Astrophysical light scattering problems

Experiments and instrumentation

*Photometry, **polarimetry**, and spectroscopy*

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- Why polarimetry
- Measuring polarization
- Polarization analyzers
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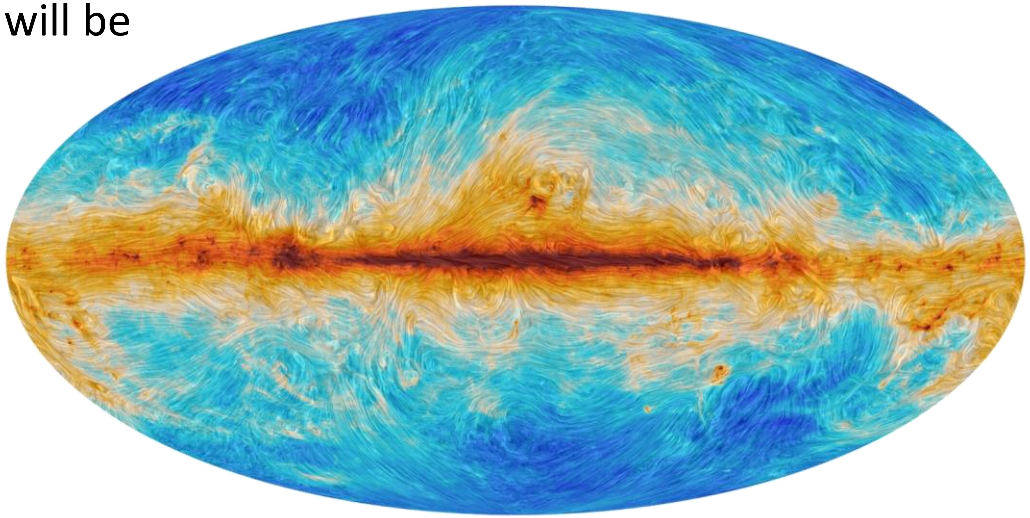
- Measuring Mueller matrix of scatterer
- Single-particle levitating scatterometer
- Granada-Amsterdam light scattering laboratory and database

Why polarimetry

- Adds information — instead of one I we can analyze up to three more properties, Q, U, and V
- The polarization state can carry information of the scattering event(s) the light has gone trough
  - Observing polarized signal on radiation that should be unpolarized
    - Light from stars is usually unpolarized
      - Observing (linear) polarization can mean that the light has gone through a cloud of interstellar dust or molecules
      - If there is preferential orientations in the particles, light will be polarized
      - Preferential orientations can be due to magnetic field

Polarized emission from Milky Way dust. Figure credit: ESA and the Planck Collaboration



Why polarimetry

- Observing changes in polarization as a function of phase or scattering angle
  - So-called polarization phase curve for surfaces, measures degree of linear polarization (DOP)

$$\frac{I_{\perp} - I_{\parallel}}{I_{\perp} + I_{\parallel}}$$

  as a function of phase angle

  - For surfaces, empirical Umov law relates degree of linear polarization to albedo of the surface
    - Maximum of DOP $\propto {}^{1}/_{p}$, where p is the albedo
    - Also valid for minimum of DOP, and the slope of DOP at inversion (when coming from negative to positive around 20°

Why polarimetry

- Observing changes in controlled polarization state
  - In radar, one can send circularly polarized radar pulses, and analyze the circular polarization state of the returned pulses
    - Every surface reflection turns the direction of circular polarization
  - Dual polarized weather radar has improved detection of targets, can filter out non-meteorological echoes better

Recap: Mueller-Stokes calculus

The *time-averaged* properties of light can be described with 4 Stokes parameters, collected in vector $\vec{S}$

$$\vec{S} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

From these 4 parameters, I describes the intensity, and the rest three the polarization state of the radiation. Q and U describe the linear polarization, and V the circular polarization.

For fully polarized coherent radiation, $I^2 = Q^2 + U^2 + V^2$, but for non-coherent, partially polarized light, $I^2 > Q^2 + U^2 + V^2$.

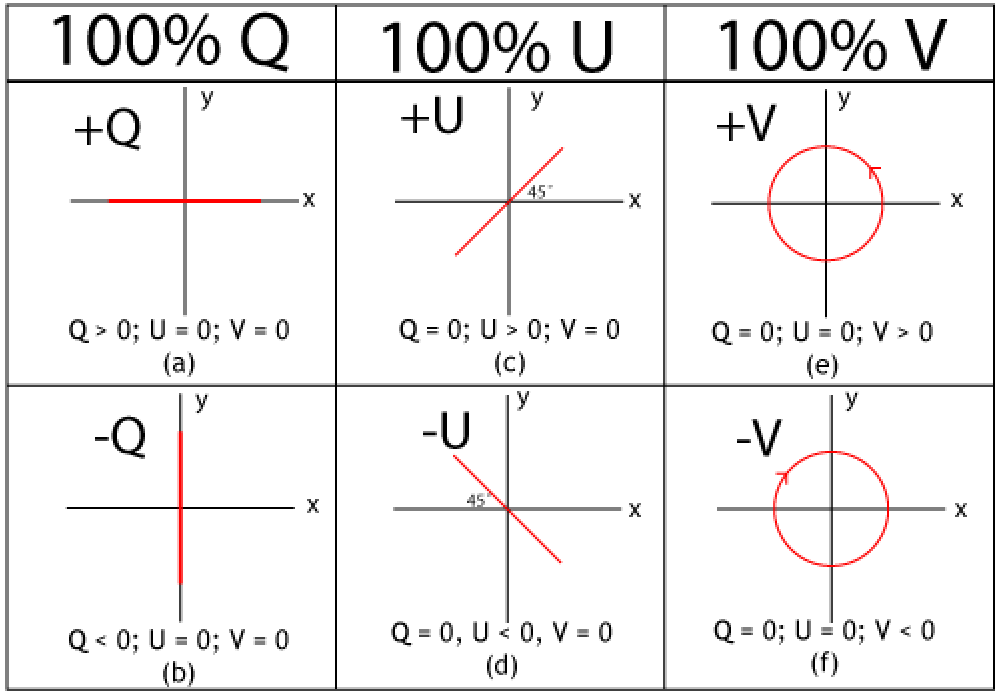


Figure source: Wikipedia, Dan Moulton

Detectors

- Detectors (for UV-Vis-NIR range, at least) are not measuring the phase of the light (EM wave), nor the amplitude variation
- Detectors are only measuring time-averaged power (intensity) of light
  - In Stokes parameters presentation, this means the component I
  - Components Q, U, and V cannot be directly measured
  - Polarization states are measured indirectly, shown later

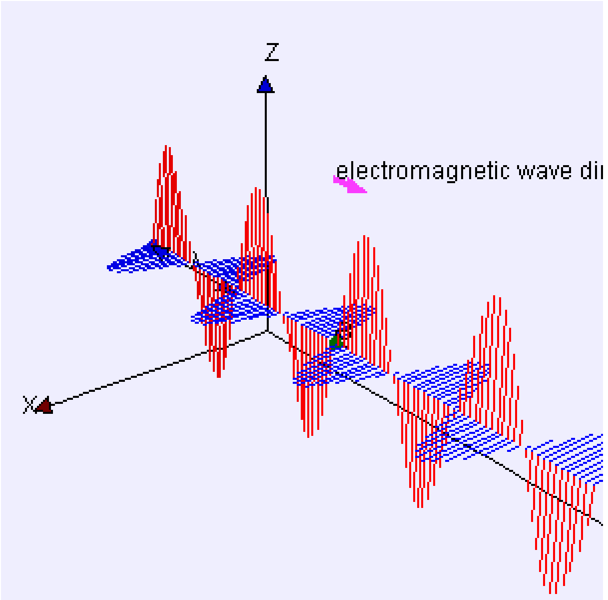


Figure source: Wikipedia, Lookang, thanks to Fu-Kwun Hwang and author of Easy Java Simulation = Francisco Esquembre

Measuring polarization

- Since only intensity can be measured, the polarization measurement needs *analyzer* that filters some polarization states
- Linear polarizer transmits only certain linear polarization state. The (theoretical) Mueller matrices for horizontal (H) and vertical (V) linear polarizers are:

$$\mathbf{M}_H = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{M}_V = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Measuring polarization

- Now, light described with Stokes vector $\vec{\boldsymbol{S}}$ going through horizontal polarizer has the Stokes vector $\vec{\boldsymbol{S}}_H$, or vertical polarizer, $\vec{\boldsymbol{S}}_V$:

$$\vec{\boldsymbol{S}}_H = \mathbf{M}_H \cdot \vec{\boldsymbol{S}} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} (I+Q)/2 \\ (I+Q)/2 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{\boldsymbol{S}}_V = \mathbf{M}_V \cdot \vec{\boldsymbol{S}} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} (I-Q)/2 \\ (-I+Q)/2 \\ 0 \\ 0 \end{pmatrix}$$

and, when a detector measures only the I component, it will detect signals $P_H = (I+Q)/2$ and $P_V = (I-Q)/2$. Now, $P_H - P_V = Q$. Component U can be measured similarly, with 45° linear polarizers.

Measuring polarization

- For circular polarization, we need a retarder, so-called quarter wave plate, which creates phase difference in orthogonal field components. Effectively, a quarter wave plate converts circular (V) into linear (Q) polarization
- We need both quarter wave plate $\mathbf{M}_{qw}$ and 45° linear polarizers $\mathbf{M}_{45+}$ and $\mathbf{M}_{45-}$:

$$\mathbf{M}_{qw} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \quad \mathbf{M}_{45+} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \mathbf{M}_{45-} = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Now, from $\mathbf{M}_{45+} \cdot \mathbf{M}_{qw} \cdot \vec{\boldsymbol{S}}$ we get measured signals $P_{45+} = (I + V)/2$ and $P_{45-} = (I - V)/2$, and therefor, V

Measuring polarization

- Note that when using polarization analyzers, we always get less signal than without. Measured signals like $(I+Q)/2$ and $(I-V)/2$ are always smaller than I, and their differences are even smaller
- This means that even with perfect polarizers (there are not any), signal-to-noise ratio of the measurement will decrease when measuring polarization
- Positive sides in measuring polarization — if studying ratios of polarizations such as degree of linear polarization DOP — the absolute intensity (magnitude) level is canceled away
  - Useful in asteroid or comet measurements if absolute calibration in photometry is difficult

Polarization analyzers

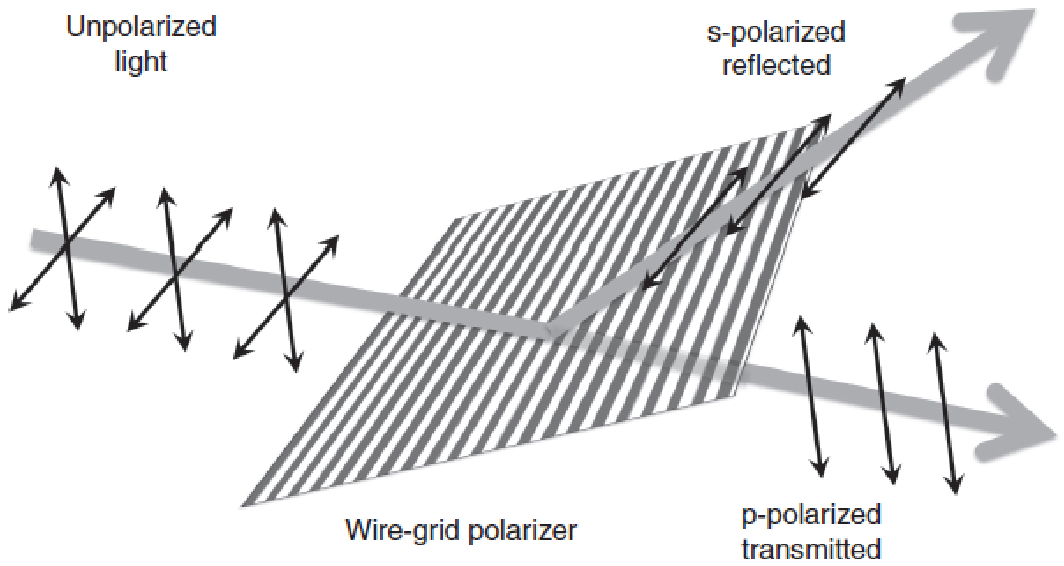


FIGURE 4.1 Wire-grid polarizer.

Linear polarizers. Prisms and plates use birefringent materials.

Polarizers can be attached to (manually or automatically) rotating holder to change the degree of polarization axis.

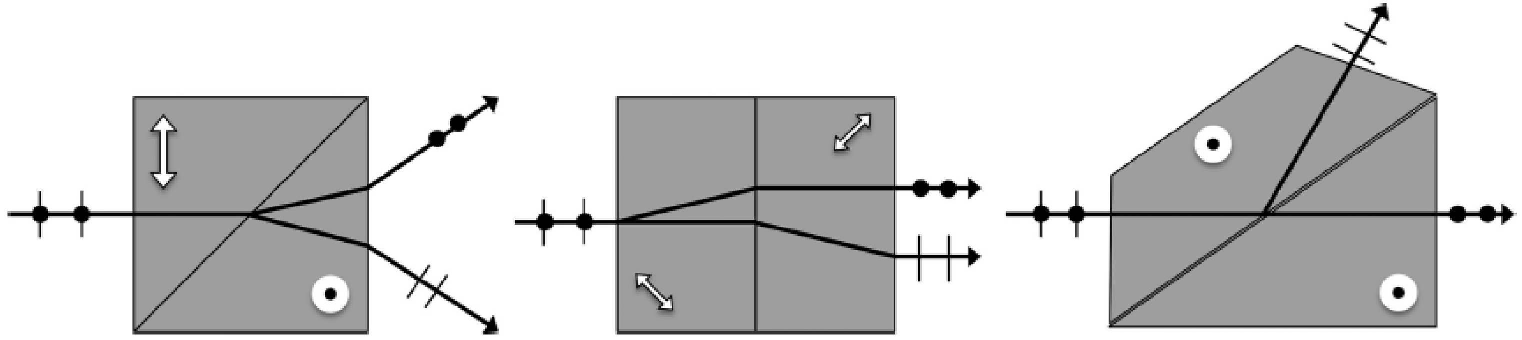
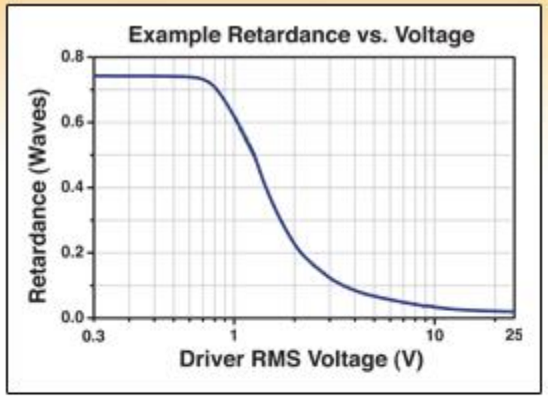


FIGURE 4.2 Crystal polarizing beamsplitters: Wollaston prism (left), modified Savart plate (middle), and Foster prism (right).

Polarization analyzers



Static retarders (quarter and half-wave plates) are also made of birefringent crystal materials.

Nowadays, liquid crystal retarders can be manipulated with voltage. This enables very fast changes in polarization.

Combining analyzers and detector

- The complete set to measure polarization properties of light consists of analyzer (one or more polarizing components) and a detector
- In order to derive U, Q, V, one needs to modulate the detected signal
- Modulation can be spatial, temporal, or spectral
- Spatial modulation:
  - Polarizing prisms separate different signals into different light paths. Finally, these beams are imaged on different parts of detector, or with different detectors
- Temporal modulation
  - Light goes trough polarizer which properties change with time
    - Classical implementation: different polarizers in filter wheel
    - Alternate implementation: rotating polarizer
    - Modern implementation: fast liquid crystal rotating polarizer

Combining analyzers and detector

- Spectral modulation
  - Most recent advancement. Achromatic QWP and a single multiple-order retarder can be used in such a way that degree of linear polarization modulates the spectral signal
  - Signal needs to be detected with a (sensitive) spectrometer, and demodulated
  - Benefits: one measurement in one light path minimizes the problems of temporal (different seeing conditions between measurements of different components) or spatial (different optical distortions between two separate beam paths) modulations

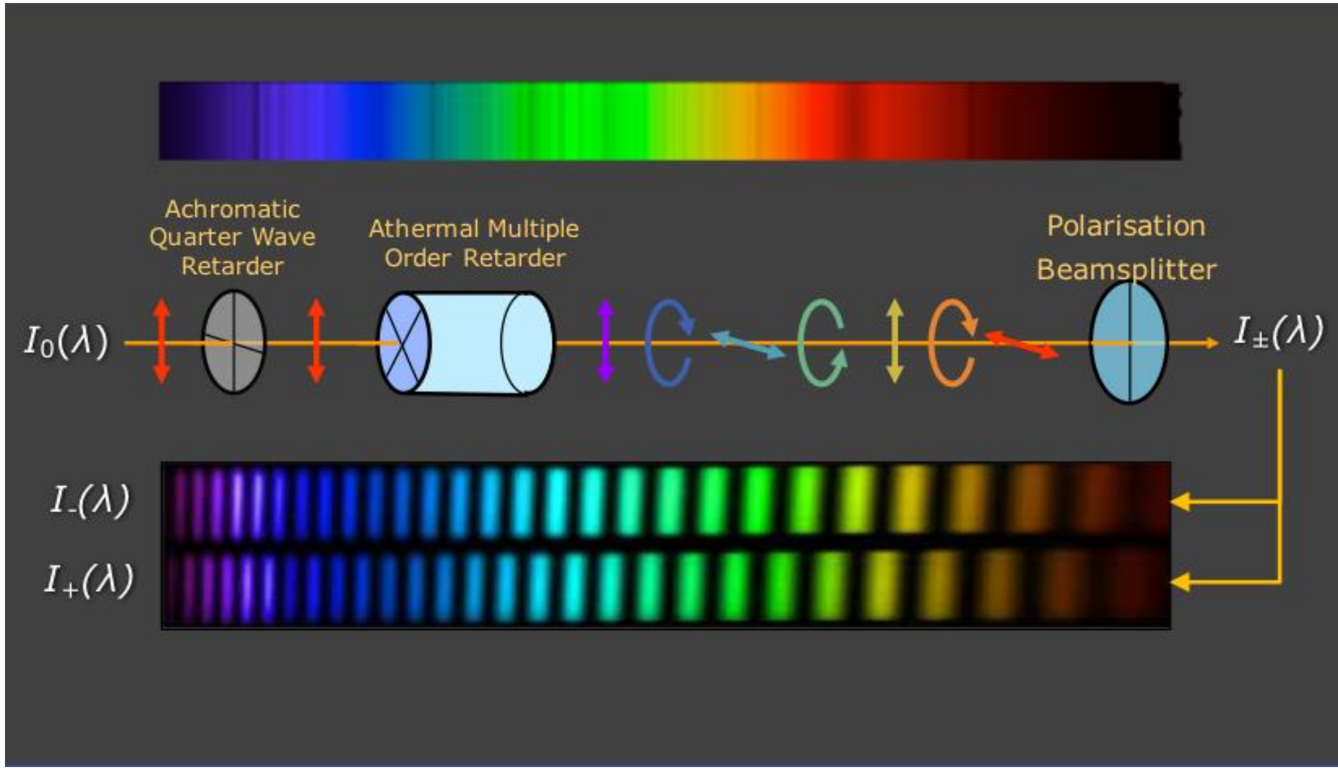


Figure credits: SRON, SPEX instrument

Measuring Mueller matrix of scatterer

- When measuring the Mueller matrix of a scatterer, same rule applies as with Stokes components – detectors can measure only intensity
- In a scattering event, all four elements of the first row of the Mueller matrix contribute to intensity, and the other elements do not contribute

$$\begin{pmatrix} I_{sca} \\ Q_{sca} \\ U_{sca} \\ V_{sca} \end{pmatrix} = \frac{\lambda^2}{4\pi^2 D^2} \begin{pmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{pmatrix} \begin{pmatrix} I_{inc} \\ Q_{inc} \\ U_{inc} \\ V_{inc} \end{pmatrix}$$

- Key for measuring Mueller matrix elements is to control both the incident light and the scattered light polarization

- For example, filtering purely horizontal polarized light into sample gives:

$$\begin{pmatrix} F_{11}+F_{12} \\ F_{21}+F_{22} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

which then is measured as $P_1 = F_{11} + F_{12}$. Similarly, using vertical polarized light we get $P_2 = F_{11} - F_{12}$. Now, $(P_1 - P_2)/2 = F_{12}$.

- Other elements can be retrieved in similar manner, but we also need analyzer after scattering to bring the scattered Q,U, or V contributions into I component before the detector. In Mueller matrix formulation, our measurement can be written as

$$[1\ 0\ 0\ 0] \cdot \mathbf{M}_A \cdot \mathbf{F} \cdot \mathbf{M}_F \cdot [1\ 0\ 0\ 0]^T\ ,$$

where the vector on the right is the incident unpolarized beam, $\mathbf{M}_F$ the filter, $\mathbf{F}$ the scatterer, $\mathbf{M}_A$ the analyzer, and the last vector product picks up the intensity component.

- Our laboratory scatterometer employs the Mueller matrix techniques to resolve the 2×2 upper left submatrix of the scatterers' Mueller matrix

$$\mathbf{M}_p(\theta) = \frac{1}{2}\begin{pmatrix} 1 & \cos(2\theta) & \sin(2\theta) & 0 \\ \cos(2\theta) & \cos^2(2\theta) & \sin(2\theta)\cos(2\theta) & 0 \\ \sin(2\theta) & \sin(2\theta)\cos(2\theta) & \sin^2(2\theta) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Thus, for one single measurement with the scatterometer, the chained transformation becomes:

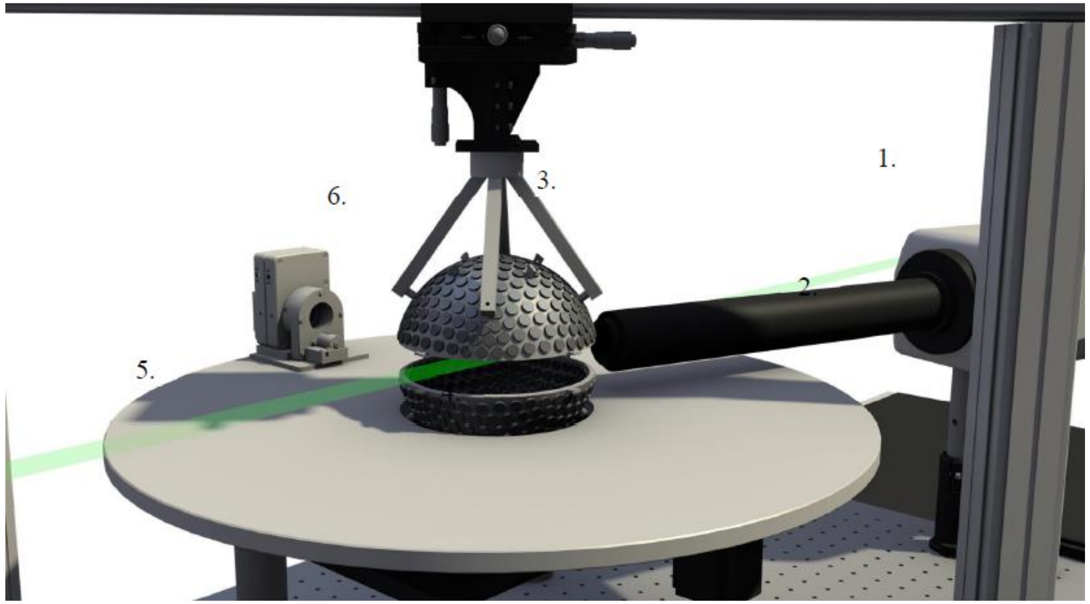
$$\boldsymbol{I}_s = \mathbf{M}_p(\theta_2)\mathbf{M}\,\mathbf{M}_p(\theta_1)\boldsymbol{I}_i\ ,$$

where $\mathbf{M} = k^{-2}R^{-2}\mathbf{S}$. The following configurations are needed to construct the 2×2 submatrix:

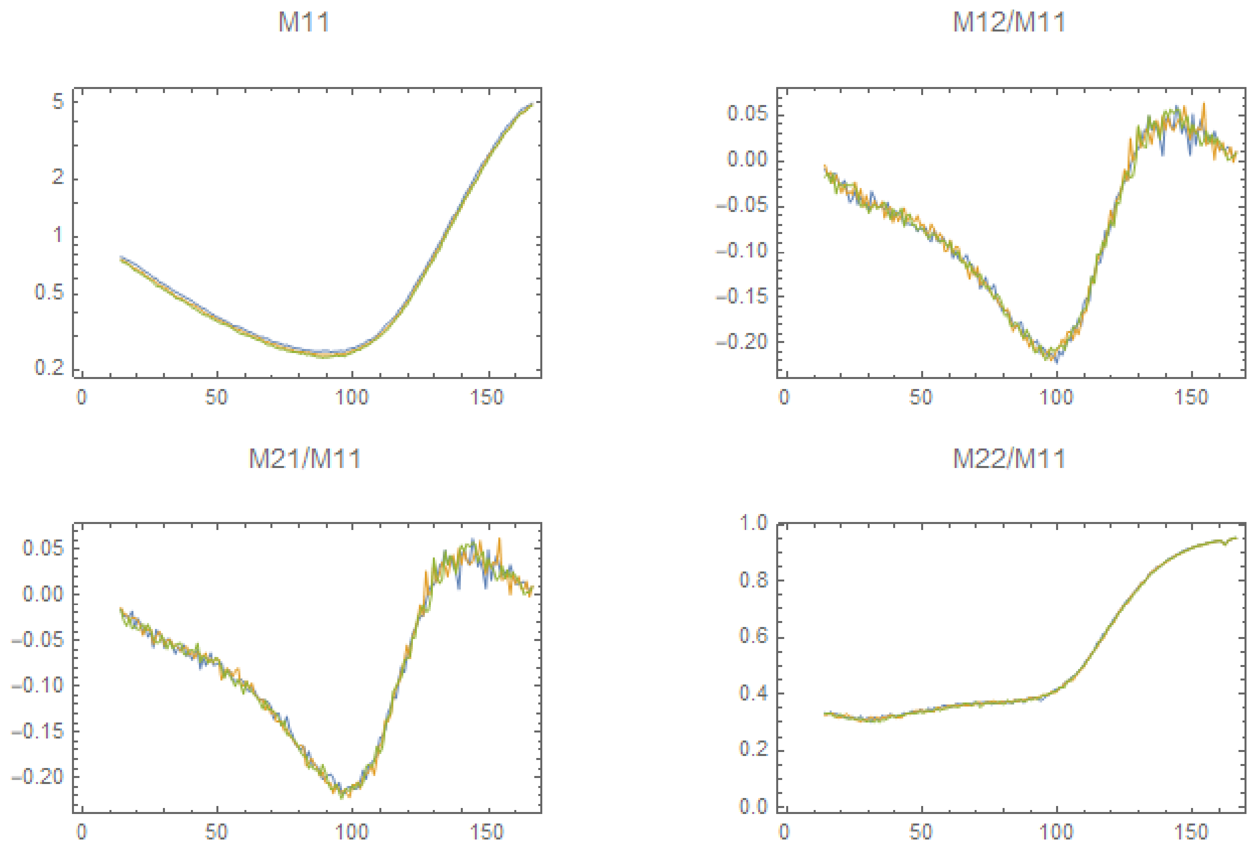
$$\begin{aligned} I_1&: \theta_1 = 0°; \theta_2 = 0° \\ I_2&: \theta_1 = 90°; \theta_2 = 90° \\ I_3&: \theta_1 = 0°; \theta_2 = 90° \\ I_4&: \theta_1 = 90°; \theta_2 = 0° \end{aligned}.$$

From these measurements, the Mueller matrix elements can be calculated as:

$$\begin{aligned} M_{11} &= I_1 + I_2 + I_3 + I_4 \\ M_{12} &= I_1 - I_2 + I_3 - I_4 \\ M_{21} &= I_1 - I_2 - I_3 + I_4 \\ M_{22} &= I_1 + I_2 - I_3 - I_4 \end{aligned}.$$

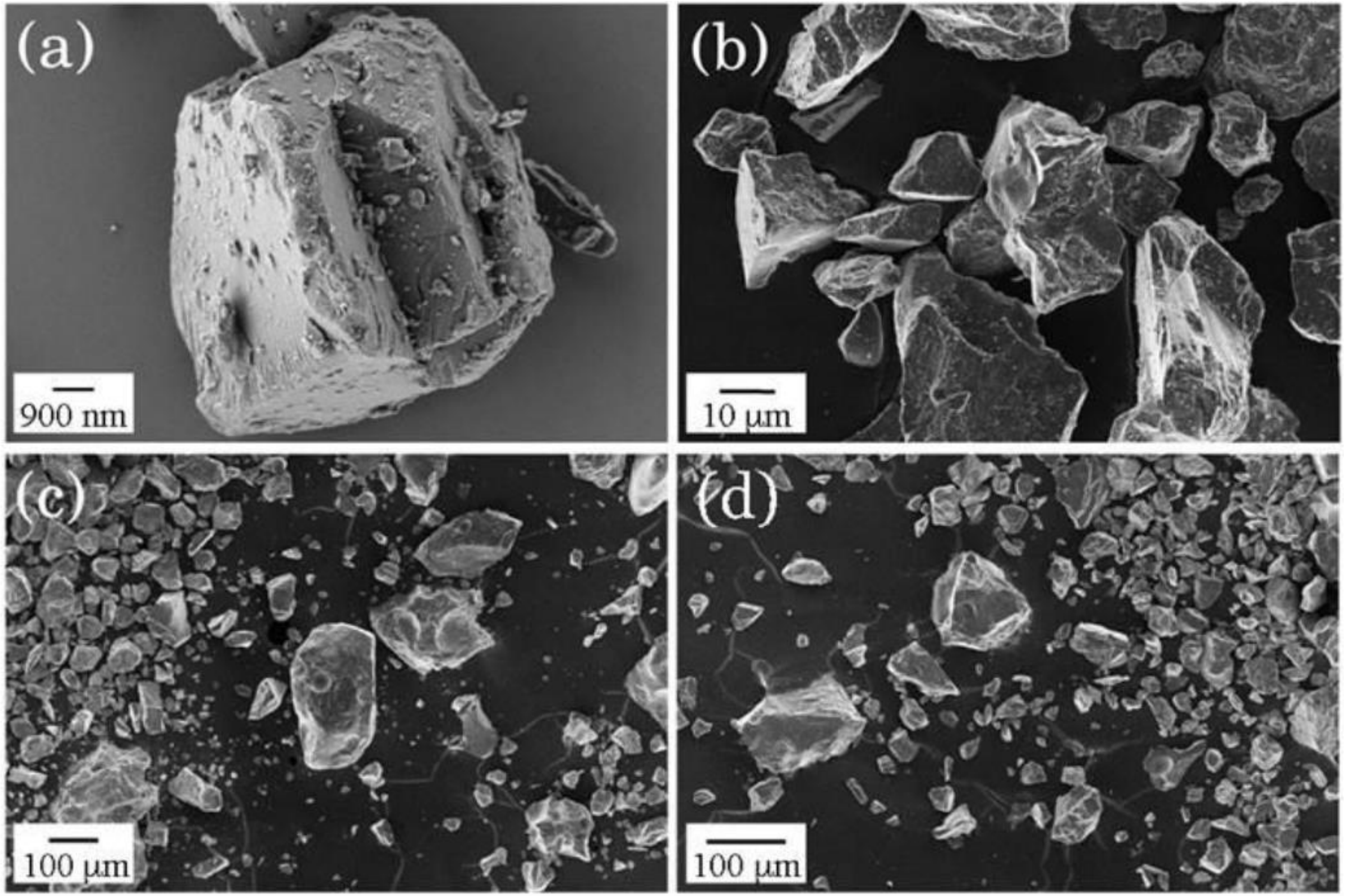


First results, calibration with 3 mm silica sphere, matte surface

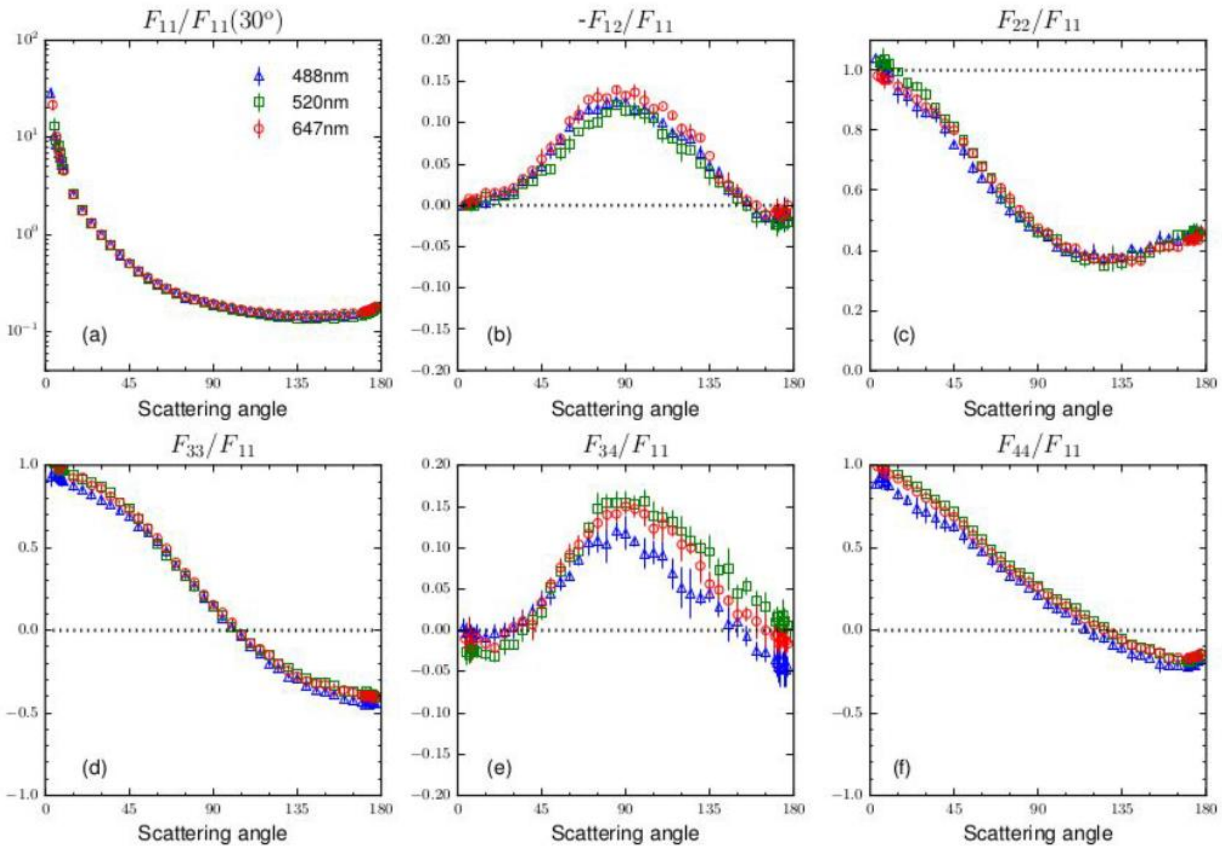


Final calibrations with the instrument still waiting — developments with acoustic levitator and then the COVID pandemic closing laboratory operations have delayed the process.

- Lunar simulant shapes in SEM microscope image



- Lunar simulant phase function and other Mueller matrix elements



Follow-up task

- Watch recorded presentations on the SOFIA Rock, Dust and Ice: Interpreting planetary data –workshop. Links will be posted as the recorded presentations are published
  - Irina Belskaya (Kharkiv National University): Probing Surface Properties of Asteroids by Polarimetric Observations, https://youtu.be/LVw8Q_rxda8
  - Nikolai Kiselev (Crimean Astrophysical Observatory): Peculiar Polarization of Icy Surfaces: Observations of Jovian Satellite Europa and their Interpretation, https://youtu.be/PxzWSzpOMHg