Computational light scattering, fall 2022 (PAP315, 5 cr), Exercise 3

The answers are due on **September 28, 2022**. Please return them to Anne Virkki via e-mail (anne.virkki@helsinki.fi).

1. Fraunhofer diffraction by a spherical particle ($x = 2\pi a/\lambda$, where a is the radius and λ is the wavelength) can be approximated with

$$D(x, \theta) = x^2 \cos\theta \left[\frac{2J_1(x\sin\theta)}{x\sin\theta}\right]^2 \Theta(90^\circ - \theta) + J_0(x)^2 + J_1(x)^2,$$

where θ is the scattering angle, J_1 is a Bessel function of the first kind and of the order 1, and Θ is the Heaviside step function,

$$\Theta(s) = 1, s \geq 0$$
$$\Theta(s) = 0, s < 0.$$

Show that

$$\int_{(4\pi)} \frac{d\Omega}{4\pi} D(x, \theta) = 1. \qquad (1)$$

What is your interpretation of the term $D(x, \theta)/(4\pi)$?

The following relationships are valid for the Bessel functions:

$$J_0'(y) = -J_1(y)$$
$$J_{n-1}(y) = \frac{n}{y} J_n(y) + J_n'(y)$$

(6 p)

2-4. Derive the scattering matrix elements for the particle system in Exercise 2.1 in a fixed orientation. What is the scattering matrix for the specific configurations where the vector **d** is

(i) perpendicular to the scattering plane;
(ii) within the scattering plane but perpendicular to the wave vector of the incident field.

(18 points)