Rough-surface scattering modeling

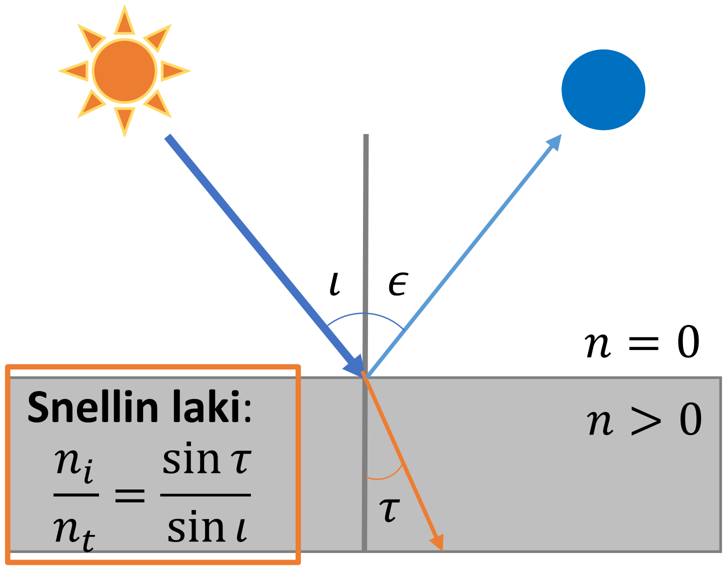
Computational Light Scattering (PAP315)

Lecture 06

Rough surfaces

- The topographic expression of a surface
- Scales can be anything, but similar to all scattering scenarios, what matters the most is the scale relative to the wavelength
- Natural surfaces have typically roughness in many scales; for example, an ideally fractal surface has the same level of roughness in all scales
- Quasi-deterministic shape + pseudo-random variation

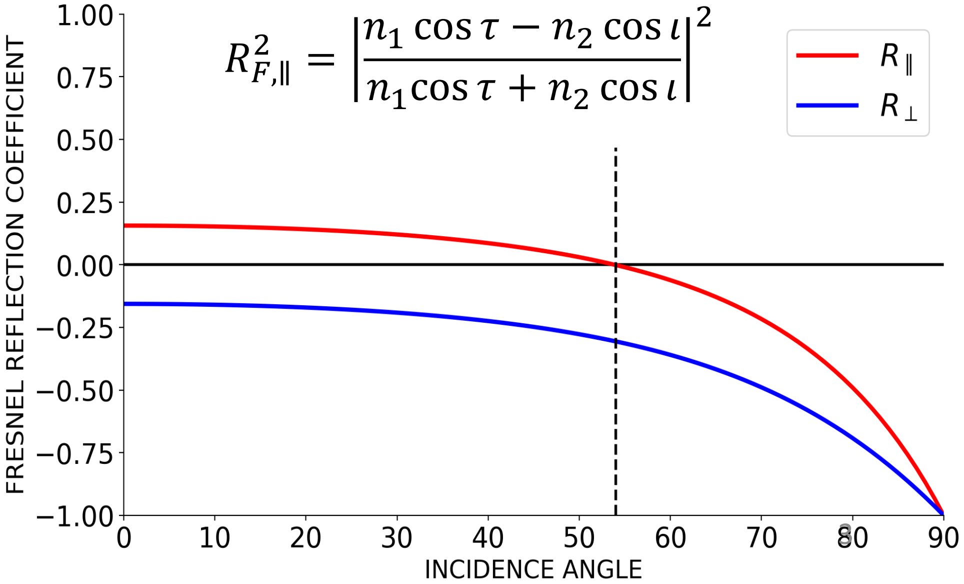
Fresnel reflection and refraction

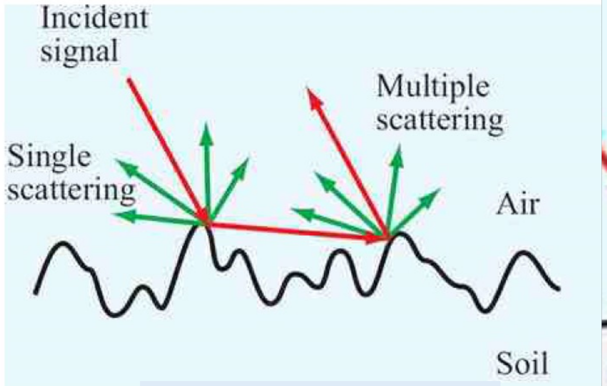


- Let us start from the simplest case of no roughness
- Fresnel reflection and refraction coefficients define how much light we observe from the reflection
- The coefficients in each polarization are a relatively simple function of the refractive index and the reflection and refraction angles

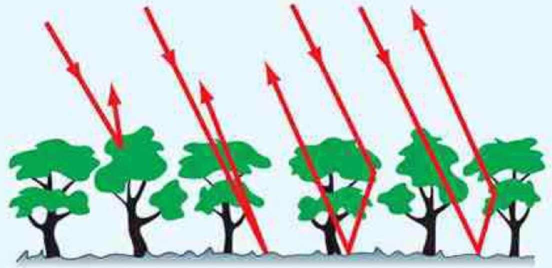
$$R_{F,\perp}^2 = \left| \frac{n_1 \cos \iota - n_2 \cos \tau}{n_1 \cos \iota + n_2 \cos \tau} \right|^2$$

$$R_{F,\parallel}^2 = \left| \frac{n_1 \cos \tau - n_2 \cos \iota}{n_1 \cos \tau + n_2 \cos \iota} \right|^2$$

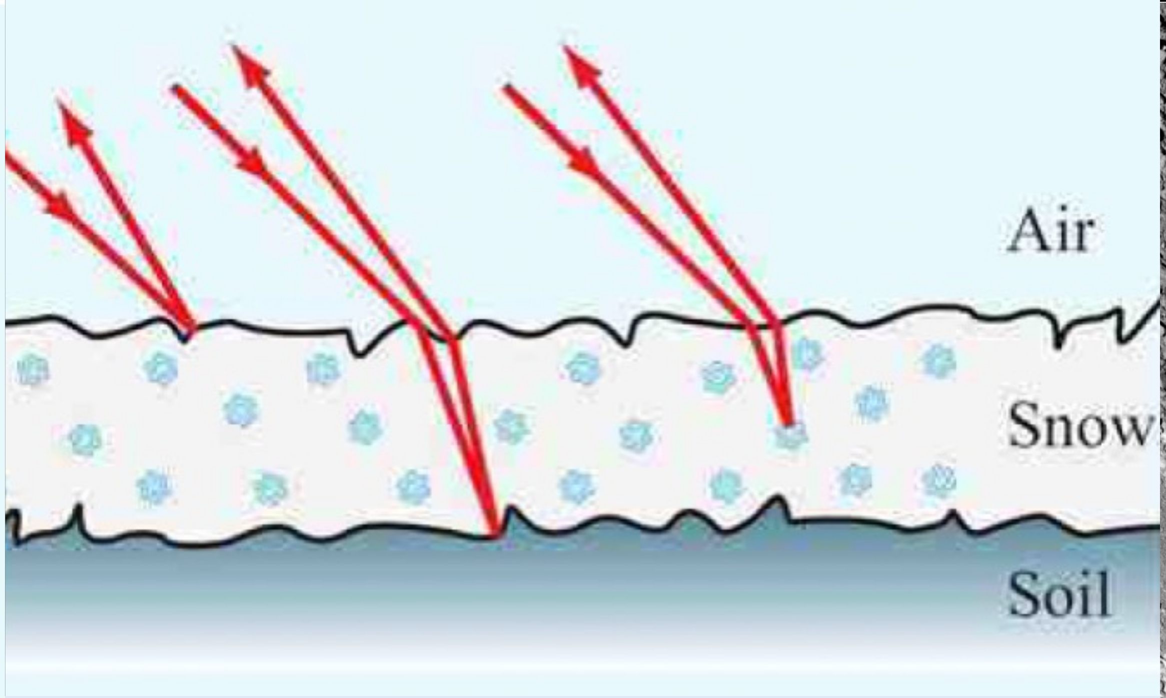




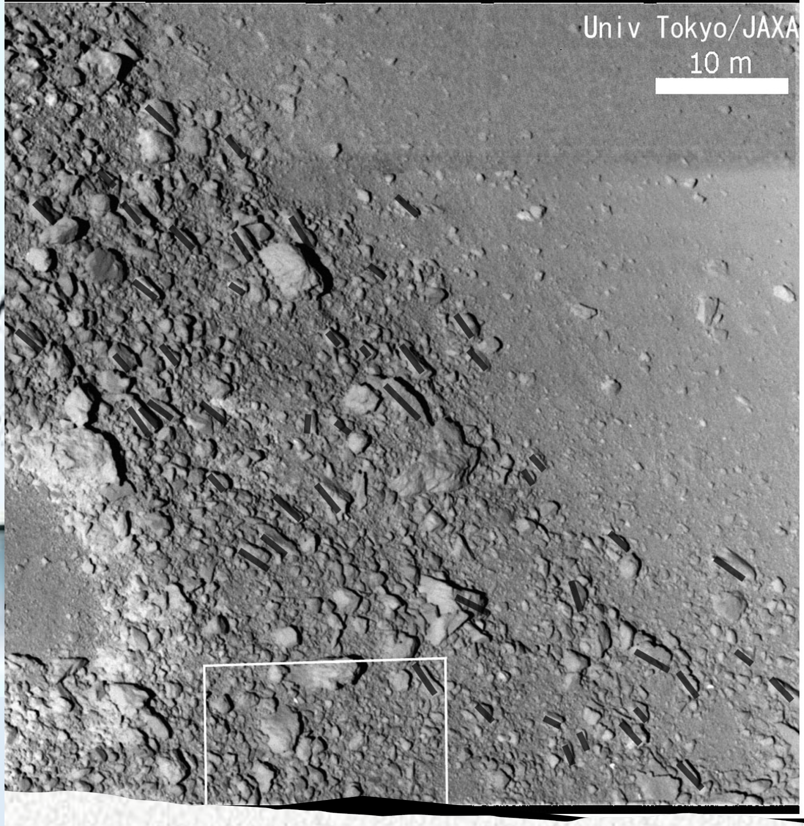
Random surface



Vegetation canopy



Multiple layers of different materials, complex interfaces and diffuse scatterers



But what if the scattering surface looks like one of these?

Relevant especially in microwave remote sensing! 😱

Random surfaces

- Here we focus on quasi-deterministically planar surfaces with pseudo-random roughness
- For most natural terrains, the height variations and their correlation function follow Gaussian distribution or an exponential distribution
- We discuss how the height variation distributions and the autocorrelation function (to be explained in detail) affect the observed scattering
- Vegetation canopies and diffuse scatterers inside the layers are considered as volume scattering which is a different can of worms

[Shepard et al. 1999, Icarus]

Developing surface-scattering models

- Modeling single-particle scattering with methods such as the discrete-dipole approximation are for individual, customized shapes
- For rough surfaces, the approach is more statistical: We perform a mathematical averaging process equivalent to
    1. generating a large number of synthetic surfaces and volumes based on the assumed statistics,
    2. computing for each synthetic surface the scattering cross section in the orientation of interest, and then
    3. performing an ensemble average.
- Step 2 is equivalent to implementing a Monte-Carlo simulation, but it is carried out mathematically by injecting the assumed statistical distributions into the scattering formulation and then calculating the mean radar cross section.

Developing surface-scattering models

All scattering models of terrain are, at best, good approximations of the true scattering process observable by an instrument observing a real-life target. They serve as guides to explain experimental observations. Surface scattering models presented here are limited to simulating the contribution of the surface in specific scenarios.

No model involving random elements is perfect – but some models can be made better than others.

Surface-roughness parameters

- **Degree of roughness** (or simply **roughness**): a set of statistical parameters for characterising the surface undulations
- **Standard deviation or root-mean-square of the height variations** (“**Rms height**”): The vertical scale of the variations
- **Correlation length**: The horizontal scale of the variations
- **Rms slope**: The rms inclination of the surface elements
- **Reference surface**: The mean or unperturbed surface

Rms height – continuous surface

A random surface whose mean is coincident with the x,y plane has a height distribution z(x,y) that can be characterized by a Gaussian probability density function given as:

$$p(z) = \frac{1}{\sqrt{2\pi h^2}} e^{-z^2/(2h^2)}$$

where the rms height is

$$h = \sqrt{\langle z^2 \rangle} = \left[\int_{-\infty}^{\infty} z^2 p(z) dz \right]^{1/2}$$

Rms height – discretized surface

If the height distribution z(x,y) is discretized to N elements, the rms height is

$$h = \left[\frac{1}{N-1}\left(\sum_{i=1}^{N} z_i^2 - N\bar{z}^2\right)\right]^{1/2}$$

where

$$\bar{z} = \frac{1}{N}\sum_{i=1}^{N} z_i$$

Surface correlation length

- The autocorrelation function describes the correlation, or in practice, the lateral separation between two locations (x_1, y_1) and (x_2, y_2):

$$\rho(\xi) = \frac{\langle z(x_1,y_1)z(x_2,y_2)\rangle}{h^2},$$

where $\xi = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

- **The surface correlation length** is the mean lateral separation between two locations where the height has decreased to $1/e$, *i.e.*,

$\rho(\xi) = e^{-1}$, so let's define that at this point $\xi = l$

Surface correlation length – discretised surface

- For a discretised surface, the autocorrelation function can be written as

$$\rho(\xi) = \frac{\sum_{i=1}^{N+1-j} z_i z_{j+i-1}}{\sum_{i=1}^{N} z_i^2},$$

where $\xi = j - 1$ and i and j are integer indices of each element $(i, j > 0)$

Surface correlation length

- The two most commonly used autocorrelation functions take Gaussian and exponential forms

- Gaussian autocorrelation function:

$$\rho(\xi) = e^{-\xi^2/l^2}$$

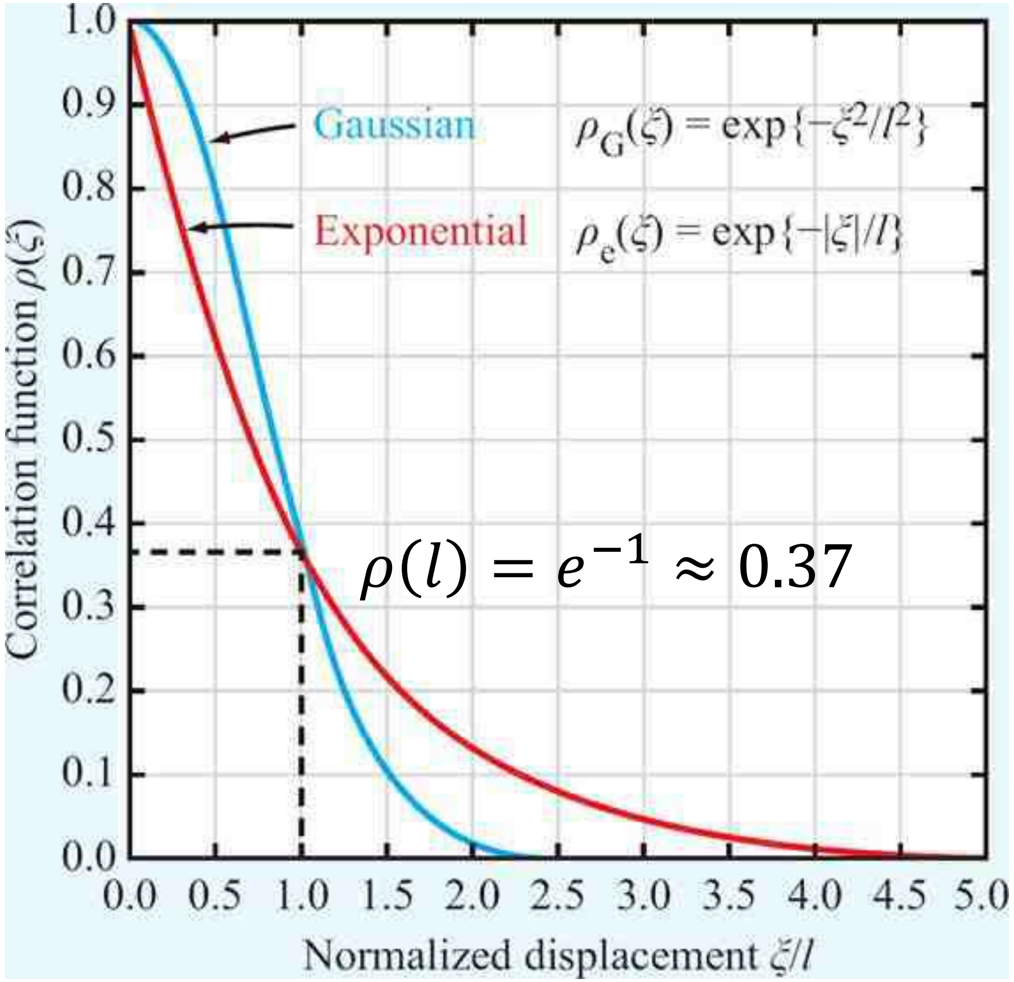
- Exponential autocorrelation function:

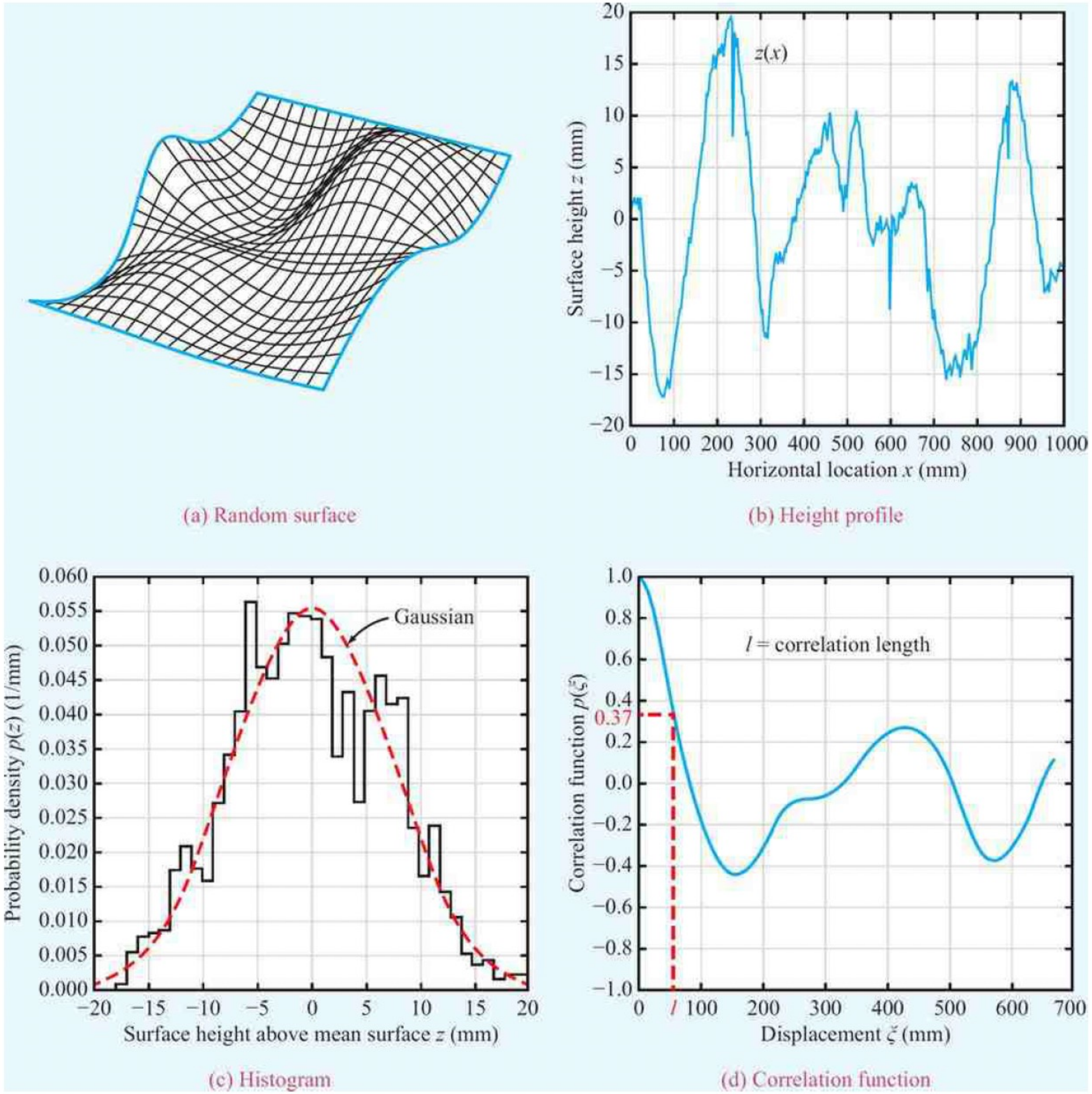
$$\rho(\xi) = e^{-|\xi|/l}$$

where l is the correlation length.

- These can be generalized to **x-exponential**:

$$\rho(\xi) = e^{-(|\xi|/l)^x}$$





(a) Random surface (b) Height profile (c) Histogram (d) Correlation function

Ulaby et al. (2014)

Rms slope

- The ensemble-averaged slope of a two-dimensional height distribution z(x) at location x is given by

$$\langle Z_x^2 \rangle = \lim_{\Delta x \to 0} \left\langle \frac{[z(x+\Delta x) - z(x)]^2}{\Delta x^2} \right\rangle$$
$$= 2h^2 \lim_{\Delta x \to 0} \left[\frac{1-\rho(\Delta x)}{\Delta x^2} \right]$$

where s is the rms height and $\rho(\Delta x)$ is the correlation function.
Expanding about Δx=0 in Taylor series, we can approximate

$$\approx -h^2 \rho''(0),$$

The rms slope is then $s_{rms} = \langle Z_x^2 \rangle^{1/2} = [-h^2 \rho''(0)]^{1/2} = \tan \theta_{rms}$.
Note: $\rho''(0) < 0$

Rms slope – discretised surface

- For a discretized surface, we can also calculate the rms variance using a sum over the N discretised elements:

$$\langle Z_x^2 \rangle = \frac{1}{N}\sum_{i=1}^{N}[z(x_i + \Delta x) - z(x_i)]^2$$

This is the square of the **Rms or Allan deviation**.

Then the **rms slope** is, which depends here on the chosen step size Δx:

$$s_{rms}(\Delta x) = \frac{\langle Z_x^2 \rangle^{\frac{1}{2}}}{\Delta x} = \tan\theta_{rms}$$

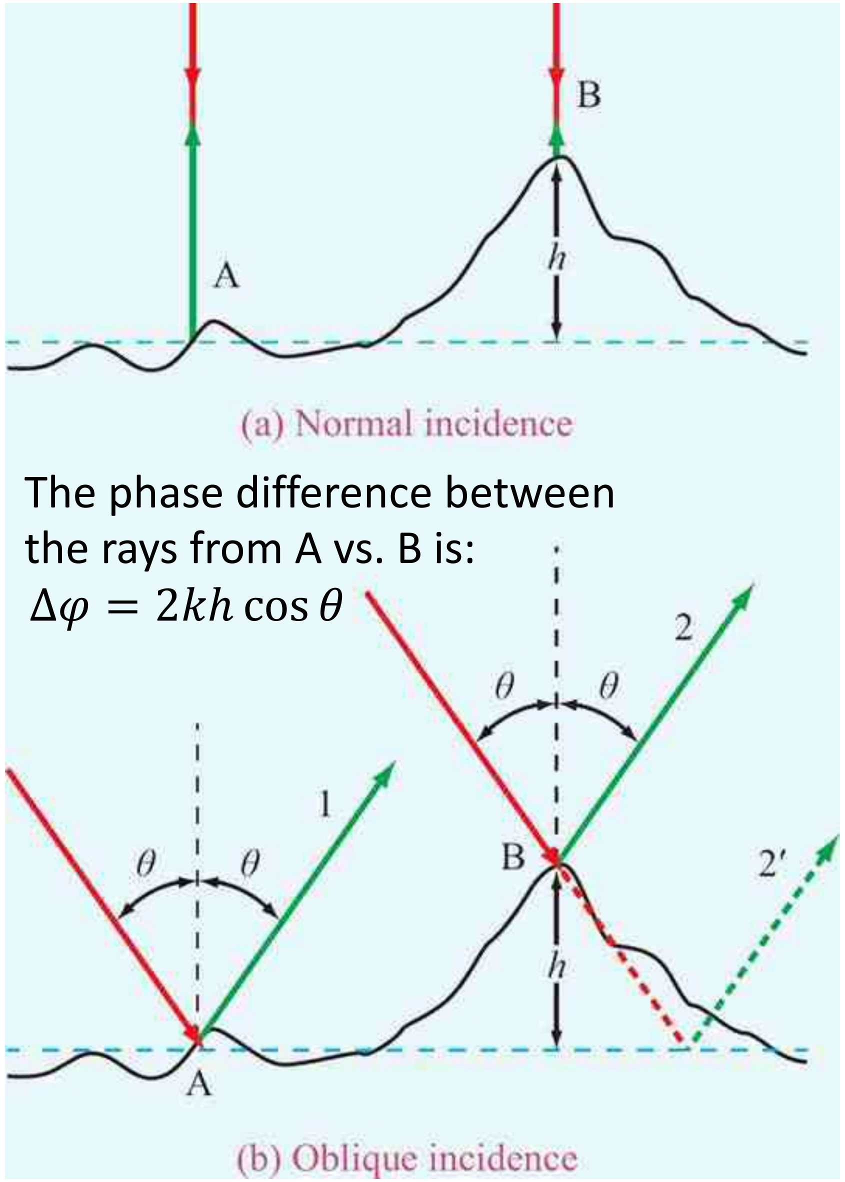
An **effective slope** is sometimes defined more simply as h/l (*e.g.*, Campbell and Garvin, 1993).

Rayleigh roughness criterion

- How to define the limit for a smooth surface?
- Similar to single particles, we can define a Rayleigh scattering limit, here based on the phase difference between the rays reflected from the opposite extremes being small enough ($< \pi/2$):

$$h < \frac{\lambda}{8\cos\theta}$$

where h is the maximum height difference. The Rms height can be used as well for a random surface.



(a) Normal incidence

(b) Oblique incidence

The phase difference between the rays from A vs. B is:
$\Delta\varphi = 2kh\cos\theta$

Roughness at different profile lengths

- In natural surfaces, the Rms height can be measured using different profile lengths (L), and the results would depend on the selected length so that

$$h(L) = h_0 L^H$$

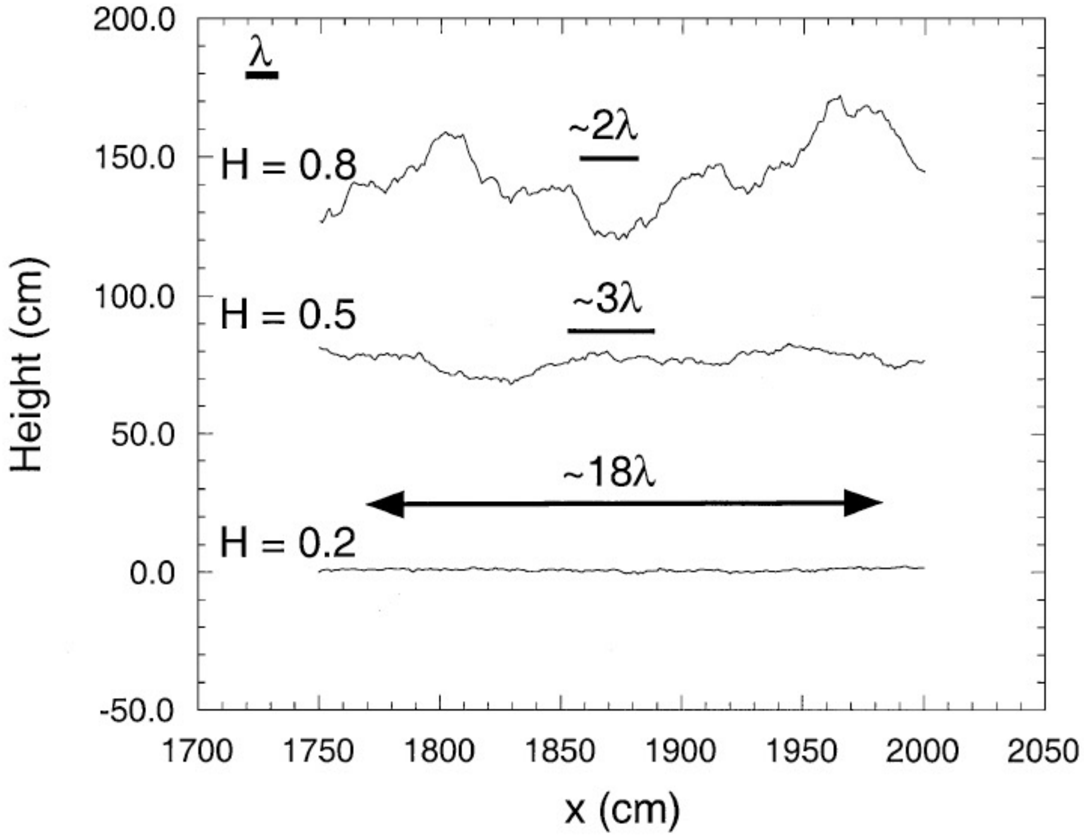
where h_0 is the Rms height at a selected scale (e.g., N wavelengths). H is **the Hurst exponent** (a.k.a. Hausdorff dimension), a value in the range [0,1].

- If the Rms height is scalable according to this equation, the surface is called **fractal** or **self-affine**.
- $H = 0$: $h(L)$ is constant; a uniform distribution
- $H = 0.5$: **Brownian** surface (the most common one in the nature)
- $H = 1$: a "**self-similar**" surface, in which case the horizontal and vertical dimensions scale at the same rate.

Roughness at different profile lengths

- The Rms slope is correspondingly scalable by $s(\Delta x) = s_0 \Delta x^{1-H}$
- The correlation length depends on the profile length, so it is not scalable nor unique for self-affine surfaces!
- Effective aperture (the annotated values in the image) describes the size of the region contributing to the coherent near-nadir echo*:

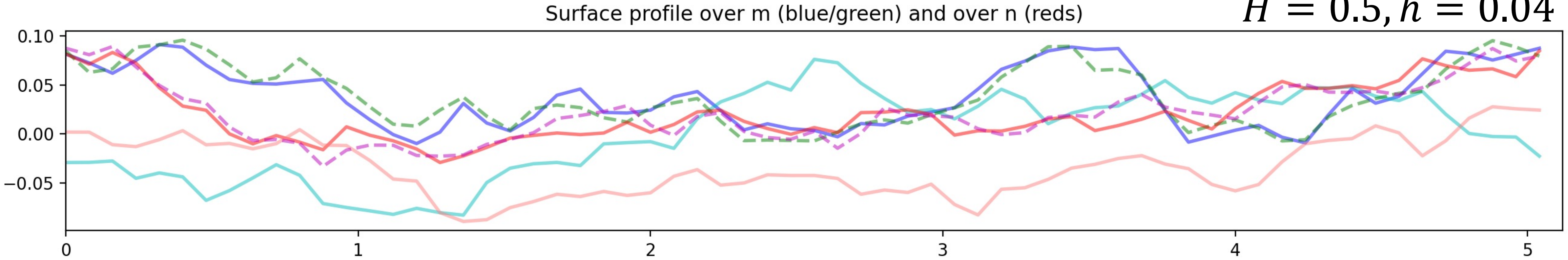
$$r_{eff} = \left[\frac{n}{4\pi^2 s_\lambda^2 \cos^2\theta}\right]^{1/(2H)} \text{(In [λ])}$$

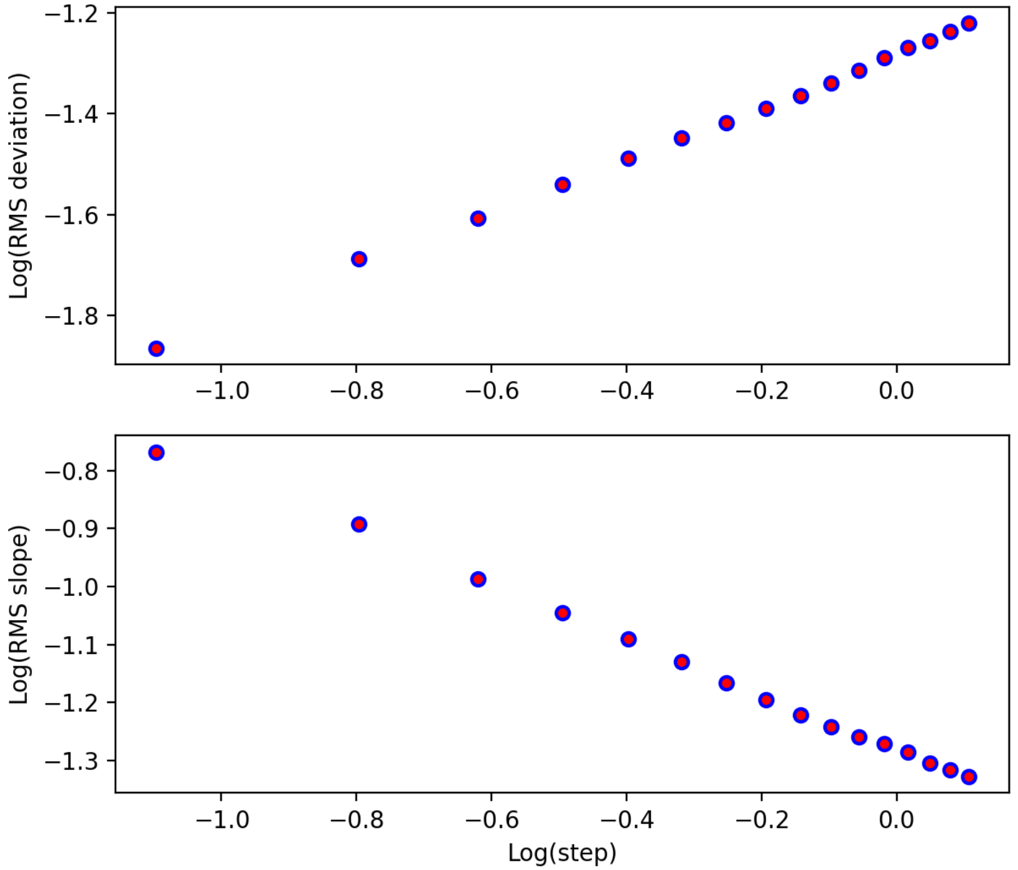


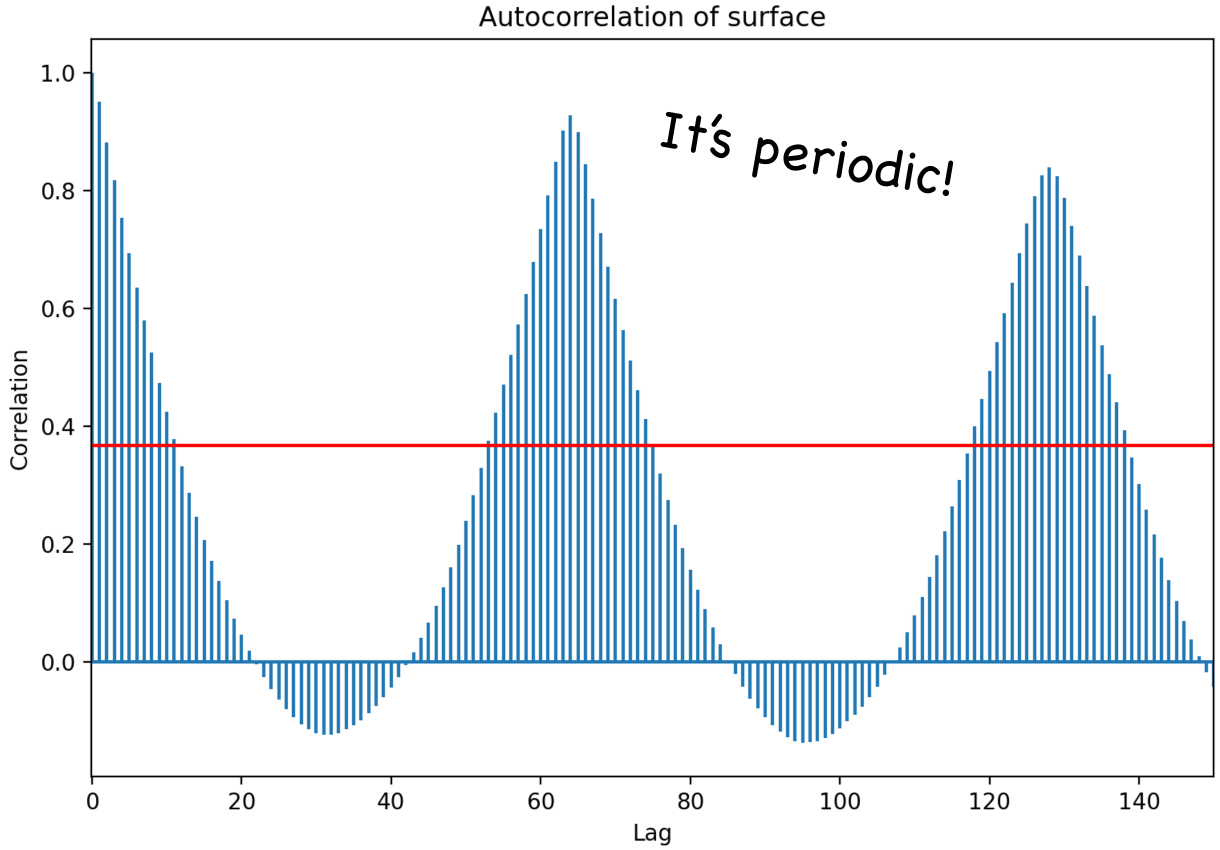
[Shepard et al. 1999, Icarus]

*$where\ the\ annular\ constructive\ interference\ becomes\ less\ than\ e^{-n}$ $(n = 1{:}\,0.37, n = 5{:}\,0.01)$

$H = 0.5, h = 0.04$


Surface profile over m (blue/green) and over n (reds)


Log(RMS deviation)
Log(RMS slope)
Log(step)

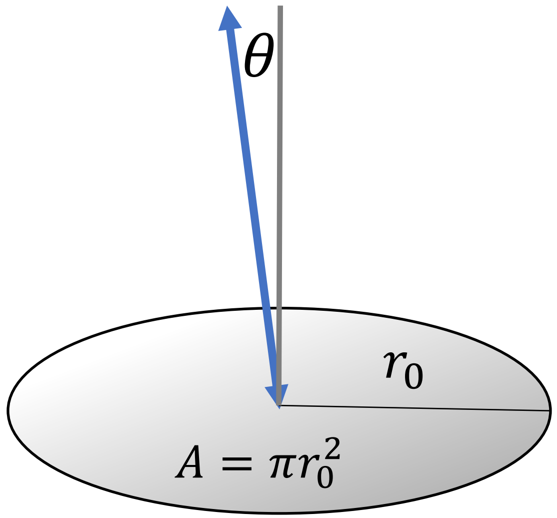

Autocorrelation of surface
Correlation
Lag
It's periodic!

Backscattering by smooth surfaces

- The amplitude of an EM wave's near-nadir backscattering from a circular, smooth, dielectric surface area can be written as

$$E = \frac{iE_0RA}{\lambda z} e^{-ikz} \frac{2J_1(kr_0 \sin\theta)}{kr_0 \sin\theta}$$

- E_0 is the incident electric field amplitude
- $R \approx R_F$ is the Fresnel reflection coefficient
- r_0 is the radius of the circular area
- θ is the incident angle
- z is the distance of the plane from the observer
- $J_1(k, r, \theta)$ is the first-order Bessel function

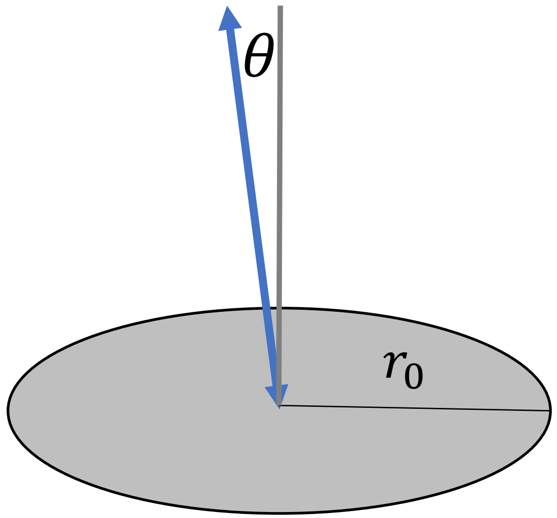


Backscattering by rough surfaces

- Roughness randomises the phases of the received set of EM waves
- The scattered electric field is modified by a phase density function Φ, so that

$$\bar{E}_{\text{rough}} = \langle \bar{E}_{\text{smooth}} \, \Phi(k, h, \theta) \rangle$$

- The phase density function is defined by the slope probability density function of the surface; the latter characterises the probability that an element is pointing to a direction that enables a Fresnel reflection from the illumination source to the observer.



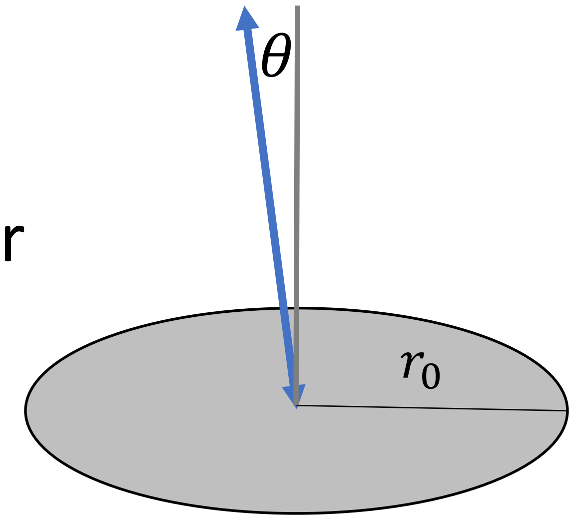
Backscattering by rough surfaces

- The power received by the detector is

$$\mathrm{P} = \frac{1}{2\eta}|\bar{E}\bar{E}^*|^2 \text{ [W/m}^2\text{]}$$

(where η is impedance of free space.

- Backscatter coefficient (σ_0) is defined as the ratio of power density scattered from that surface to the power density scattered from a perfect isotropic conducting scatterer of the same area, at the same distance, and under the same illumination and viewing conditions:



$$\sigma_0 = P/P_{\mathrm{ref}}, \text{ where } P_{\mathrm{ref}} = \frac{\lambda^2 E_0^2}{8\eta\pi z^2}$$

[Elachi 1987, Ulaby et al. 1981]

Surface scattering models

Small perturbation model, Kirchhoff model

- Serious efforts to develop mathematical models for randomly rough surfaces began in the 1950s
- The first one, developed by Rice (1951) for slightly rough surfaces whose rms heights and correlation lengths are both smaller than the incident wavelength, became known as the **small perturbation model**. It assumes that $kh < 0.3$, $kl < 3$, and $h/l < 0.3$
- Beckman and Spizzichino (1963) developed the **Kirchhoff model** to describe EM scattering by surfaces with gentle undulations whose average horizontal dimensions are large compared to the wavelength ($kl \geq 3$).
  - $kh \geq 3$ would be a **geometric optics model**

Surface scattering models
Hagfors and Gaussian models

- In 1960s-1980s, several models were published, inspired by the newly emerging radar observations to model the backscattering efficiency of the Moon and other planetary bodies
- **Hagfors' model** (Hagfors, 1964) used Gaussian height distribution and exponential autocorrelation function to model **empirically** the radar scattering of the Moon
- **Gaussian model** was published by Simpson and Tyler (1982) utilizing Gaussian autocorrelation function in addition to the Gaussian height distribution

Surface scattering models

Hagfors, Gaussian, and cosine models

- Hagfors model:

$$\sigma_0(\theta) = \frac{RC}{2}(\cos^4\theta + C\sin^2\theta)^{-3/2}$$

- Gaussian model:

$$\sigma_0(\theta) = RC\sec^4\theta\, e^{-C\tan^2\theta}$$

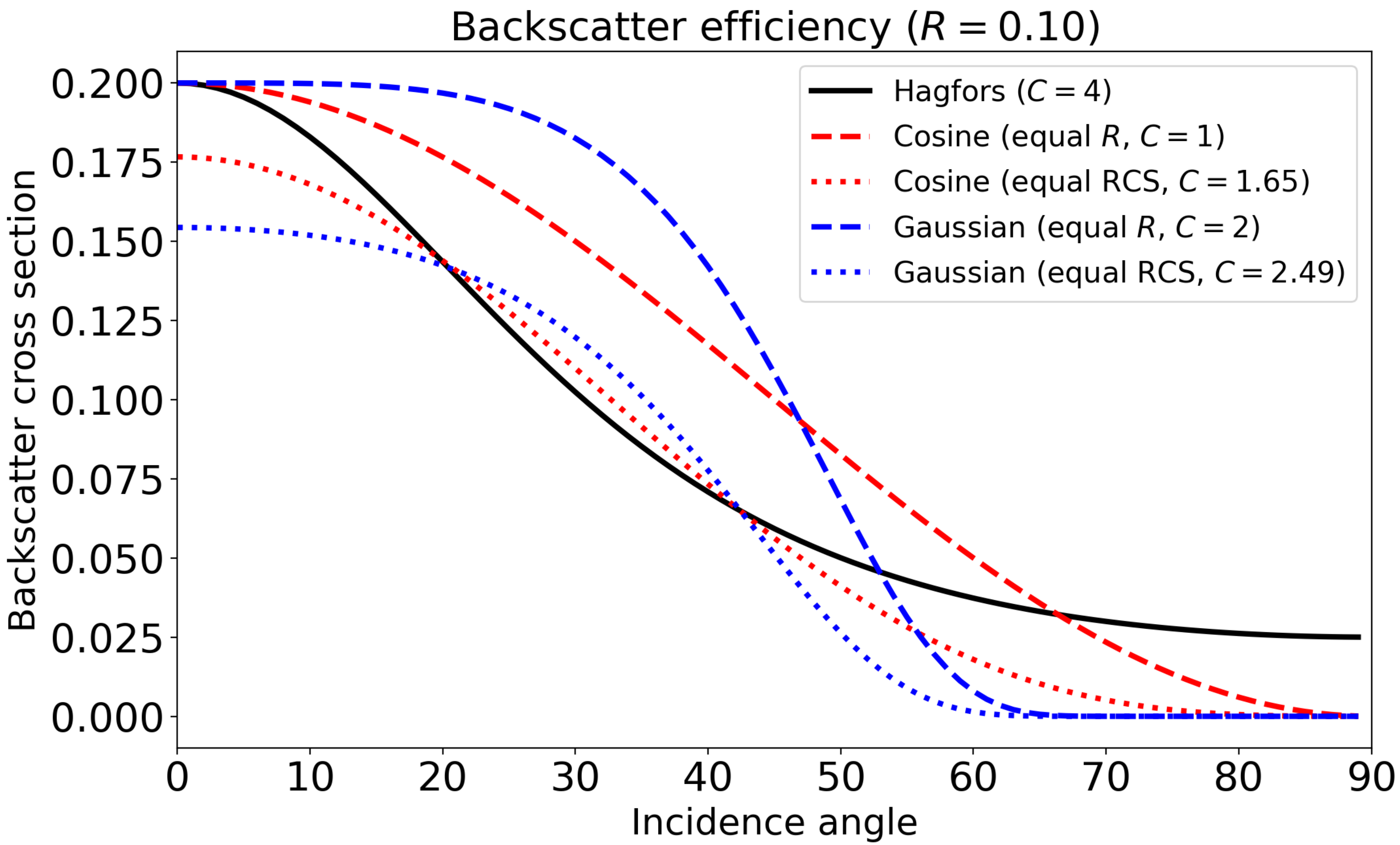
- Cosine model:

$$\sigma_0(\theta) = R(C+1)\cos^{2C}\theta$$

Hagfors defined R as the Fresnel reflection coefficient at normal incidence for an ideally smooth interface. C is the roughness factor related to the slope, the rms height, and the correlation function as

$$s_{rms} = \frac{4\pi h^2}{l\lambda} = 1/\sqrt{C}$$

The same interpretation cannot be valid for all the scattering models!



Backscatter efficiency ($R = 0.10$)
Backscatter cross section
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
0
10
20
30
40
50
60
70
80
90
Incidence angle
Hagfors ($C = 4$)
Cosine (equal R, $C = 1$)
Cosine (equal RCS, $C = 1.65$)
Gaussian (equal R, $C = 2$)
Gaussian (equal RCS, $C = 2.49$)

Surface scattering models
Shepard's Hurst-exponent model

$$\sigma_0(H) = 16\pi^3|R_F|^2 \times \left[\int_{\hat{r}=0}^{\infty} \exp\left(-4\pi^2 s_\lambda^2 \hat{r}^{2H} \cos^2\theta\right) \hat{r} J_0(4\pi\hat{r}\sin\theta) d\hat{r}\right]^2$$

[Shepard et al. 1999, Icarus]

- R_F is the Fresnel reflection coefficient
- s_λ is the rms slope in the wavelength scale
- $\hat{r} = r_{eff}$ as defined on slide 20
- $J_0(4\pi\hat{r}\sin\theta)$ is the zeroth-order Bessel function

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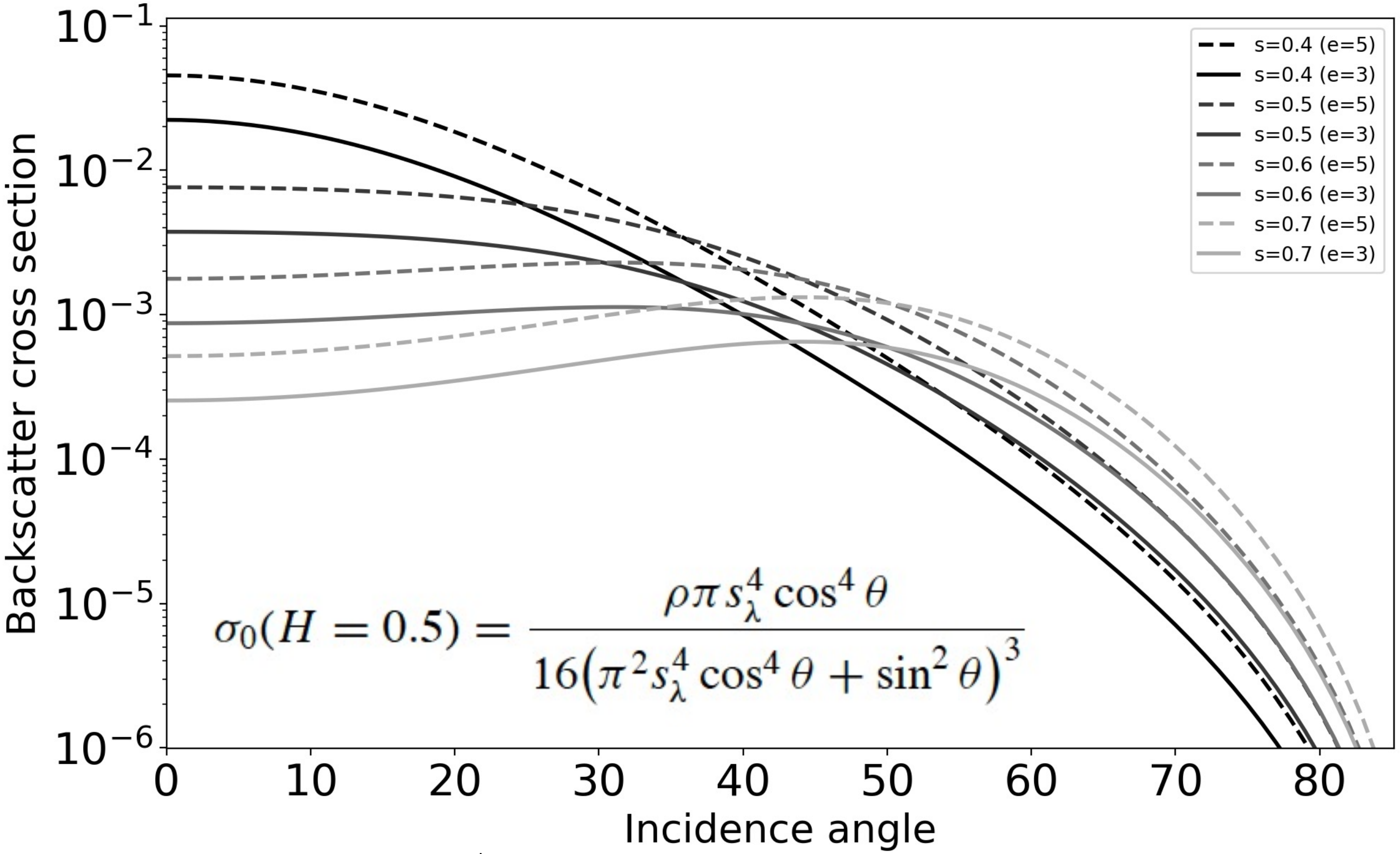
Pros:

+ Self-affine surfaces are more realistic than those using one value for h and l
+ A better physical handling for roughness

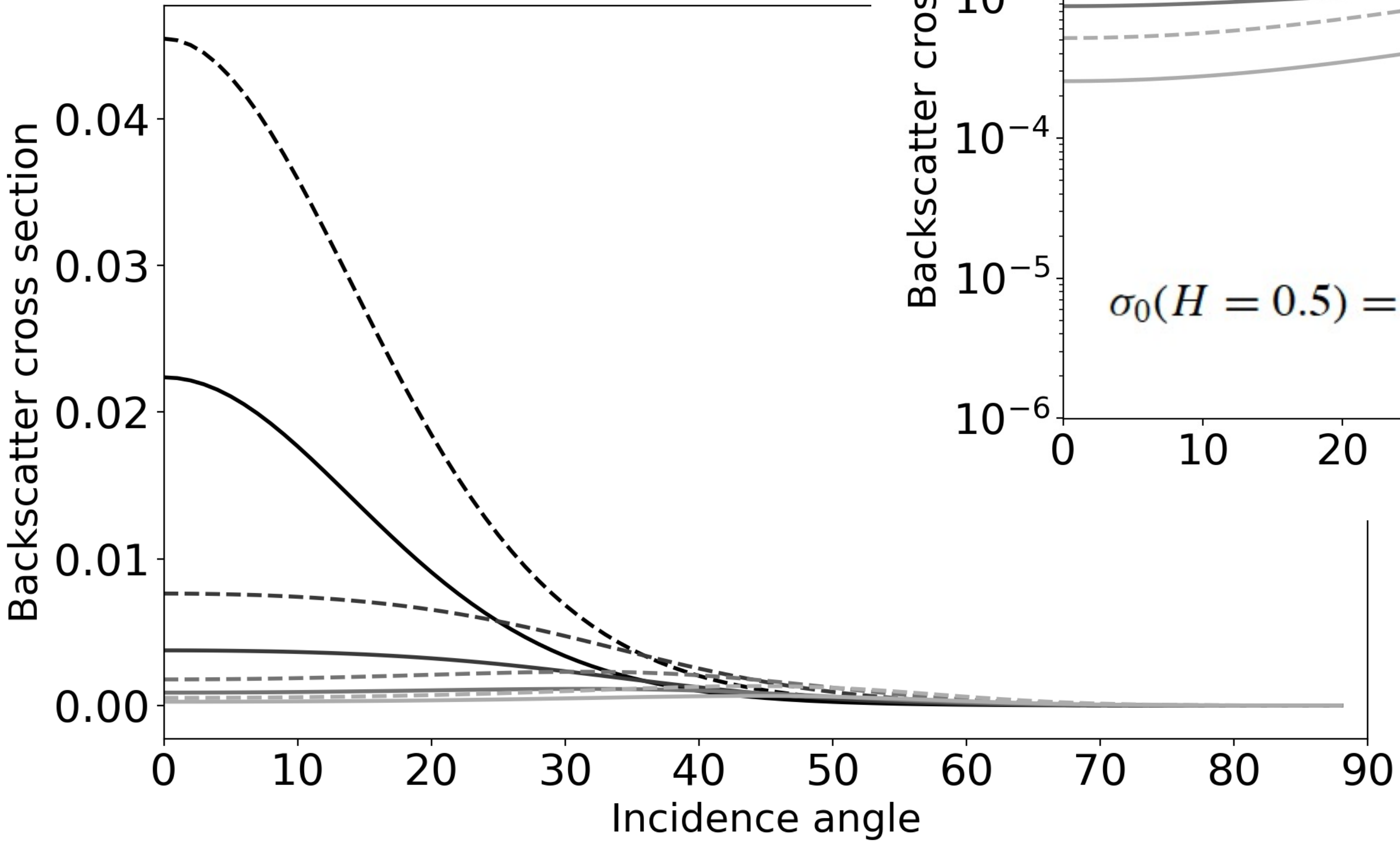
Cons:

- The Fresnel reflection component is questionable (constrained to near-nadir scattering)
- Shadowing not included

A set of examples of $\sigma_0(H = 0.5)$ in log scale (on the right) and in linear scale (below) using different s and ε



$$\sigma_0(H = 0.5) = \frac{\rho \pi s_\lambda^4 \cos^4 \theta}{16\left(\pi^2 s_\lambda^4 \cos^4 \theta + \sin^2 \theta\right)^3}$$



- Rms slope $s \in [0.4, 0.7] \approx [22°, 35°]$ (shade from dark to light)
- Electric permittivity $\varepsilon = 3$ (solid lines) or $\varepsilon = 5$ (dashed lines)

Surface scattering models
Integral equation model (IEM)

- In 1992, Adrian Fung published an improved attempt to be less constrained to small perturbations: the **integral equation model (IEM)**
- He continued this effort with his colleagues by publishing in 2002 the **Bidirectional** or **Improved integral equation model (IEM-B / I^2EM)**
    - To be presented as published in Fung & Chen (2010), *Microwave Scattering and Emission models for Users*.

Surface scattering models
Integral equation model (IEM)

The general forms of the backscattering coefficients for vertically, σ^0_{vv}, horizontally, σ^0_{hh}, and cross-polarized, σ^0_{vh}, scattering based on the integral equation method [2] are given below by (3.1) and (3.5).

$$\sigma^0_{pp} = \frac{k^2}{4\pi}\exp[-2k^2\sigma^2\cos^2\theta]\sum_{n=1}^{\infty}|I^n_{pp}|^2\frac{w^{(n)}(2k\sin\theta, 0)}{n!} \qquad (3.1)$$

where $I^n_{pp} = (2k\sigma\cos\theta)^n f_{pp}\exp[-k^2\sigma^2\cos^2\theta] + (k\sigma\cos\theta)^n F_{pp}$, $p = v, h$

Surface scattering models
Integral equation model (IEM)

k: the wavenumber
R_v, R_h: Fresnel reflection coefficients in vertical and horizontal polarizations

$$I_{pp}^n = (2k\sigma\cos\theta)^n f_{pp}\exp[-k^2\sigma^2\cos^2\theta] + (k\sigma\cos\theta)^n F_{pp}, \; p = v, h$$

where $f_{vv} = \frac{2R_v}{\cos\theta}$, $f_{hh} = \frac{-2R_h}{\cos\theta}$, $T_p = 1 + R_p$, $T_{pm} = 1 - R_p$, $sq = \sqrt{\mu_r\varepsilon_r - \sin^2\theta}$

$$F_{vv} = \left(\frac{\sin^2\theta}{\cos\theta} - \frac{sq}{\varepsilon_r}\right)T_v^2 - 2\sin^2\theta\left(\frac{1}{\cos\theta} + \frac{1}{sq}\right)T_vT_{vm} + \left(\frac{\sin^2\theta}{\cos\theta} + \frac{\varepsilon_r(1+\sin^2\theta)}{sq}\right)T_{vm}^2$$

$$F_{hh} = -\left[\left(\frac{\sin^2\theta}{\cos\theta} - \frac{sq}{\mu_r}\right)T_h^2 - 2\sin^2\theta\left(\frac{1}{\cos\theta} + \frac{1}{sq}\right)T_hT_{hm} + \left(\frac{\sin^2\theta}{\cos\theta} + \frac{\mu_r(1+\sin^2\theta)}{sq}\right)T_{hm}^2\right]$$

Surface scattering models
Integral equation model (IEM)

And the quantity $w^{(n)}$ is the surface spectrum corresponding to the two-dimensional Fourier transform of the surface correlation coefficient $\rho(x, y)$ raised to its nth power, $\rho^n(x, y)$. It is defined as follows in polar form:

$$w^{(n)}(\kappa, \varphi) = \int_0^{2\pi}\int_0^{\infty} \rho^n(r, \phi) e^{-j\kappa r \cos(\varphi - \phi)} r dr d\phi \qquad (3.2)$$

If the surface roughness is independent of the view direction, the correlation coefficient is isotropic depending only on r. In this case (3.2) reduces to

$$w^{(n)}(\kappa) = 2\pi \int_0^{\infty} \rho^n(r) J_0(\kappa r) r dr \qquad (3.3)$$

where $J_0(\kappa r)$ is the zeroth-order Bessel function.

Surface scattering models

Integral equation model (IEM)

The cross-polarisation term for multiple scattering (zero for first-order scattering):

$$\sigma^0_{vh} = \frac{S(\theta)k^4}{8\pi}\int_0^{1-}\int_0^{2\pi}\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\exp[-2k_z\sigma^2]\frac{k_z\sigma^{2m+n}}{m!n!}\right.$$

$$\left. S(\nu)\left|F_{vh}(\nu,\varphi)\right|^2 W^{(m)}W^{(n)}\right)d\varphi d\nu \qquad (3.5)$$

where $W^{(m)} = W^{(m)}[k(\nu\cos\varphi - \sin\theta), k\nu\sin\varphi]$, $S(\theta), S(\nu)$ are the shadowing functions, $W^{(n)} = W^{(n)}[k(\nu\cos\varphi + \sin\theta), k\nu\sin\varphi]$, $k_z = k\cos\theta$, $q = \sqrt{k^2 - \nu^2}$, $q_t = \sqrt{k^2\varepsilon_r - \nu^2}$,

$$F_{vh} = \frac{\nu^2\cos\varphi\sin\varphi}{\cos\theta}\left\{\left(\frac{1-R}{q} - \frac{1+R}{q_t}\right)(1-3R) - \left(\frac{1-R}{q} - \frac{1+R}{\varepsilon_r q_t}\right)(1+R)\right.$$

$$\left. + \left(\frac{1+R}{q} - \frac{1-R}{q_t}\right)(1+3R) - \left(\frac{1+R}{q} - \varepsilon_r\frac{1-R}{q_t}\right)(1-R)\right\}$$

There are various options for **shadowing functions**. For example, B. Smith (1967), *IEEE Trans. On Antennas and Propagation, 15*

Surface scattering models

Scattering matrix representation (bidirectional)

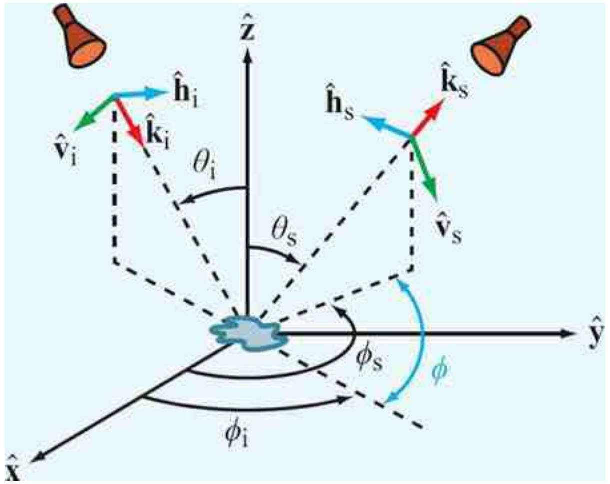
$$\overline{\overline{R}}(\theta_s,\phi_s;\pi-\theta_i,\phi_i)=\begin{bmatrix}\langle|S_{vv}|^2\rangle & \langle|S_{vh}|^2\rangle & \mathrm{Re}\langle S_{vv}S_{vh}^*\rangle & -\mathrm{Im}\langle S_{vv}S_{vh}^*\rangle \\ \langle|S_{hv}|^2\rangle & \langle|S_{hh}|^2\rangle & \mathrm{Re}\langle S_{hv}S_{hh}^*\rangle & -\mathrm{Im}\langle S_{hv}S_{hh}^*\rangle \\ 2\,\mathrm{Re}\langle S_{vv}S_{hv}^*\rangle & 2\,\mathrm{Re}\langle S_{vh}S_{hh}^*\rangle & \mathrm{Re}\langle S_{vv}S_{hh}^*+S_{vh}S_{hv}^*\rangle & \mathrm{Im}\langle S_{vh}S_{hv}^*-S_{vv}S_{hh}^*\rangle \\ 2\,\mathrm{Im}\langle S_{vv}S_{hv}^*\rangle & 2\,\mathrm{Im}\langle S_{vh}S_{hh}^*\rangle & \mathrm{Im}\langle S_{vv}S_{hh}^*+S_{vh}S_{hv}^*\rangle & \mathrm{Re}\langle S_{vv}S_{hh}^*-S_{vh}S_{hv}^*\rangle\end{bmatrix}$$

Here, $\langle S_{qp}S_{rs}^*\rangle=\dfrac{k^2}{8\pi}\exp\left[-\delta^2\left(k_z^2+k_{sz}^2\right)\right]\cdot\sum_{n=1}^{\infty}\delta^{2n}\left(I_{qp}^{n}I_{rs}^{n*}\right)\dfrac{W^{(n)}\left(k_{sx}-k_x,k_{sy}-k_y\right)}{n!}$ $\quad(\delta=h)$

where

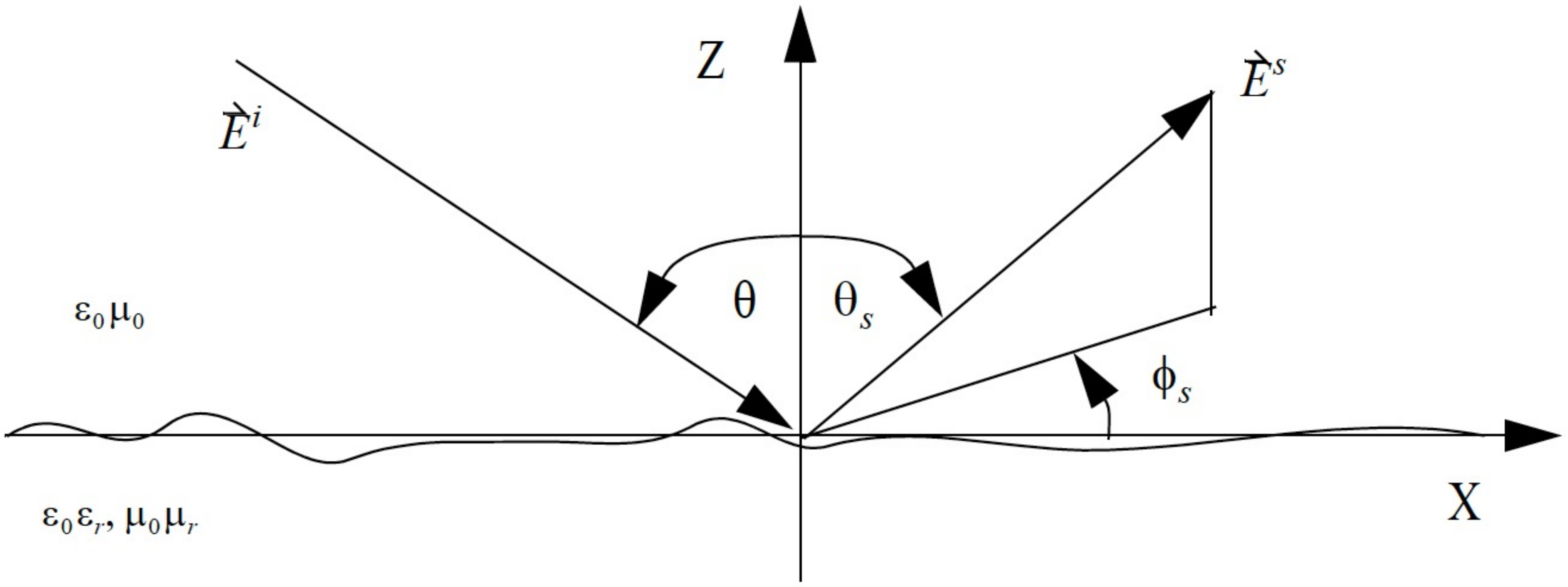
$$I_{\alpha\beta}^{n}=(k_{sz}+k_z)^n f_{\alpha\beta}\exp\left(-\delta^2k_zk_{sz}\right)+\frac{(k_{sz})^nF_{\alpha\beta}(-k_x,-k_y)+(k_z)^nF_{\alpha\beta}(-k_{sx},-k_{sy})}{2}$$

($f_{\alpha\beta},F_{\alpha\beta}$ on slide 33)



[Fung et al. 1994, Fa et al. 2011]

Surface scattering models
Integral equation model (I^2EM / IEM-B)



Fung & Chen (2010), *Microwave Scattering and Emission models for Users*, Chapter 4, p. 161-166.

Surface scattering models
Integral equation model (I^2EM / IEM-B)

- The model is available as a Python code through GitHub (part of a package)
  - https://github.com/ibaris/pyrism
- Includes the following radar models:
  - **Rayleigh**: the extinction coefficients in terms of Rayleigh scattering.
  - **Mie**: the extinction coefficients in terms of Mie scattering.
  - **Dielectric Constants**: the dielectric constants of different materials.
  - **I2EM**: RADAR soil scattering model to compute the backscattering coefficient (VV and HH polarized).
  - **Emissivity**: Calculate the emissivity for single-scale random surface (Bistatic and Monostatic)
- and a number of optical models for: Leaf reflectance, canopy reflectance, simple Lambertian soil reflectance, and volume scattering functions and interception coefficients for given solar zenith, viewing zenith, azimuth and leaf inclination angle.

Scattering model applications

- One practical application for the scattering models is shape modeling of asteroids
- Needs a simpler model than IEM or I^2EM
- Most commonly used model is the cosine model, although Hagfors and Gaussian models are also options at least in a widely used software SHAPE
- Cosine model: $R(C+1)\cos^{2C}\theta$
    - R and C are fitting parameters
    - I^2EM can be used later for help in interpretation (does not include a volume scattering component)

Surface scattering models

Fitting cosine law to I^2EM models