**Computational light scattering**, fall 2022 (PAP315, 5 cr), Exercise 5

The answers are due on **October 19, 2022**. Please return them to Anne Virkki via e-mail (anne.virkki@helsinki.fi).

1-2. Use the PyRISM software's I2EM model (github.com/ibaris/pyrism) to compute the HH and VV polarization components of the backscattering coefficients as a function of incidence angle in the following cases, when $f = 3$ GHz and $\epsilon = 5 + i0.01$:

- $l = 10$ cm, Gaussian correlation function. Compare rms heights 0.5 cm, 1.0, and 2.0 cm.
- $h = 1$ cm, Gaussian correlation function. Compare correlation lengths 3 cm, 10, and 30 cm.
- $h = 1$ cm, $l = 10$ cm. Compare Gaussian and exponential correlation functions.

Plot and evaluate the results qualitatively. Hint: You can use Figures 10-11 and 10-12 in Ulaby et al. Microwave Radar and Radiometric Remote Sensing (2014) for consistency checks. (12 points)

3-4. Consider Monte Carlo radiative transfer within a scattering and absorbing medium in the case of a Mie-type block-diagonal scattering phase matrix with nonzero matrix elements $P_{11} = P_{22}$, $P_{12} = P_{21}$, $P_{33} = P_{44}$, and $P_{34} = -P_{43}$. Show that, in the scattering process, a new scattering direction can be derived by using the marginal probability density

$$p(\theta) \quad = \quad \frac{1}{2}P_{11}(\theta) \tag{1}$$

to obtain the sample $\tilde{\theta}$ and, subsequently, by generating the azimuthal angle $\tilde{\phi}$ from (with $y \in ]0,1[$ a uniform random deviate)

$$(2\tilde{\phi} + \gamma) - e\sin(2\tilde{\phi} + \gamma) \quad = \quad 4\pi y + \gamma - e\sin\gamma, \tag{2}$$

that is, Kepler's equation $E - e\sin E = M$ with "eccentric anomaly" $E = 2\tilde{\phi} + \gamma$, "eccentricity" $e$, and "mean anomaly" $M = 4\pi y + \gamma - e\sin\gamma$. Here

$$\begin{aligned} e\cos\gamma &= -\frac{P_{12}(\tilde{\theta})}{P_{11}(\tilde{\theta})}\frac{Q_1}{I_1}, \\ e\sin\gamma &= -\frac{P_{12}(\tilde{\theta})}{P_{11}(\tilde{\theta})}\frac{U_1}{I_1}, \end{aligned} \tag{3}$$

where $\underline{I}_1 = (I_1, Q_1, U_1, V_1)^T$ denotes the Stokes vector (in an arbitrary reference system) of radiation incident on the scatterer. Literature: K. Muinonen, *Waves in Random Media* **14**, 365, 2004. (12 points)