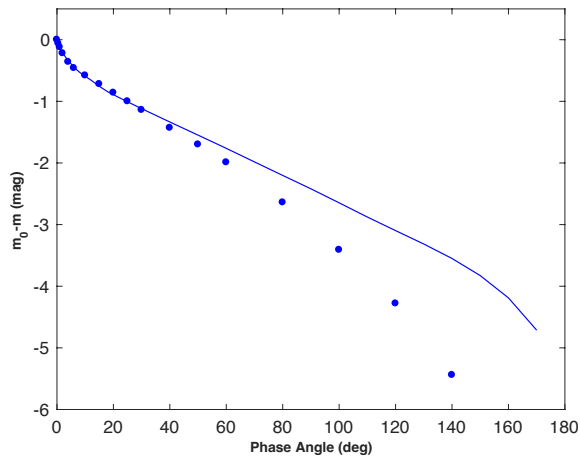
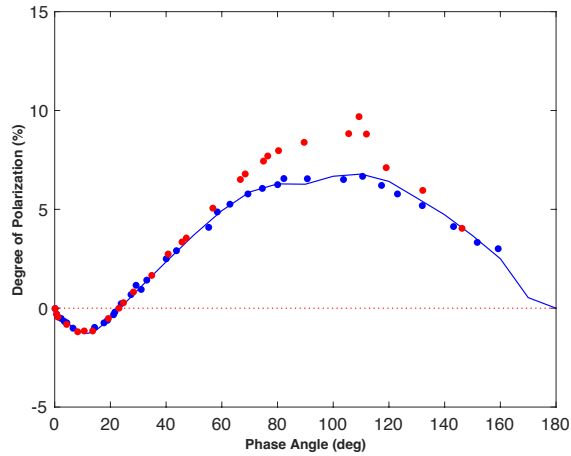
- To be computed!
- Keep  $\omega$ ,  $l$ , and  $w$  unchanged
- Scale original small-particle  $-S_{12}/S_{11}$  by  $s=0.95$
- Increase number of rays

# Intermediate conclusions

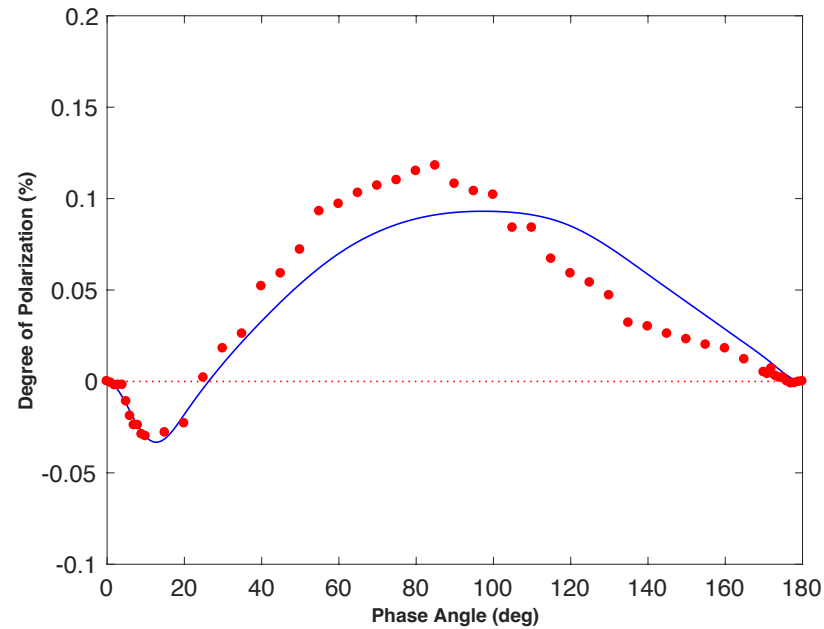
- Promising for the Moon but iteration requires attention at each step
- Proceed with discussion at each iteration!

# Demonstration: Lunar photometry and polarimetry (2/2)

Approximate scattering matrix decomposition



Example single-scattering polarization



Photometric data from Rougier (1933)  
averaged by Bowell et al. (1989)  
Polarimetric data from Lyot (1929)

Full scattering matrix decomposition

